GAS DYNAMICS ALGORITHMS APPLIED TO THE ANALYSIS OF FLUID SUBSONIC AND SUPERSONIC MOVEMENT THROUGH THE LAVAL NOZZLES

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ABSTRACT: The convergent or convergent – divergent nozzle are often used in gas-dynamic technique. The study of these nozzles is complex and it requires complex mathematical models. The paper generates a Laval nozzle contour model 1 for critical diameter of 20 mm, that are known angle of convergence q_1, divergence angle q_2 and end point abscissa. Using gas dynamic functions determine ideal fluid motion parameters Laval nozzle in the convergence and divergence. The stagnation pressure is $p_0 = 6.10^6$ Pa and the stagnation temperature $T_0 = 600$ K . Supersonic air flow inside Laval nozzle is model using Fluent software.
KEYWORDS: Fluid Mechanics, Dynamics of gas, Laval nozzle.

1. INTRODUCTION

A Laval nozzle or a propelling nozzle is the component of a jet engine that operates to constrict the flow, to form an exhaust jet and to maximize the velocity of propelling gases from the engine.
Propelling nozzles can be subsonic, sonic, or supersonic [Bat67[.Physically the nozzles can be convergent, or convergent-divergent. Convergent-divergent nozzles can give supersonic jet velocity within the divergent section, whereas in a convergent nozzle the exhaust fluid cannot exceed the speed of sound within the nozzle. Propelling nozzles can be fixed geometry, or they can have variable geometry, to give different throat and exit diameters so as to deal with differences in ambient pressure, flow and engine pressure; thus permitting improvement of thrust and efficiency.
A Laval nozzle will only choke at the throat if the pressure and mass flow through the nozzle is sufficient to reach sonic speeds, otherwise no supersonic flow is achieved and it will act as a Venturi tube; this requires the entry pressure to the nozzle to be significantly above ambient at all times (equivalently, the stagnation pressure of the jet must be above ambient).

2. LAVAL NOZZLE GEOMETRY

Laval nozzle is considered represented in figure 1.

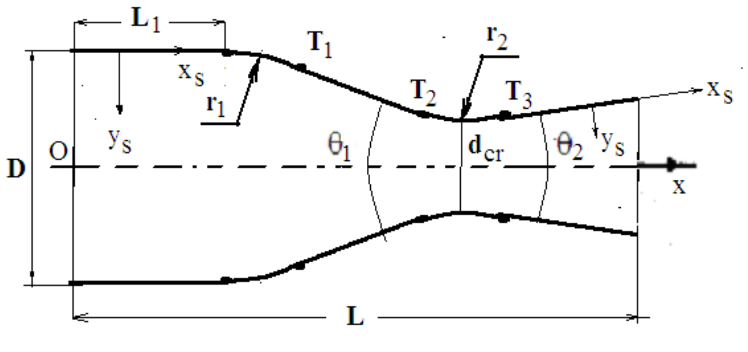

Fig.1 Geometry Laval nozzle

Computing version:

- Critical diameter $d_{cr} = 20\ [mm]$,
- Diameter nozzle entry section $D = 50\ [mm]$,
- Length of the straight portion of the nozzle $L1 = 50\ [mm]$.
- Calculated length Laval nozzle $L = 189.723\ [mm]$,

- Radius curve in the convergent $r_1 = 20\ [mm]$,
- Radius of curvature of the divergent part $r_2 = 10\ [mm]$, ,
- Angle of convergence $\theta_1 = 32\ [degrees]$,
- Angle of divergence $\theta_2 = 10\ [degrees]$,
- Stagnation pressure $p_0 = 6 \cdot 10^5 \left[\frac{N}{m^2}\right]$,
- Stagnation temperature $T_0 = 600\ [K]$.

2.1. Generating contour Laval nozzle into Mathcad. Laval nozzle model 1

Laval nozzle to generate contour model 1 (Fig. 2) is considered known coordinates $A_1(x_{11}, y_{11})$, $B_1(x_{21}, y_{21})$, $C_1(x_{31}, y_{31})$ the angle of convergence θ_1 and divergence angle θ_2, abscissa $D_1(x_{41})$. Contour connecting the points T_1, T_2, T_3. From equations connecting the circles conditions rays r_1 and r_2, determine the coordinates of points of connection $T_1(x_{T11}, y_{T11})$, $T_2(x_{T21}, y_{T21})$, $T_3(x_{T31}, y_{T31})$ and ordered point $D_1(y_{41})$.
It adopts dimensions in mm and angles in degrees:

$x_{11} = 0$, $y_{11} = \frac{D1}{2} = 25$, $x_{21} = L_{11} = 50$, $y_{21} = 50$, $x_{O11} = 50$, $y_{O11} = 5$ coordinates of the center of curvature O_{11},

$x_{31} = x_{C1} = 129.723$, $y_{31} = y_{C1} = \frac{d_{critic}}{2} = 10$, $x_{41} = 189.723$.

