AN ALGORITHM FOR CONSTRUCTING SYMMETRIC ((r+1)v, kr, kλ) BIBDs FROM AFFINE RESOLVABLE (v, b, r, k, λ) BIBDs

Kazeem A. Osuolale [1], Oluwaseun A. Otekunrin [2]

[1] Department of Statistics, Faculty of Science, University of Ilorin, Nigeria
[2] Department of Statistics, Faculty of Science, University of Ibadan, Nigeria

ABSTRACT: This work deals with the construction of symmetric ((r+1)v, kr, kλ) BIBDs from affine resolvable (v, b, r, k, λ) BIBDs. A MATLAB program was written to construct resolution and parallel classes from ((4, 6, 3, 2, 1) and (9, 12, 4, 3, 1) affine resolvable designs. A technique was thereafter used, via the MATLAB program to obtain their corresponding incidence matrices which gave rise to symmetric ((r+1)v, kr, kλ) BIBDs.
The two SBIBDs ((16, 6, 2) and (45, 12, 3) constructed obeyed the mathematical property of a symmetric matrix and this makes the technique employed in this research unique since the terminology SBIBD does not imply true mathematical symmetry.
KEYWORDS: Affine Resolvable BIBDs, MATLAB Program, Incidence Matrices, Symmetric BIBDs and Symmetric Matrix.

1. INTRODUCTION

Balanced incomplete block designs (BIBDs) are one of the most studied structures in combinatorial design theory. A BIBD is defined as an arrangement of v distinct objects into b blocks such that each block contains exactly k distinct objects, each object occurs in exactly r different blocks, and every two distinct objects occur together in exactly λ blocks ([GPP06]). The variables v, b, r, k, λ are referred to as the parameters of the BIBDs. BIBDs are variance balanced like the randomized complete block designs. Also, among the binary block designs with block size k less than number of treatments v, they are the only designs which are variance balanced, [Rao58].
Balanced incomplete block designs were introduced in the statistics literature by Yates ([Yat36]). However, combinatorial structures which we recognize as BIBDs were known even in the 19th century. For instance, Kirkman ([Kir50]) solved the Kirkman's school girl problem, originally proposed by Woolhouse ([Woo44]).
There are necessary conditions for the existence of a (v, b, r, k, λ)- BIBDs.

Theorem 1. In a (v, b, r, k, λ) - design,
i) $r(k-1) = \lambda(v-1)$
ii) $bk = vr$ ([Sti03])

Theorem 2. (Fisher's Inequality):
Fisher's Inequality, $b \geq v$ is another necessary condition for the existence of a (v, b, r, k, λ)- BIBDs ([Fis40]).

1.1 Incidence Matrices

It is always convenient to represent a BIBD by means of an incidence matrix. This is especially useful for computer programs.

Definition 1. Let (X, A) be a design, Stinson ([Sti03]) where $X = \{x_1 \dots x_v\}$ and $A = \{A_1 \dots A_b\}$.The incidence matrix of (X, A) is the v×b 0−1 matrix M = $(m_{i,j})$ defined by the rule

$$m_{i,j} = \begin{cases} 1 \text{ if } x_i \in A_j \\ 0 \text{ if } x_i \notin A_j. \end{cases}$$

The incidence matrix, M of a (v, b, r, k and λ)-BIBD satisfies the following properties:

1. every column of M contains exactly k "1"s;
2. every row of M contains exactly r "1"s;
3. two distinct rows of M both contain"1"s in exactly λ columns.

Definition 2. A block design with at least one zero entry in its incidence matrix is called an incomplete block design Dey ([Alo10]).

2. SYMMETRIC BALANCED INCOMPLETE BLOCK DESIGN (SBIBD)

Symmetric balanced incomplete block design (SBIBD) is a type of BIBDS where b = v and r=k. In a symmetric (v, k, λ) design every two distinct blocks have λ points in common. Besides, a necessary condition for the existence of an SBIBD is that r−λ be a perfect square when v is even. The (7,3,1), (11,5,2), (16,6,2) and (19,9,4) designs are good examples of SBIBDs ([Sti03]). Certain designs such as derived and residual designs can be obtained from SBIBDs. The derived design is obtained by omitting a block and retaining only the treatments of the omitted block in the remaining blocks, while in the residual design we omit a block and in the remaining blocks retain only these treatments which do not occur in the omitted block. The non-existence of certain SBIBDs was demonstrated by Chowla and Ryser ([CR50]). Any incidence matrix that equals its transpose is symmetric.

2.1 Applications of SBIBD

They are the single most important and well studied subclass of BIBDs. Projective planes, biplanes and Hadamard 2-designs are all SBIBDs. They are of particular interest since they are the extremal examples of Fisher's inequality ($b \geq v$).For many industrial applications, the useful designs belong to the symmetrical type. O. Lee et al ([L+06]) employed Symmetric Balanced Incomplete Block Design (SBIBD) in the design of efficient load balancing algorithm on distributed networks. They generated a special incidence structure using the SBIBD and then propose a new load balancing algorithm which executes well for an arbitrary number of nodes.

