SOME RESULTS ON THE NUMERICAL SIMULATION OF SEIRS EPIDEMIC MODEL WITH SATURATED INCIDENCE RATE CONSIDERING THE SATURATION TERM FOR THE SUSCEPTIBLE INDIVIDUAL

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ABSTRACT: In this research paper, the numerical simulation of a Susceptible-Exposed-Infected-Recovered- Susceptible (SEIRS) epidemic model with saturated incidence rate with the saturated terms for the susceptible individuals was analyzed. We established the disease-free equilibrium and endemic equilibrium states of the model. We also investigated the local and global stabilities of the disease free equilibrium using matrix and Lyapunov function methods when the basis reproduction number, $1 < R_0 < 1$. The work is an extension Kolawole and Olayiwola [KO16a] to investigate the effect of saturation term on the susceptible individual. We used maple for the simulation of the analysis and the result we obtained are in good agreement with other results in the literature.

KEYWORDS: Reproductive number, SEIRS, saturated incidence rate, susceptible individual.

1. INTRODUCTION

Mathematical models can give us good understanding of how infectious diseases spread, discover general principles governing the transmission dynamics of the diseases and identify more importance and sensitive parameters to make reliable predictions and provide useful control strategies and guidance.

Several authors have worked in this area. Kolawole and Olayiwola [KO16a] investigated the numerical solution effect of saturation term on the susceptible individual using variational iteration method. Greenhalgh [Gre92], Liu [LHL87] discussed dynamical behavior of epidemiological models with nonlinear incidence rates. Li and Muldowney [LM95] and Liu et al. [LHL87] studied the global dynamics of the SEIR models with a non-linear incidence rate and with a standard incidence, respectively. Greenhalgh [Gre97] considered SEIR models that incorporate density dependence in the death rate. Li and Jin [LJ05] considered the global stability of the SEI and SEIR model with infectious force in latent and infected period with non permanent immunity.

Cooke and van den Driessche [CD96] studied bifurcations in models of the SEIRS type with density dependent contact rate and death rate with delays. Li, Graef et al. [L+91] studied a SEIR model for the transmission of an infectious disease that spreads in a population through direct contact of the hosts. The force of infection is of proportionate mixing type.

Hethcote and Tudor [HT80] studied endemic infectious disease models for which infection conferred permanent immunity with no disease-related mortality but with vaccination. In [Kor04], Korobeinikov considers the global properties for SEIR and SEIS by means of Lyapunov functions.

Kuniya and Nakata [KN12] studied the long time behavior of a nonautonomous SEIRS epidemic model. They obtained new sufficient conditions for the permanence (uniform persistence) and extinction of infectious population of the model.

In [OK16, KO16b-14], recent works were also presented in the current trend of SEIRS epidemic model. In this paper, we extend the work done by Kolawole and Olayiwola [KO16a] to analyse the effect of saturation term on a susceptible individual using theorems and proves. We present our result in form of basic reproduction number and theorems are used to prove the local and global stabilities of the disease free equilibrium. We also presented the maple software simulation results of the model.

A population of size $N(t)$ is partitioned into subclasses of individuals who are susceptible, exposed (infected but not yet infectious) infectious and recovered with sizes denoted by $S(t)$, $E(t)$, $I(t)$ and $R(t)$ respectively. The sum $E(t)+I(t)$ is the total infected population. If is assumed that all immigrant individuals are susceptible and vertical

transmission can be assumed to acquire temporary immunity in which recovered individual goes back to the susceptible class again.

2. SEIRS EPIDEMIC MODEL

$$\left.\begin{aligned}\frac{dS}{dt} &= \wedge - \frac{\beta SI}{1+m_1S+m_2I} - \mu S + \delta R \\ \frac{dE}{dt} &= \frac{\beta SI}{1+m_1S+m_2I} - (\mu+\varepsilon)E \\ \frac{dI}{dt} &= \varepsilon E - (\mu+\gamma)I \\ \frac{dR}{dt} &= \gamma I - (\mu+\delta)R)\end{aligned}\right\} \quad (1)$$

The parameter $\Lambda(t) > 0$ is the birth rate, $\beta(t) > 0$ is the disease transmission coefficient, $\mu(t) > 0$ is the mortality/death rate, $\varepsilon(t) > 0$ is the rate of developing infectivity, $\gamma(t) > 0$ is the recovery rate, $\delta(t) > 0$ is the rate of losing immunity, with initial value