The results are:

$x_{T11C} = 55.513$, $y_{T11C} = 24.225$
$x_{T21C} = 126.967$, $y_{T21C} = 10.387$
$x_{T31C} = 130.595$, $y_{T31C} = 10.038$
$D_1(x_{41}, y_{41})$, $x_{41} = 189.723$, $y_{41} = 15.211$.

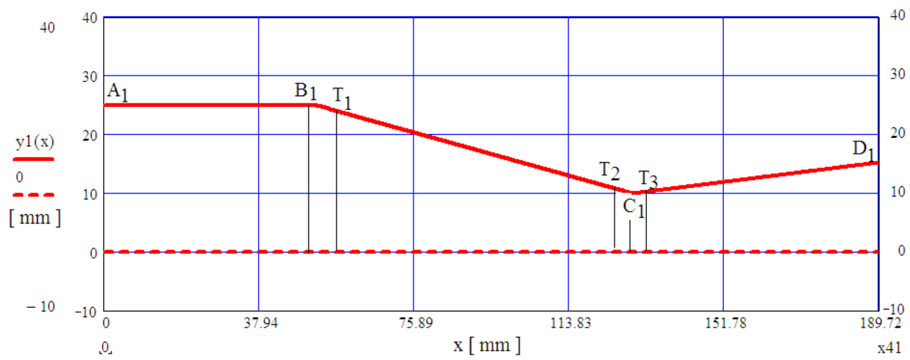

Fig. 2 Outline Laval nozzle model 1

3. GAS DYNAMIC FUNCTIONS USED IN THE STUDY OF IDEAL FLUID MOTION

The flow is considered one-dimensional, thermodynamic, and working fluid is considered compressible. Gas dynamic functions are as independent variable speed ratio $\lambda = {}^{v}/{a_c}$ or number of Ceaplâghin (or critical Mach $M_c = {}^{v}/{a_c}$).
Gas dynamic functions are also called λ functions and are divided into three groups. Get a group functions are reports of fluid movement parameters and parameters corresponding stagnation. Assuming an adiabatic evolution of the fluid we have:

$$\tau(() = T/T_{\downarrow}0 \ = 1 - (\gamma - 1)/(\gamma + 1) \cdot (^{\uparrow}2\ a \qquad (1)$$

$$\pi(() = p/p_{\downarrow}0 \ = (T/T^{\uparrow}0\ )^{\uparrow}(\gamma/(\gamma - 1)) = (1 - (\gamma - 1)/(\gamma + 1) \cdot (^{\uparrow}2\ )^{\uparrow}(\gamma/(\gamma - 1)) \qquad (2)$$

$$\varepsilon(() = \rho/\rho_{\downarrow}0 \;= (T/T^{\uparrow}0\,)^{\uparrow}(1/(\gamma-1)) = (1-(\gamma-1)/(\gamma+1)\cdot(^{\uparrow}2\,)^{\uparrow}(1/(\gamma-1)) \tag{3}$$

From equation (1) is obtained $T \to \infty \; (M \to \infty)$ for $\gamma = 1.4$ and the maximum value of the parameter λ :

$$(_{\downarrow}max = (\gamma+1)/(\gamma-1) = 2.449 \tag{4}$$

Functions of group II is used to calculate the gas mass flow and functions are called Dynamic Gas Flow. For flow through devices on a permanent stabilized regime, the continuity equation is:

$$Q_m = \rho \cdot v \cdot \sigma \tag{5}$$

where $Q_m \left[{}^{kg}/_{s}\right]$ is the mass flow of fluid, $\rho \left[{}^{kg}/_{m^3}\right]$ fluid-density, $\sigma\ [m^2]$ cross-sectional area of the channel.