Noshad and Brandt-Pearce ([NB12]) also used Symmetric Balanced Incomplete Block Designs (SBIBDs) to obtain symbols for a pulse position modulation (PPM) scheme, called expurgated PPM (EPPM), for application in peak power limited, communication systems, such as impulse radio (IR) ultra wide band (UWB) systems and free space optical (FSO) communications and generally SBIBDs have interesting applications in information theory.

2.2 Construction methods for symmetric balanced incomplete block designs

Several construction methods for BIBDs exist in literature. These include the method of cyclical development of initial blocks ([Bos39]), difference methods techniques e.g. cyclic development of difference sets, the method of symmetrically repeated differences ([HK05]), use of finite permutation groups ([Hal69]) and the trade-off method ([HL79]).

2.3 Resolvable BIBDs

An important concept in design theory is resolvability, which arises frequently in many applications such as tournament scheduling. Resolvability was the basis of the schoolgirls' problem proposed by Kirkman ([Kir50]). Analysing a resolvable design as a randomized complete block experiment gives an unbiased estimate of the error for treatment comparisons. An incomplete block design that is not resolvable may be less efficient than randomized complete blocks. A resolvable design must always be as efficient as randomized complete blocks. The advantage of resolvable designs also concerns management. If a design is resolvable, one or more complete replicates can be dealt with at each session and possible differences caused by different times of sowing or harvesting are balanced for all varieties. The United Kingdom has for some time required the use of resolvable designs in agricultural field trials; see [PS80]. Some examples of resolvable BIBDs are (4,2,1), (6,2,1), (15,3,1) and (16,4,1) designs respectively.

Definition 3. Stinson ([Sti03]): Suppose (X, A) is a (v, b, r, k, λ)-BIBD. A parallel class in (X, A) is a subset of disjoint blocks from A whose union is X. A partition of A into r parallel classes is called a resolution and (X, A) is said to be a **resolvable BIBD** if A has at least one resolution. Note that a parallel class contains v/k blocks, and therefore a BIBD can have a parallel class only if $v \equiv 0 \bmod k$.

A **resolution class** in a BIBD is a set of blocks which together contain each variety of the design precisely once.

A resolvable BIBD is one whose blocks can be partitioned into mutually disjoint resolution classes; this partition can in some cases be carried out in several different ways, ([SS1987]).

Definition 4. A resolvable design is said to be **affine resolvable** if any two blocks from different resolution classes intersect in q_2 varieties ([SS87]).

Theorem 3. (Bose's Inequality)

If there exists a resolvable (v, b, r, k, λ) BIBD, then $b \geq v+r-1$ ([Bos42]). Bose's Inequality strengthens Fisher's Inequality whenever the BIBD is resolvable.

Lemma 1: For a BIBD, $b \geq v + r - 1$ iff $r \geq k + \lambda$ ([Bos42])

Proof: Murty ([Mur61]): If $b \geq v + r - 1$ then $b > v$ and so $r > k$. Then $\frac{vr}{k} \geq v + r - 1$

Definitions 5. ([SS87]): A design is said to be **α-resolvable** if the set of blocks can be partitioned into classes such that each variety appears α times in each class. In addition to this, if any two blocks in the same class intersect in q_1 varieties and any two blocks in different classes intersect in q_2 varieties, then the design is called **affine α-resolvable.** The major concern of this work is the construction of symmetric balanced incomplete block design from affine resolvable designs.

2.4 Construction of symmetric ((r+1)v, kr, kλ) BIBDS from affine resolvable BIBDS

Having dealt with the relevant discussions on the BIBDs, SBIBDs and resolvable BIBD including affine resolvable BIBD, the major concern of this work is to construct symmetric [(r+1) v, kr, kλ] BIBDs from affine resolvable BIBDs.

Theorem 4. ([Sti03])

If there exists an affine resolvable (v, b, r, k, λ) BIBD then there exists a symmetric ((r+1) v, kr, kλ)-BIBD.

3. MATERIALS AND METHODS

An SBIBD algorithm is written in MATLAB based on five cell executions to generate incidence matrices for the Symmetric Balanced Incomplete Block Designs (SBIBDs). There are basically five steps taken to achieve this construction.

The first step creates a set of variables for the SBIBD algorithm which constructs a symmetric matrix and determines the size of the row vector to be created based on ((r+1)v, kr, kλ). The parameters of BIBDs are defined at this stage as $v = k^2$, b = k (k+1) and r = k+1.

The second step is for creating resolution classes using MATLAB's in-built function. Resolution classes are generated from the set of (k+1) mutually orthogonalised Latin squares. Each resolution class is formed in such a way that each point appears once in a row and once in a column. This is repeated for subsequent resolution classes in random permutation process, creating a unique set of numbers from 1 to the set maximum value (v value).

In the third step, we create parallel classes from the resolution classes generated in the second step such that the rows of the matrix in each cell are re-arranged to a combined row vector of dimension 1 x v. There are as many parallel classes as there are resolution classes.