S (0) > 0 , E (0) ≥ 0 , I (0) > 0, R (0) ≥ 0

where

S (t) ------ is the size of susceptible population
E (t) ------ is the exposed (not infections but infected) population
I (t) ------ is the infectious population
R (t) ------ is the recovered population at time ≥ 0 respectively
λ ------- is the birth rate
β ------- is the disease transmission coefficient
μ ------ the mortality
ε ------ the rate of developing infectivity
γ ------ the recovery rate
δ ------ the rate of losing immunity at time t

A special case of [KO16a] is the Kuniya and Nakata [KN12] model where $m_1 = m_2 = 0$.

Considering the above autonomous system (1) we present a SEIRS epidemic model using $\frac{\beta SI}{1+m_1S+m_2I}$ as the incidence rate. Where $m_1 > 0$ and $m_2 > 0$ are saturation terms to the susceptible and infected individual respectively. Where m_1 and m_2 are saturation terms to the susceptible and infected individuals respectively. It is also the parameter that measures the sociological and psychological effects of disease in the model. In this research work, the effect of saturation term to the susceptible individual m_1 is considered. i.e when m_2 =0 , hence the new model is shown below:

$$\left.\begin{aligned}\frac{ds}{dt} &= \wedge - \frac{\beta SI}{1+m_1s} - \mu s + \delta R \\ \frac{dE}{dt} &= \frac{\beta SI}{1+m_1s} - (\mu+\varepsilon)E \\ \frac{dI}{dt} &= \varepsilon E - (\mu+\gamma)I \\ \frac{dR}{dt} &= \gamma I - (\mu+\delta)R\end{aligned}\right\} \quad (2)$$

$$At \text{ the equilibrum }, \frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0 \quad (3)$$

Then, we obtained

$$\left.\begin{aligned} &\wedge - \frac{\beta SI}{1+m_1s} - \mu S + \delta R = 0 \\ &\frac{\beta SI}{1+m_1s} - (\mu+\varepsilon)E = 0 \\ &\varepsilon E - (\mu+\gamma)I = 0 \\ &\gamma I - (\mu+\delta)R = 0\end{aligned}\right\} \quad (4)$$

Equation (4) implies

$$\gamma I = (\mu+\delta)R \quad (5)$$

From equations (4) and (5) we obtained:

$$R = 0 \text{ or } \frac{\beta(\mu+\delta)}{\gamma} S - \frac{(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta)}{\varepsilon\gamma} - \frac{m_1(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta)S}{\varepsilon\gamma} = 0 \quad (6)$$

$$\left.\begin{aligned} &\mathrm{I} = \left(\frac{\mu+\delta}{\gamma}\right)(0) = 0 \qquad \therefore \ \mathrm{I} = 0 \\ &\mathrm{E} = \frac{(\mu+\gamma)(\mu+\delta)}{\varepsilon\gamma}(0) = 0 \quad \therefore \ \mathrm{E} = 0 \\ &\wedge - \mu S = 0 \qquad \therefore \quad \mathrm{S} = \frac{\wedge}{\mu}\end{aligned}\right\} \quad (7)$$

For disease free equilibrium;

$$(\mathrm{DFE}) = (\mathrm{S}^0, \mathrm{E}^0, \mathrm{I}^0, \mathrm{R}^0) = (\frac{\wedge}{\mu}, 0, 0, 0) \quad (8)$$

And for the endemic equilibrium $\mathrm{I} \neq 0$ equation (4) implies that

$$R=\frac{(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta)(\mu+\wedge m_1)-\wedge\varepsilon\beta(\mu+\delta)\gamma\varepsilon}{(\varepsilon\beta(\mu+\delta)-m_1(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta))(\gamma\varepsilon\delta-(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta))}=R^{**}$$

$$I=\frac{(\mu+\delta)}{\gamma}\left(\frac{(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta)(\mu+\wedge m_1)-\wedge\varepsilon\beta(\mu+\delta)\gamma\varepsilon}{(\varepsilon\beta(\mu+\delta)-m_1(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta))(\gamma\varepsilon\delta-(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta))}\right)=I^{**}$$

$$E=\frac{(\mu+\gamma)(\mu+\delta)}{\varepsilon\gamma}\left(\frac{(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta)(\mu+\wedge m_1)-\wedge\varepsilon\beta(\mu+\delta)\gamma\varepsilon}{(\varepsilon\beta(\mu+\delta)-m_1(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta))(\gamma\varepsilon\delta-(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta))}\right)=E^{**}$$

$$S=\frac{(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta)}{\varepsilon\gamma\beta-m_1(\mu+\varepsilon)(\mu+\gamma)(\mu+\delta)}=S^{**} \quad (9)$$