Of expression $\lambda = {}^{v}/a_c$ is determined $v = \lambda \cdot a_c$, where:

$$a_c = \sqrt{\frac{2\gamma}{\gamma+1} R T_0} \tag{6}$$

and mass flow expression (5) is written:

$$Q_{\downarrow}m = \rho \cdot \sigma \cdot (\cdot\, a_{\downarrow}c = \rho \cdot \sigma \cdot (\cdot \sqrt{}(2\gamma/(\gamma+1)\; RT_{\downarrow}0\,) \tag{7}$$

By multiplying the relation (7) with a_c the result:

$$Q_{\downarrow}m \cdot a_{\downarrow}c = 2\gamma/(\gamma+1) \cdot \sigma \cdot \lambda \cdot p_{\downarrow}0\;(1-(\gamma-1)/(\gamma+1)\cdot(^{\uparrow}2\,)^{\uparrow}(1/(\gamma-1)) \tag{8}$$

Let the function $q(\lambda)$ Gas flow dynamics,

$$q(\lambda) = ((\gamma+1)/2\,\cdot)^{\uparrow}(1/(\gamma-1)) \cdot \lambda \cdot (1-(\gamma-1)/(\gamma+1)\cdot(^{\uparrow}2\,)^{\uparrow}(1/(\gamma-1)) \tag{9}$$

which is equal to the unit value $\lambda = 1$, $q(l) = 1$ and the physical meaning of dimensionless mass density of the form:

$$q(\lambda) = \frac{\rho v}{\rho_c \cdot a_c} \tag{10}$$

4. CALCULATION IDEAL FLUID MOTION IN THE CORE FLOW IN FIRST APPROXIMATION

Movement in Laval nozzle (Fig. 1) is considered with one dimension.

In the convergent part of nozzle the flow is subsonic $(= v\,/\,a_{\downarrow}c < 1$, the minimum section spare nozzle speed is achieved $(v = v_c = a_c, \lambda = 1)$, and in the divergent part the flow is supersonic $(\lambda > 1)$.

From the equation of continuity $Q_m = \rho \cdot v \cdot \sigma = \rho_c \cdot v_c \cdot \sigma_c = const.$ and relations (9), (10) follows:

$$q(\lambda) = \sigma_{\downarrow}c/\sigma = ((\gamma+1)/2\,\cdot)^{\uparrow}(1/(\gamma-1)) \cdot \lambda \cdot (1-(\gamma-1)/(\gamma+1)\cdot(^{\uparrow}2\,)^{\uparrow}(1/(\gamma-1)) \tag{11}$$

Flow rate for the core function is expressed by the relation,

$$q(\lambda) = \left(\frac{d_c - 2\delta_c^*}{d - 2\delta^*}\right)^2 \tag{12}$$

where d and d_c diameter are analyzed and the critical section, δ^* and δ_c^* are the boundary layer displacement thickness in the two sections considered.

Calculation function $q(\lambda)$ in the first approximation is performed under the assumption $\delta^{*}=0$ and $\delta_c^{*}=0$, while in the second and subsequent approximations of values δ^{*} and δ_c^{*} determine the boundary layer calculation.

Solving the equation $q(\lambda)-{}^{\sigma_c}/_{\sigma}=0$ is determined by numerical methods 1 depending on the size σ, parameter known. It shall use **root** Mathcad function.

4.1. The Laval nozzle convergent model 1

Divide the interval $[0, x_{C1}=x_{31}]$, into 15 equal parts, and notes

$k1 = 0..15$, $x(k1) = k1\cdot\frac{x_{C1}}{15}$, $y1(k1) = y1(x(k1))$

$$f(\lambda,k1) = ((\gamma+1)/2\cdot)^{\uparrow}(1/(\gamma-1))\cdot\lambda\cdot(1-(\gamma-1)/(\gamma+1)\cdot(^{\uparrow}2\,)^{\uparrow}(1/(\gamma-1))-(d1/2)^{\uparrow}2/(y1(k1))^{\uparrow}2 \quad (13)$$

For the Laval nozzle convergent adopted $\lambda = 0.6$. Applies root function:

$\lambda(k1) = soln(k1)$, $soln(k1) = root(f(\lambda,k1),\lambda)$, $T_{01} = 600\ [K]$, $\gamma = 1.4$, $R_{aer} = 287.11\left[\frac{J}{kg\cdot K}\right]$.