The fourth step generates the incidence matrix of dimension v x v for each parallel class. The first matrix is a zero matrix of dimension v x v which will be used in generating an incidence matrix for the SBIBDs.

In the last step we combine each of the incidence matrix into a single matrix based on the matrix arrangement as stated in Theorem 6. This matrix is the incidence matrix of a symmetric BIBD denoted by M and is of order (r+1) v x (r+1) v. The affine resolvable BIBD (v, b, r, k, λ) therefore produces symmetric [(r+1) v, kr, kλ]-BIBDs. This implies that (4, 6, 3, 2, 1)-BIBD produces a symmetric (16, 6, 2)-BIBD, (9, 12, 4, 3, 1)-BIBD produces symmetric (45, 12, 3)-BIBD and so on and so forth.

The steps taken in the construction of symmetric balanced incomplete block designs can be illustrated with the aid of the program flowchart as displayed adjacent.

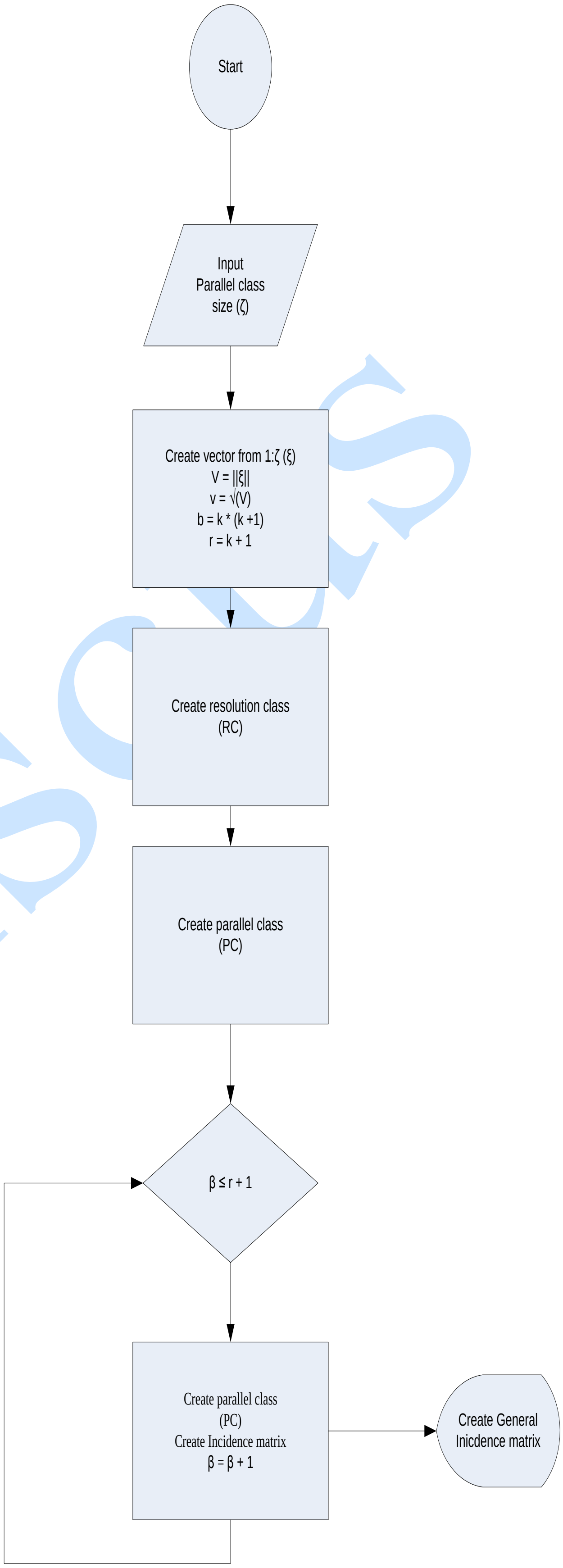

Figure 1: Program flowchart for the construction of symmetric ((r+1)v, kr, kλ)-BIBD

3.1 MATLAB Code for the Construction of Symmetric [(r+1)v, kr, kλ] BIBDs

Cell 1

```
%% (1) Create variables
clear
clc
% Setup random stream
s=RandStream('mt19937ar');
RandStream.setDefaultStream(s)
reset(s) %reset seed
v_value = 16; %u can change this value to your needs
V = 1:v_value;
v = length(V);
k = sqrt(length(V));
b = k*(k+1);
r = k+1;
```

Cell 2

```
%% (2) Create resolution class
for u = 1:r
   RC{u} = reshape(randperm(s,v),k,k);
end
```

Cell 3

```
%% (3) Create Parallel class
for u = 1:r
   PC{u} = reshape((RC{u})',1,v);
End
```