Therefore the endemic equilibrium points are (S,E,I,R)= $S^{**},E^{**},I^{**},R^{**})$

3. DERIVATION OF R_0 USING THE NEXT GENERATION MATRIX

Let G be a next generation matrix. It comprises of two parts F and V^{-1} where

$$F=\left[\frac{\partial f_i(x_0)}{dx_j}\right] \quad (10)$$

$$V=\left[\frac{\partial V_i(x_0)}{\partial x_j}\right] \quad (11)$$

F_i is the new infections, while the V_i are transfers of infections from one compartment to another. X_0 is the disease free equilibrium state.
R_0 is the dominant Eigen value of the matrix

$$G=FV^{-1} \quad (12)$$

To calculate the R_0 using (10) where G is the next generation matrix for the SEIRS model in Equation (2) in E and I compartments of the model, we proceed thus;

$$\left.\begin{aligned}\frac{dE}{dt}&=\frac{\beta SI}{1+m_1S}-(\mu+\varepsilon)E\\ \frac{dI}{dt}&=\varepsilon E-(\mu+\gamma)I\end{aligned}\right] \quad (13)$$

$$F=\begin{pmatrix}\frac{\beta\Lambda}{\mu+m_1\wedge} & 0\\ 0 & 0\end{pmatrix} \quad \text{and}$$

$$V=\begin{pmatrix}0 & (\mu+\varepsilon)\\ (\mu+\gamma) & -\varepsilon\end{pmatrix} \quad (14)$$

The inverse of V is obtained as

$$V^{-1}=\frac{-1}{(\mu+\gamma)(\mu+\varepsilon)}\begin{pmatrix}-\varepsilon & -(\mu+\varepsilon)\\ (\mu+\gamma) & 0\end{pmatrix} \quad (15)$$

and

$$G=\begin{pmatrix}\frac{\beta\wedge\varepsilon}{(\mu+m_1\wedge)(\mu+\gamma)(\mu+\varepsilon)} & \frac{\beta\wedge}{(\mu+m_1\wedge)(\mu+\gamma)}\\ 0 & 0\end{pmatrix} \quad (16)$$

The characteristic equation of matrix G above is;

$$|G-\lambda I|=0 \quad (17)$$

The dominant eigenvalue is our R_0 therefore,

$$R_0=\frac{\beta\wedge\varepsilon}{(\mu+m_1\wedge)(\mu+\gamma)(\mu+\varepsilon)} \quad (18)$$

4. ESTABLISHING THE LOCAL STABILITY OF DISEASE FREE EQUILIBRIUM

The system of equation (2) can be linearised by setting

$$S-S_1=x,\ \mathrm{E}=\mathrm{E},\ \mathrm{I}=\mathrm{I},\ \mathrm{R}=\mathrm{R} \quad (19)$$

Which gives;

$$\left.\begin{aligned}\frac{dx}{dt}&=-\mu x+(\beta{S_1}^2m_1-\beta S_1)I+\delta R+\text{non-linear terrms}\\ \frac{dE}{dt}&=-(\mu+\varepsilon)E+(\beta S_1I-\beta{S_1}^2m_1)I+\text{non-linear terrms}\\ \frac{dI}{dt}&=\varepsilon E-(\mu+\gamma)I\\ \frac{dR}{dt}&=\gamma I-(\mu+\delta)R\end{aligned}\right] \quad (20)$$

Therefore the resulting characteristic equation is $|A-\lambda I|=0$ i.e

$$\begin{vmatrix}-\mu-\lambda & 0 & \frac{\beta\wedge}{\mu}\left(\frac{\wedge m_1-1}{\mu}\right) & 0\\ 0 & -(\mu+\varepsilon)-\lambda & \frac{-\beta\wedge}{\mu}\left(\frac{\wedge m_1-1}{\mu}\right) & 0\\ 0 & \varepsilon & -(\mu+\gamma)-\lambda & 0\\ 0 & 0 & \gamma & -(\mu+\delta)-\lambda\end{vmatrix}=0 \quad (21)$$