Speed critical $a_c = \sqrt{\frac{2}{\gamma+1}\cdot\gamma\cdot R_{aer}\cdot T_{01}}$, $a_c = 448.227\left(\frac{m}{s}\right)$.

Air velocity through the convergent nozzle $v_c(k1) = a_c\cdot\lambda(k1)$.

Mach number in the convergent nozzle $M(k1) = \dfrac{\sqrt{\frac{2}{\gamma+1}}\cdot\lambda(k1)}{\sqrt{1-\frac{\gamma-1}{\gamma+1}\cdot\lambda(k1)^2}}$.

4.2. The divergent Laval nozzle model 1

Divide the interval $[x_{C1}=x_{31}, x_{41}]$ into 15 equal parts. It notes

$k2 = 0...15$, $x(k2) = x_{C1} + k2\cdot\frac{x_{41}-x_{C1}}{15}$, $y1(k2) = y1(x(k2))$,

$$fd(\lambda,k2) = ((\gamma+1)/2\cdot)^{\uparrow}(1/(\gamma-1))\cdot\lambda\cdot(1-(\gamma-1)/(\gamma+1)\cdot(^{\uparrow}2\,)^{\uparrow}(1/(\gamma-1))-(d1/2)^{\uparrow}2/(y1(k2))^{\uparrow}2 \quad (14)$$

For the divergent Laval nozzle is adopted $\lambda = 1.2$.
Applies Mathcad root function:

root : $\lambda(k2) = soln(k2)$,
$soln(k2) = root(f(\lambda,k2),\lambda)$. (15)

Air velocity through the nozzle $v_{1d}(k2) = a_c\cdot\lambda(k2)$. Divergence Mach number at the nozzle

$$M(k2) = \frac{\sqrt{\frac{2}{\gamma+1}}\cdot\lambda(k2)}{\sqrt{1-\frac{\gamma-1}{\gamma+1}\cdot\lambda(k2)^2}}. \quad (16)$$

Resulting variation in nozzle velocity in Figure 3.

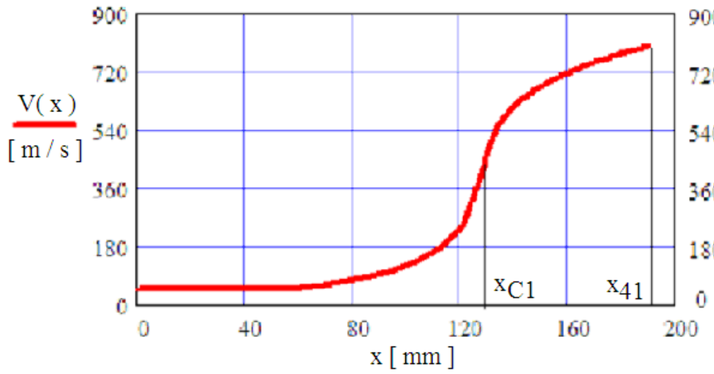

Fig. 3 Variation of fluid velocity in Laval nozzle model 1

4.3. Variation of condition parameters Laval nozzle model 1

Determine the temperature, pressure and density of the fluid in the nozzle according to the abscissa x, through variable $\lambda(x)$ or $M(x)$:

$$T/T\downarrow 0 = 1-((\gamma-1)/(\gamma+1)\cdot(\uparrow 2)/(1-(\gamma-1)/(\gamma+1)\cdot(\uparrow 2) = 1/(1-(\gamma-1)/(\gamma+1)\cdot(\uparrow 2) = 1+(\gamma-1)/2\cdot M\uparrow 2 \quad (17)$$

resulting

$$\frac{T}{T_0} = 1 - \frac{\gamma-1}{\gamma+1}\cdot\lambda^2 \quad (18)$$

and provided izentropic development

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma-1}},\ \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{\frac{1}{\gamma-1}}. \quad (19)$$

For $T_{01} = 600\,[K]$, $p_{01} = 6\cdot 10^5\,[Pa]$ the numerical values obtained are plotted in Fig. 4, 5 and 6. There are represented variations in temperature, pressure and density of the working fluid in Laval nozzle depending on axis positions.