Cell 4

```
%% (4) Create incidence matrix
for ii = 1:r+1
   if ii == 1
      M{ii} = zeros(v,v);
   else
      for jj = 1:v
         a = RC{ii-1};
         ind = find(any(jj == a,2));
         data = zeros(k,k);
         data(a(ind,:)) = 1;
         M{ii}(jj,:) = reshape(data,1,v);
      end
   end
end
```

Cell 5

```
%% (5) General Incidence matrix
ii = 1:length(M);
n = 1;
for vv = 1:length(M)
   f = ii(n:end);
   if n == 1
      b = [];
   else
      b = 1:n-1;
   end
   len = [f b];
   MM{vv,:} = cell2mat(M(len));
   n = n + 1;
end
MM = cell2mat(MM);
```

4. RESULTS AND DISCUSSION

4.1 Results

4.1.1 Construction of symmetric (16, 6, 2) - BIBD from an affine resolvable (4, 6, 3, 2, 1) – BIBD

```
RC{1}          >> RC{2}         >> RC{3}

ans =          ans =            ans =
   4  3           4  2             2  1
   2  1           3  1             3  4

                                 >> PC{3}
>> PC{1}       >> PC{2}
                                 ans =
ans =          ans =
  4  3  2  1      4  2  3  1        2  1  3  4
```

```
>> M{1}

ans =
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
```

```
>> M{2}

ans =
     1     1     0     0
     1     1     0     0
     0     0     1     1
     0     0     1     1
```

```
>> M{3}

ans =
     1     0     1     0
     0     1     0     1
     1     0     1     0
     0     1     0     1
```

```
>> M{4}

ans =
     1     1     0     0
     1     1     0     0
     0     0     1     1
     0     0     1     1
```

```
>> MM
MM =

  Columns 1 through 11

     0     0     0     0     1     1     0     0     1     0     1
     0     0     0     0     1     1     0     0     0     1     0
     0     0     0     0     0     0     1     1     1     0     1
     0     0     0     0     0     0     1     1     0     1     0
     1     1     0     0     1     0     1     0     1     1     0
     1     1     0     0     0     1     0     1     1     1     0
     0     0     1     1     1     0     1     0     0     0     1
     0     0     1     1     0     1     0     1     0     0     1
     1     0     1     0     1     1     0     0     0     0     0
     0     1     0     1     1     1     0     0     0     0     0
     1     0     1     0     0     0     1     1     0     0     0
     0     1     0     1     0     0     1     1     0     0     0
     1     1     0     0     0     0     0     0     1     1     0
     1     1     0     0     0     0     0     0     1     1     0
     0     0     1     1     0     0     0     0     0     0     1
     0     0     1     1     0     0     0     0     0     0     1

  Columns 12 through 16

     0     1     1     0     0
     1     1     1     0     0
     0     0     0     1     1
     1     0     0     1     1
     0     0     0     0     0
     0     0     0     0     0
     1     0     0     0     0
     1     0     0     0     0
     0     1     1     0     0
     0     1     1     0     0
     0     0     0     1     1
     0     0     0     1     1
     0     1     0     1     0
     0     0     1     0     1
     1     1     0     1     0
     1     0     1     0     1
```

```
MM'

ans =

Columns 1 through 12

     0     0     0     0     1     1     0     0     1     0     1     0
     0     0     0     0     1     1     0     0     0     1     0     1
     0     0     0     0     0     0     1     1     1     0     1     0
     0     0     0     0     0     0     1     1     0     1     0     1
     1     1     0     0     1     0     1     0     1     1     0     0
     1     1     0     0     0     1     0     1     1     1     0     0
     0     0     1     1     1     0     1     0     0     0     1     1
     0     0     1     1     0     1     0     1     0     0     1     1
     1     0     1     0     1     1     0     0     0     0     0     0
     0     1     0     1     1     1     0     0     0     0     0     0
     1     0     1     0     0     0     1     1     0     0     0     0
     0     1     0     1     0     0     1     1     0     0     0     0
     1     1     0     0     0     0     0     0     1     1     0     0
     1     1     0     0     0     0     0     0     1     1     0     0
     0     0     1     1     0     0     0     0     0     0     1     1
     0     0     1     1     0     0     0     0     0     0     1     1

Columns 13 through 16

     1     1     0     0
     1     1     0     0
     0     0     1     1
     0     0     1     1
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
     1     1     0     0
     1     1     0     0
     0     0     1     1
     0     0     1     1
     1     0     1     0
     0     1     0     1
     1     0     1     0
     0     1     0     1
```