Hence

$$\left.\begin{aligned}
&\lambda_1 = -\mu, \quad \lambda_2 = -(\mu+\delta)\\
&\lambda_3 = \frac{1}{\mu}\left(-\mu^2 - \frac{\mu\gamma}{2} - \frac{\varepsilon\mu}{2} + \frac{1}{2}\sqrt{D}\right)\\
&\lambda_4 = \frac{1}{\mu}\left(-\mu^2 - \frac{\mu\gamma}{2} - \frac{\varepsilon\mu}{2} - \frac{1}{2}\sqrt{D}\right)
\end{aligned}\right] \quad (22)$$

Where

$$\begin{aligned}
D = {} & \mu^2\gamma^2 - 2\varepsilon\gamma\mu^2 + \varepsilon^2\mu^2 + 4\varepsilon\gamma R_0\mu^2 - 4\varepsilon\gamma R_0\lambda^2 m_1^2 + \\
& + 4\mu^4 R_0 + 4\mu^2 R_0\lambda^2 m_1^2 + 4\varepsilon\mu^2 R_0 + \\
& + 4\mu^3\gamma R_0 - 4\mu\gamma R_0\lambda^2 m_1^2 - 4\varepsilon\mu R_0\lambda^2 m_1^2
\end{aligned} \quad (23)$$

Theorem 1: If $R_0 < \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} < 1,$ The disease free equilibrium is locally asymptotically stable and if $R_0 \geq \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} > 1$ the disease free equilibrium is unstable.

Proof: Since $\mu > 0, \varepsilon > 0, \delta > 0, \gamma > 0, m > 0$ and if

$$R_0 < \frac{\mu^2}{\mu^2 - \wedge^2 m_1^2} < 1$$

Hence the disease free equilibrium point $(S_1, E_1, I_1, R_1)(\frac{\wedge}{\mu}, 0,0,0)$ is unstable if

$$R_0 \geq \frac{\mu^2}{\mu^2 - \Lambda^2 m_1^2} > 1$$

6. LOCAL STABILITY OF THE ENDEMIC EQUILIBRIUM

Let

$$S - S^* = w, E - E^* = x, I - I^* = y, R - R^* = z \quad (29)$$

$$\frac{dS}{dt} = \frac{dw}{dt}, \frac{dE}{dt} = \frac{dx}{dt}, \frac{dI}{dt} = \frac{dy}{dt}, \frac{dR}{dt} = \frac{dz}{dt} \quad (30)$$

Hence
The linearised equations are as follows:

5. GLOBAL STABILITY OF THE DISEASE FREE EQUILIBRIUM

Consider the Lyapunov function defined thus

$$\mathrm{L} = (\mu+\varepsilon)I + \varepsilon E\ (\mu+\varepsilon)(\mu+\gamma) \quad (24)$$

$$L^1 = (\mu+\varepsilon)\varepsilon E - (\mu+\varepsilon)(\mu+\gamma)I + \frac{\varepsilon\beta SI}{1+m_1 S} - (\mu+\varepsilon)\varepsilon E \quad (25)$$

At disease free $S = \frac{\Lambda}{\mu}$ and $I \geq 0$ then:

$$\mathrm{L}^1 = (S_1, E_1, I_1, R_1) = \left(\frac{\wedge}{\mu}, 0, 0, 0\right)$$

$$= \left(\frac{\varepsilon\beta\frac{\wedge}{\mu}}{1+m_1\frac{\wedge}{\mu}} - (\mu+\varepsilon)(\mu+\gamma)\right)I \quad (26)$$

Equation (26) simplified to;

$$\mathrm{L}^1 = (\mu+\varepsilon)(\mu+\gamma)\left[R_0 - 1\right]I \quad (27)$$

Therefore if $R_O \leq 1$

$$L^1 \leq 0 \quad (28)$$

Hence the disease free equilibrium is globally asymptotically stable.