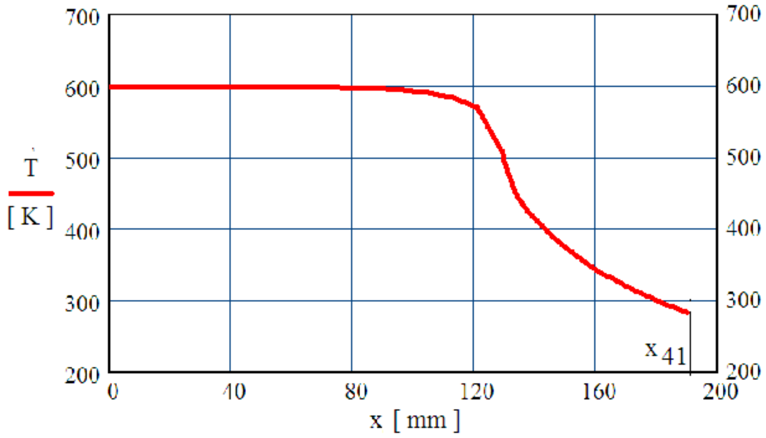

Fig. 4 Variation of temperature along the Laval nozzle axis

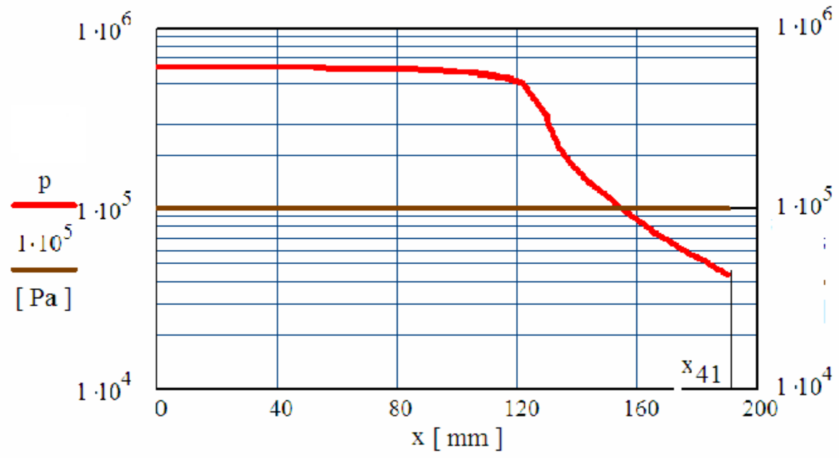

Fig. 5 Variation of pressure along the Laval nozzle axis

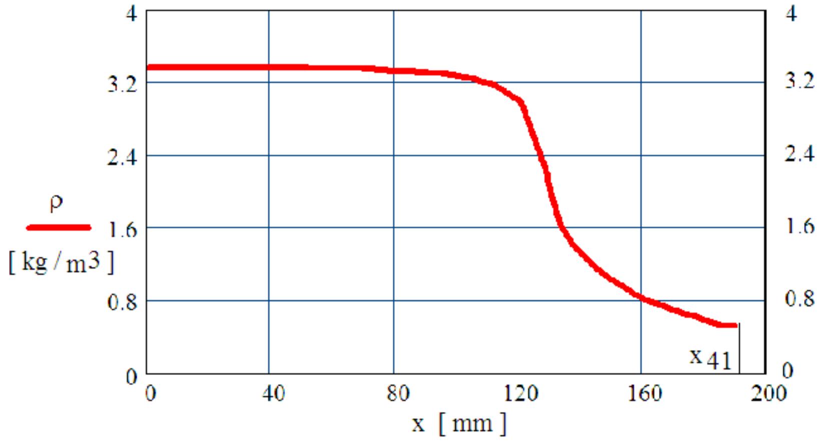

Fig. 6 Variation of density along the Laval nozzle axis

5. NUMERICAL DETERMINATIONS WITH FLUENT SOFTWARE

The figure 7 presents the fluid domain. For numerical determination is consider three nozzles diameters and for each nozzle an mass flow rate, the stagnation temperature and pressure:

- nozzle diameter 20 mm – mass flow rate 0,311 kg/s, $p_0 = 6 \cdot 10^5 \left[\frac{N}{m^2}\right]$ and $T_0 = 600\ [K]$;
- nozzle diameter 30 mm – mass flow rate 0,864 kg/s, $p_0 = 8 \cdot 10^5 \left[\frac{N}{m^2}\right]$, and $T_0 = 700\ [K]$;
- nozzle diameter 40 mm – mass flow rate 1,796 kg/s, $p_0 = 10 \cdot 10^5 \left[\frac{N}{m^2}\right]$, and $T_0 = 800\ [K]$.