4.1.2 Construction of symmetric (45, 12, 3) - BIBD from an affine resolvable (9, 12, 4, 3, 1)-BIBD

```
RC{1}

ans =
     7     6     4
     2     9     3
     8     1     5
```

```
>> RC{2}

ans =
     4     7     8
     9     5     1
     3     6     2
```

```
>> RC{3}

ans =
     7     6     8
     1     5     2
     3     4     9
```

```
>> RC{4}

ans =
     9     6     5
     7     2     3
     4     8     1
```

```
>> PC{1}

ans =
7    6    4    2    9    3    8    1    5

>> PC{2}

ans =
4    7    8    9    5    1    3    6    2

>> PC{3}

ans =
7    6    8    1    5    2    3    4    9

>> PC{4}

ans =
9    6    5    7    2    3    4    8    1

>> M{1}

ans =
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0

>> M{2}

ans =
    1    0    0    0    1    0    0    1    0
    0    1    1    0    0    0    0    0    1
    0    1    1    0    0    0    0    0    1
    0    0    0    1    0    1    1    0    0
    1    0    0    0    1    0    0    1    0
    0    0    0    1    0    1    1    0    0
    0    0    0    1    0    1    1    0    0
    1    0    0    0    1    0    0    1    0
    0    1    1    0    0    0    0    0    1

>> M{3}

ans =
    1    0    0    0    1    0    0    0    1
    0    1    1    0    0    1    0    0    0
    0    1    1    0    0    1    0    0    0
    0    0    0    1    0    0    1    1    0
    1    0    0    0    1    0    0    0    1
    0    1    1    0    0    1    0    0    0
    0    0    0    1    0    0    1    1    0
    0    0    0    1    0    0    1    1    0
    1    0    0    0    1    0    0    0    1

>> M{4}

ans =
    1    1    0    0    1    0    0    0    0
    1    1    0    0    1    0    0    0    0
    0    0    1    1    0    0    0    0    1
    0    0    1    1    0    0    0    0    1
    1    1    0    0    1    0    0    0    0
    0    0    0    0    0    1    1    1    0
    0    0    0    0    0    1    1    1    0
    0    0    0    0    0    1    1    1    0
    0    0    1    1    0    0    0    0    1

>> M{5}

ans =

    1    0    0    1    0    0    0    1    0
    0    1    1    0    0    0    1    0    0
    0    1    1    0    0    0    1    0    0
    1    0    0    1    0    0    0    1    0
    0    0    0    0    1    1    0    0    1
    0    0    0    0    1    1    0    0    1
    0    1    1    0    0    0    1    0    0
    1    0    0    1    0    0    0    1    0
    0    0    0    0    1    1    0    0    1
```

>> MM

MM =

Columns 1 through 11

0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	1	0	0	1	0	1	0
0	1	1	0	0	0	0	0	1	0	1
0	1	1	0	0	0	0	0	1	0	1
0	0	0	1	0	1	1	0	0	0	0
1	0	0	0	1	0	0	1	0	1	0
0	0	0	1	0	1	1	0	0	0	1
0	0	0	1	0	1	1	0	0	0	0
1	0	0	0	1	0	0	1	0	0	0
0	1	1	0	0	0	0	0	1	1	0
1	0	0	0	1	0	0	0	1	1	1
0	1	1	0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0	0	0	0
0	0	0	1	0	0	1	1	0	0	0
1	0	0	0	1	0	0	0	1	1	1
0	1	1	0	0	1	0	0	0	0	0
0	0	0	1	0	0	1	1	0	0	0
0	0	0	1	0	0	1	1	0	0	0
1	0	0	0	1	0	0	0	1	0	0
1	1	0	0	1	0	0	0	0	1	0
1	1	0	0	1	0	0	0	0	0	1
0	0	1	1	0	0	0	0	1	0	1
0	0	1	1	0	0	0	0	1	1	0
1	1	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0
0	0	0	0	0	1	1	1	0	0	1
0	0	0	0	0	1	1	1	0	1	0
0	0	1	1	0	0	0	0	1	0	0
1	0	0	1	0	0	0	1	0	0	0
0	1	1	0	0	0	1	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	1	0	0	0	1	0	0	0
0	0	0	0	1	1	0	0	1	0	0
0	0	0	0	1	1	0	0	1	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	1	0	0	0	1	0	0	0
0	0	0	0	1	1	0	0	1	0	0

Columns 12 through 22

0	0	1	0	0	1	0	1	0	0	0
1	0	0	0	0	0	1	0	1	1	0
1	0	0	0	0	0	1	0	1	1	0
0	1	0	1	1	0	0	0	0	0	1
0	0	1	0	0	1	0	1	0	0	0
0	1	0	1	1	0	0	0	1	1	0
0	1	0	1	1	0	0	0	0	0	1
0	0	1	0	0	1	0	0	0	0	1
1	0	0	0	0	0	1	1	0	0	0
0	0	1	0	0	0	1	1	1	0	0
1	0	0	1	0	0	0	1	1	0	0
1	0	0	1	0	0	0	0	0	1	1
0	1	0	0	1	1	0	0	0	1	1
0	0	1	0	0	0	1	1	1	0	0
1	0	0	1	0	0	0	0	0	0	0
0	1	0	0	1	1	0	0	0	0	0
0	1	0	0	1	1	0	0	0	0	0
0	0	1	0	0	0	1	0	0	1	1
0	0	1	0	0	0	0	1	0	0	1
0	0	1	0	0	0	0	0	1	1	0
1	1	0	0	0	0	1	0	1	1	0
1	1	0	0	0	0	1	1	0	0	1
0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	0	0	1	1	0
0	0	0	1	1	1	0	1	0	0	1
1	1	0	0	0	0	1	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	0	1	1	0	0	1	0	0	0	0
0	0	1	1	0	0	1	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	0	1	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	1	0

Columns 23 through 33											Columns 34 through 44											Column 45
1	0	0	0	1	1	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0
0	1	0	0	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	0	1	0	0
0	1	0	0	0	0	0	1	1	0	0	0	0	1	0	1	1	0	0	0	1	0	0
0	0	1	1	0	0	0	1	1	0	0	0	0	1	1	0	0	1	0	0	0	1	0
1	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	0	0	1
0	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	1
0	0	1	1	0	0	0	0	0	0	1	1	1	0	0	1	1	0	0	0	1	0	0
0	0	1	1	0	0	0	0	0	0	1	1	1	0	1	0	0	1	0	0	0	1	0
1	0	0	0	1	0	0	1	1	0	0	0	0	1	0	0	0	0	1	1	0	0	1
1	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	1	1	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0
1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0
1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	1	0	0	0	1
0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	1	1	0	0	1	0	0	0
0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	1	1	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	1	1	0
0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	1	0	0	0	1
0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	1	1	0	0	1	0	0	0
0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	1	1	0
0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	1	0
0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	1	0	0	0	1
1	0	0	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	1	0	0	0	0
0	0	0	0	1	0	1	1	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0
0	0	0	0	1	0	1	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	1
0	1	1	0	0	0	0	0	1	0	0	1	1	0	0	0	1	1	0	0	0	0	1
1	0	0	1	0	1	0	0	0	1	0	0	0	1	1	1	0	0	1	0	0	0	0
0	1	1	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	1	1	1	0
0	1	1	0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	1	1	1	0
1	0	0	1	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	1	1	1	0
0	0	0	0	1	1	0	0	0	1	0	0	0	1	0	0	1	1	0	0	0	0	1

MM'

Columns 1 through 12

0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	0	1	0	1	0	0
0	1	1	0	0	0	0	0	1	0	1	1
0	1	1	0	0	0	0	0	1	0	1	1
0	0	0	1	0	1	1	0	0	0	0	0
1	0	0	0	1	0	0	1	0	1	0	0
0	0	0	1	0	1	1	0	0	0	1	1
0	0	0	1	0	1	1	0	0	0	0	0
1	0	0	0	1	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	1	1	0	0
1	0	0	0	1	0	0	0	1	1	1	0
0	1	1	0	0	1	0	0	0	1	1	0
0	1	1	0	0	1	0	0	0	0	0	1
0	0	0	1	0	0	1	1	0	0	0	1
1	0	0	0	1	0	0	0	1	1	1	0
0	1	1	0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	1	1	0	0	0	0
0	0	0	1	0	0	1	1	0	0	0	0
1	0	0	0	1	0	0	0	1	0	0	1
1	1	0	0	1	0	0	0	0	1	0	0
1	1	0	0	1	0	0	0	0	0	1	1
0	0	1	1	0	0	0	0	1	0	1	1
0	0	1	1	0	0	0	0	1	1	0	0
1	1	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0
0	0	0	0	0	1	1	1	0	0	1	1
0	0	0	0	0	1	1	1	0	1	0	0
0	0	1	1	0	0	0	0	1	0	0	0
1	0	0	1	0	0	0	1	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0	0
1	0	0	1	0	0	0	1	0	0	0	0
0	0	0	0	1	1	0	0	1	0	0	0
0	0	0	0	1	1	0	0	1	0	0	0
0	1	1	0	0	0	1	0	0	0	0	0
1	0	0	1	0	0	0	1	0	0	0	0
0	0	0	0	1	1	0	0	1	0	0	0

Columns 13 through 24

0	1	0	0	1	0	1	0	0	0	1	0
0	0	0	0	0	1	0	1	1	0	0	1
0	0	0	0	0	1	0	1	1	0	0	1
1	0	1	1	0	0	0	0	0	1	0	0
0	1	0	0	1	0	1	0	0	0	1	0
1	0	1	1	0	0	0	1	1	0	0	1
1	0	1	1	0	0	0	0	0	1	0	0
0	1	0	0	1	0	0	0	0	1	0	0
0	0	0	0	0	1	1	0	0	0	1	0
0	1	0	0	0	1	1	1	0	0	1	0
0	0	1	0	0	0	1	1	0	0	1	0
0	0	1	0	0	0	0	0	1	1	0	0
1	0	0	1	1	0	0	0	1	1	0	0
0	1	0	0	0	1	1	1	0	0	1	0
0	0	1	0	0	0	0	0	0	0	0	1
1	0	0	1	1	0	0	0	0	0	0	1
1	0	0	1	1	0	0	0	0	0	0	1
0	1	0	0	0	1	0	0	1	1	0	0
0	1	0	0	0	0	1	0	0	1	0	0
0	1	0	0	0	0	0	1	1	0	0	0
1	0	0	0	0	1	0	1	1	0	0	0
1	0	0	0	0	1	1	0	0	1	0	0
0	1	0	0	0	0	0	0	0	0	1	1
0	0	1	1	1	0	0	0	0	0	1	1
0	0	1	1	1	0	0	1	1	0	0	0
0	0	1	1	1	0	1	0	0	1	0	0
1	0	0	0	0	1	0	0	0	0	1	1
1	0	0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0	0
0	1	1	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	1	0	0	0