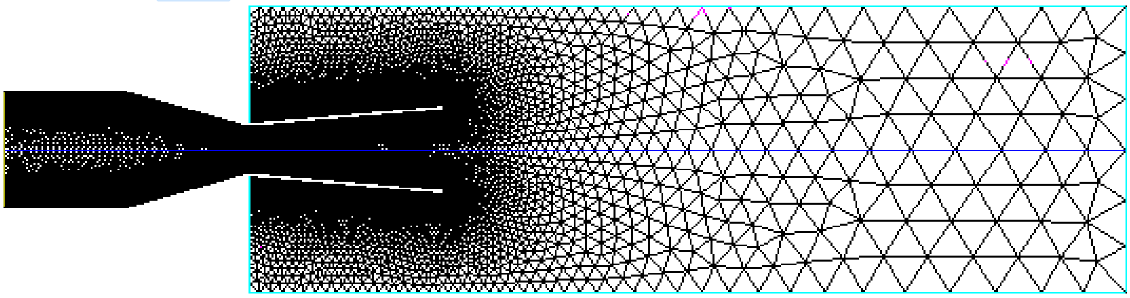

Fig. 7 The 2D domanin geometry and cell mesh

The flow is considered 2D with rotational symmetry. The working fluid is air; the considered turbulence model is Spalart-Allmaras. In figures 8 – 11 is presented the Mach number, density, static pressure and temperature fields.

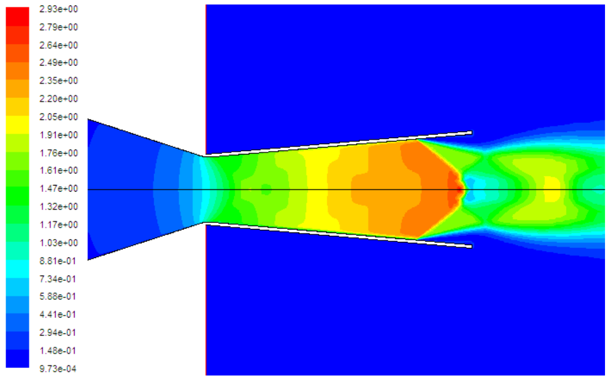

Fig. 8 Mach number

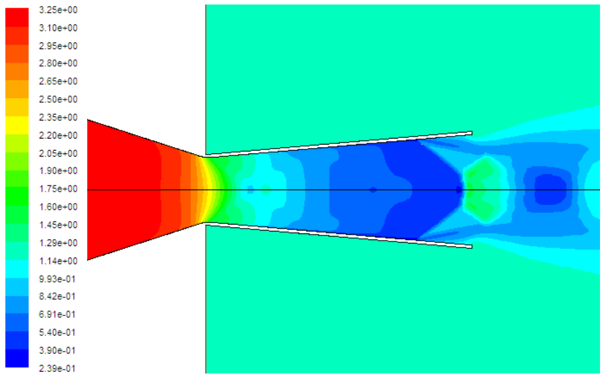

Fig. 9 Density (kg/m^3)

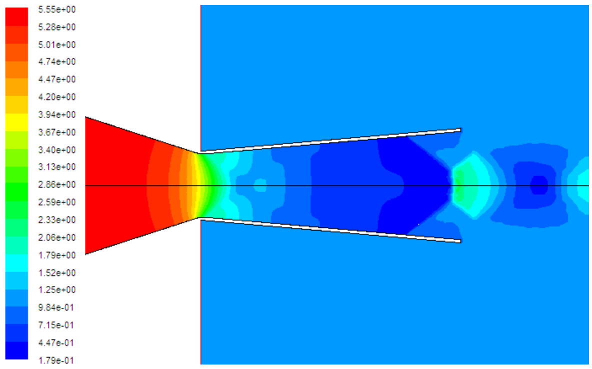

Fig. 10 Static pressure (atm)

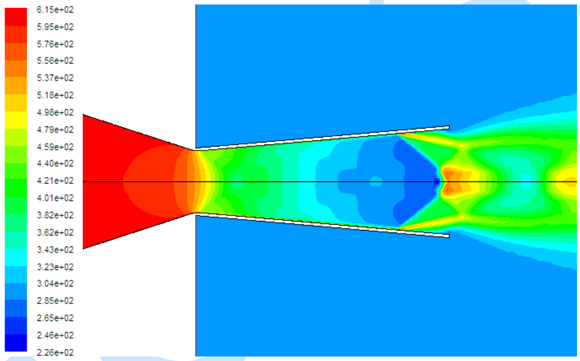

Fig. 11 Temperature (K)

In the figure 12 is presented the Mach number variations along the nozzle axis for the three cases considered.

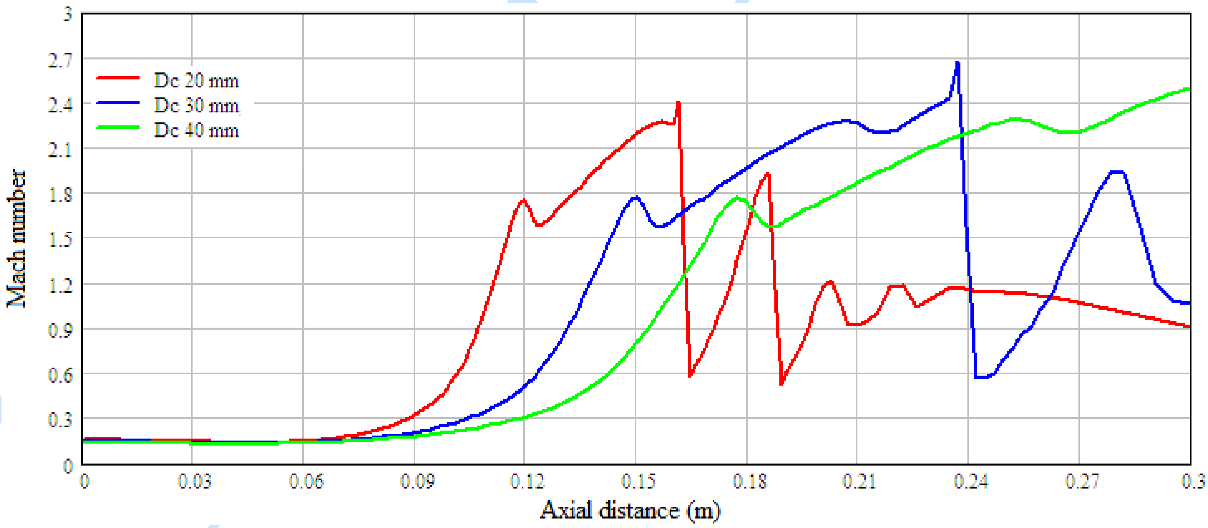

Fig. 12 Mach number along the nozzles axis

CONCLUSIONS

If consideration of boundary layer flow calculation kernel parameters is performed by using the tool gas dynamic $q(\lambda)$ defined in (11). Voltage wall friction is calculated with

$$\tau_p = \frac{1}{2} \cdot c_{fr} \cdot \rho \cdot v^2 \qquad (20)$$

where the coefficient of friction c_{fr} is determined from the boundary layer calculation.
Air viscosity depending on pressure and temperature is determined by the relation,

$$v(p,T) = (1.314 \cdot 10^{-5} + 8.785 \cdot 10^{-13} \cdot p)\left(\frac{T}{\Theta_0}\right)^{2.5} \frac{\Theta_0 + 110}{T + 110},$$
$$\Theta_0 = 273.15.$$

Mass flow through the nozzle boundary layer is calculated considering the relationship

$$Q_m = \rho_c \cdot a_c \cdot \sigma_c = \sqrt{\frac{\gamma}{R} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \cdot \frac{p_0 \cdot q(\lambda)}{\sqrt{T_0}} \cdot \sigma \qquad (21)$$

where

$$\sigma = \frac{\pi \cdot (d - 2 \cdot \delta^*)^2}{4}$$

is the cross-sectional area of the core potential shift is thick.
If turbulent motion equations of motion, continuity and momentum kept the same as in the laminar boundary layer when using average velocity components instead of instantaneous velocities.
To calculate dynamic turbulent boundary layer thickness using a standard solution Pohlhausen

$$\frac{(v_x)_y}{(v_x)_\delta} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}.$$

Displacement thickness (thickness loss of mass) is calculated with

$$\delta^* = \int_0^\delta \left(1 - \frac{(\rho_x \cdot v_x)_y}{(\rho_x \cdot v_x)_\delta}\right) dy \cong \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right) dy = \frac{\delta}{8}.$$

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