Columns 25 through 36

0	0	1	1	1	0	0	1	0	0	0	0
0	0	0	1	1	0	0	1	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	1
1	1	0	0	0	1	1	0	0	0	0	1
0	0	1	1	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	1	1	1	0
1	1	0	0	0	0	0	0	1	1	1	0
0	0	1	0	0	1	1	0	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	0
0	0	0	0	1	1	0	0	0	1	0	0
0	0	1	0	1	1	0	0	0	1	0	0
0	0	1	1	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	1	0	0	1
1	1	0	0	0	0	0	1	1	0	0	1
1	1	0	0	1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	0	0	0	1	0
0	0	1	0	0	0	0	1	1	0	0	1
0	1	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	0	0	1	0
0	0	0	0	1	1	0	0	0	0	0	1
0	0	0	0	1	1	0	0	0	0	0	1
0	0	0	0	0	0	1	0	1	1	0	0
0	0	0	1	0	0	0	1	0	0	1	0
0	0	0	0	0	0	1	0	1	1	0	0
0	0	0	0	0	0	1	0	1	1	0	0
0	0	0	1	0	0	0	1	0	0	1	0
0	0	0	0	1	1	0	0	0	0	0	1
0	1	0	1	0	0	0	1	0	0	0	1
0	0	1	0	1	1	0	0	1	0	0	0
0	0	1	0	1	1	0	0	1	0	0	0
1	0	0	0	0	0	1	0	0	1	1	0
0	1	0	1	0	0	0	1	0	0	0	1
1	0	0	0	1	1	0	0	1	0	0	0
1	0	0	0	0	0	1	0	0	1	1	0
0	1	0	0	0	0	1	0	0	1	1	0
0	0	1	1	0	0	0	1	0	0	0	1

Columns 37 through 45

1	0	0	1	0	0	0	1	0
0	1	1	0	0	0	1	0	0
0	1	1	0	0	0	1	0	0
1	0	0	1	0	0	0	1	0
0	0	0	0	1	1	0	0	1
0	0	0	0	1	1	0	0	1
0	1	1	0	0	0	1	0	0
1	0	0	1	0	0	0	1	0
0	0	0	0	1	1	0	0	1
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	1	0
0	1	1	0	0	0	0	0	1
0	1	1	0	0	0	0	0	1
0	0	0	1	0	1	1	0	0
1	0	0	0	1	0	0	1	0
0	0	0	1	0	1	1	0	0
0	0	0	1	0	1	1	0	0
1	0	0	0	1	0	0	1	0
0	1	1	0	0	0	0	0	1
1	0	0	0	1	0	0	0	1
0	1	1	0	0	1	0	0	0
0	1	1	0	0	1	0	0	0
0	0	0	1	0	0	1	1	0
1	0	0	0	1	0	0	0	1
0	1	1	0	0	1	0	0	0
0	0	0	1	0	0	1	1	0
0	0	0	1	0	0	1	1	0
1	0	0	0	1	0	0	0	1
1	1	0	0	1	0	0	0	0
1	1	0	0	1	0	0	0	0
0	0	1	1	0	0	0	0	1
0	0	1	1	0	0	0	0	1
1	1	0	0	1	0	0	0	0
0	0	0	0	0	1	1	1	0
0	0	0	0	0	1	1	1	0
0	0	0	0	0	1	1	1	0
0	0	1	1	0	0	0	0	1

>>

Table 1: Some symmetric BIBDs constructed from affine resolvable BIBDs

Affine Resolvable BIBD (v, b, r, k, λ)	**Symmetric BIBD constructed [(r+1)v, kr, kλ]**
(4, 6,3, 2,1)	(16, 6, 2)
(9, 12,4, 3,1)	(45, 12, 3)

NOTE: In SBIBDs, v = b and r = k and therefore the full parameters for the SBIBDs generated is [(r+1)v, (r+1)v, kr, kr, kλ].

4.2 Discussion

From our results, we observe the following in sections 4.1.1 and 4.1.2 as discussed below. Section 4.1.1 shows the result of construction of Symmetric (16, 6, 2)-BIBD from an affine resolvable (4, 6, 3, 2, 1) BIBD. There are three (3) resolution and parallel classes in this case and an incidence matrix of 4x4 is generated for each parallel class. A special matrix M {1} of 4x4 with zero entries is also generated and used in the incidence matrix MM following a definite pattern. The incidence matrix MM generated from the MATLAB is a 16x16 symmetric matrix. The sum of each column and the sum of each row equals 6 while each pair of distinct varieties appears together in exactly 2 of the blocks.

Section 4.1.2 is a construction of Symmetric (45, 12, 3) BIBD from an affine resolvable (9, 12, 4, 3, 1) BIBD.We generated four (4) resolution and parallel classes and an incidence matrix of 9x9 is generated for each of the classes. A special matrix M {1} of 9x9 with zero entries is also generated and used in the incidence matrix MM following a definite pattern. The incidence matrix MM generated is of order 45x45 symmetric matrix. The sum of each column and the sum of each row equals 12 while each pair of distinct varieties appears together in exactly 3 of the blocks.

The MATLAB program written has the capability to construct several SBIBDs from affine resolvable BIBDs. We cannot display all the results of our constructions in this work because it may be cumbersome for large values of v. Some results can also be better displayed with the use of Microsoft Excel Sheet.

CONCLUSION

Many authors and researchers have paid particular attention to the construction of Balanced Incomplete Block Designs (BIBDs). There are several techniques available for the construction of BIBDs. These include the trade-off method, difference methods, variety cutting and construction from finite permutation groups among others.

In this study, a MATLAB program was written to construct resolution and parallel classes from ((4, 6, 3, 2, 1) and (9, 12, 4, 3, 1)) affine resolvable designs. A technique described by Stinson (2003) was thereafter used, via the MATLAB program to obtain their corresponding incidence matrices which gave rise to ((r+1) v, kr, kλ)-SBIBD.

All the two SBIBDs ((16, 6, 2) and (45, 12, 3)) constructed obeyed the mathematical property of a symmetric matrix and this makes our technique unique since the terminology SBIBD does not necessarily imply true mathematical symmetry.

REFERENCES

[Alo10] **D. Aloke** - *Incomplete Block Designs*. World Scientific Publishing Co. Plc. Ltd., 2010.

[Bos39] R. C. Bose - *On the Construction of Balanced Incomplete Block Designs*. Annals of Eugenics 9, 353-399, 1939.

[Bos42] **R. C. Bose** - *Some New Series of Balanced Incomplete Block Designs*. Bulletin, of Calcutta Mathematics Society. Vol. 34, 17–31, 1942.

[CR50] **S. Chowla, H. J. Ryser** - *Combinatorial Problems.* Can. J. Math. 2, 93–99, 1950.

[Fis40] **R. A. Fisher** - *An Examination of the Different Possible Solutions of a Problem in Incomplete Blocks*. Annals of Eugenics. 10, 52-75, 1940.

[GPP06] **I. Gent, K. Petrie, J. F. Puget** - *Symmetry in Constraint Programming*: Handbook of Constraint Programming, Elsevier. Chapter10, 2006.

[Hal69] **M. Hall Jr.** - *Construction of Designs from Permutation Groups*. Institute of Statistics Mimeo series No. 600.10, Chapel Hill, N.C., 1969.

[HL79] **A. S. Hedayat, S. R. Li** - *The Trade off Method in the Construction of BIB Designs with Repeated Blocks*. The Annals of Statistics.Vol.7, No 6, 1277-1287, 1979.

[HK05] **K. H. Hinkelmann, O. Kempthorne** - *Design and Analysis of Experiments: Advanced Experimental Design*.Vol.2. Wiley Series in Probability and Statistics, Wiley-Interscience, 2005.

[Kir50] **T. P. Kirkman** - Query VI on *Fifteen Young Ladies*. Lady's and Gentleman's Diary, No 148, 48, 1850.

[L+06] **O. Lee, S. Yoo, B. Park, I. Chung** - *The Design and Analysis of an Efficient Load Balancing Algorithm Employing the Symmetric Balanced Incomplete Block Design.* Journal of Information Sciences. 176, 2148-2160, 2006.

[Mur61] **V. N. Murty** - *An Inequality for Balanced Incomplete Block Designs*. Annals of Mathematical Statistics. Vol. 32, No. 3, 908-909, 1961.

[NB12] **M. Noshad, M. Brandt-Pearce** - *Expurgated PPM Using Symmetric Balanced Incomplete Block Designs*. Cornell University Press, 2012.

[PS80] **H. D. Patterson, V. Silvey** - *Statutory and Recommended List Trials of Crop Varieties in the United Kingdom*. J. Roy. Statist. Soc. Ser. A143 219–252, 1980.

[Rao58] **V. R. Rao** - *A Note on Balanced Designs*. Annals of Mathematical Statistics. 29, 290–294, 1958.

[Sti03] **D. R. Stinson** - *Combinatorial Designs: Construction and Analysis*. New York: Springer, 2003.

[SS87] **A. P. Street, D.J. Street** - *Combinatorics of Experimental Design*. Oxford: Clarendon Press, 1987.

[Woo84] **W. S. B. Woolhouse** - *Prize Question 1733*. Lady's and Gentleman's Diary, 1844.

[Yat36] **F. Yates** - *Incomplete Randomized Blocks*. Annals of Eugenics. 7, 121-140, 1936.