$$\left.\begin{aligned}
&\frac{dw}{dt} = (-\beta I^* + 2\beta I^{*2} m_1 S - \mu)(w + (-\beta S^* + \beta S^* \gamma + \beta S^* m_1)\gamma + \delta z + \\
&non\ \ linear\ \ terms + cons\tan t\ \ terms \\
&\frac{dx}{dt} = (\beta I^* - 2\beta I^* m_1 S^*)w + (\beta S^* - \beta S^{*2} + \beta S^* m_1)\gamma - (\mu + \varepsilon)x + \\
&non\ \ linear\ \ terms + cons\tan t\ \ terms \\
&\frac{dy}{dt} = \varepsilon x - (\mu + \gamma)\gamma + non\ \ linear\ \ terms + cons\tan t\ \ terms \\
&\frac{dz}{dt} = \gamma y - (\mu + \delta)z + non\ \ linear\ \ terms + cons\tan t\ \ terms
\end{aligned}\right] \quad (31)$$

Equation (31) is equivalent to;

$$\begin{pmatrix} w^1 \\ x^1 \\ y^1 \\ z^1 \end{pmatrix} = \begin{pmatrix} (-\beta I^* + 2\beta I^* m_1 S^* - \mu) & 0 & (-\beta S^* + \beta S^* m_1) & \delta \\ (\beta I^* - 2\beta I^* m_1 S^*) & -(\mu + \varepsilon) & (\beta S^* - \beta S^* m_1) & 0 \\ 0 & \varepsilon & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \delta) \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} + non\ linear\ terms \quad (32)$$

Let $f(\lambda) = \lambda^4 - A_3\lambda^3 - A_4\lambda^2 - A_5\lambda - A_6 = 0$, then (S^*, E^*, I^*, R) is locally asymptotically stable if $A_3 < 0, A_4 < 0, A_5 < 0, A_6 < 0$.

The Jacobian matrix of above equation is

$$A^* = \begin{pmatrix} (-\beta I^* + 2\beta I^* m_1 S^* - \mu) & 0 & (-\beta S^* + \beta S^{*2} m_1) & \delta \\ (-\beta I^* - 2\beta I^* m_1 S^*) & -(\mu + \varepsilon) & \beta S^* - \beta S^* m_1 & 0 \\ 0 & \varepsilon & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \delta) \end{pmatrix} \quad (33)$$

Therefore the characteristics equation is

$$\left|A^* - \lambda I\right| = 0 \quad (34)$$

$$\begin{vmatrix} (-\beta I^* + \beta I^* m_1 s - \mu) & 0 & (\beta S^* + \beta S^* m_1) & \delta \\ (\beta I^* + 2\beta I^* m_1 S - \mu) & -(\mu + \varepsilon) & (\beta S^* - \beta S^* m_1) & 0 \\ 0 & \varepsilon & -(\mu + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \delta) \end{vmatrix} = 0 \quad (35)$$

Evaluating (35) and let

$$A_0 = -\delta\varepsilon\gamma\left(\beta I^* + 2\beta I^* m_1 S^*\right),\ \ A_1 = (-\beta I^* + 2\beta I^* m_1 S^* - \mu),\ \ A_2 = \beta S^* - \beta S^* m_1 \quad (36)$$

$$A_0 - [(\mu + \delta) + \lambda]\left[-([\mu + \gamma] + \lambda)((-(\mu + \gamma) + \lambda)(A_1 - \lambda)) - \varepsilon(\beta S^* - 2\beta S^* m_1)\left[\frac{-A_0}{\varepsilon\gamma\delta} + (A_1 - \lambda)\right]\right] A_0 - \left((\mu + \delta) + \lambda\right)$$

$$\left[-((\mu + \varepsilon) + \lambda)\left[-(\mu + \gamma)A_1 + (\mu + \gamma + A_1)\lambda - \lambda^2\frac{A_0 A_2}{\varepsilon\gamma\delta} - A_2 A_1 + A_2\lambda\right]\right] = 0 \quad (37)$$

Expanding, factorizing and re-arranging gives,

$$\lambda^4-(\gamma+A_1-\varepsilon+A_2-(\mu+\delta)\lambda^4-\begin{pmatrix}(\mu+\varepsilon)(\mu+\gamma+A_1)+A_2(\mu+\varepsilon)-(\mu+\gamma)A_1\\ -\frac{A_0A_2}{\varepsilon\gamma\delta}-A_2A_1+(\mu+\delta)(\gamma+A_1-\varepsilon+A_2\end{pmatrix}\lambda^2$$

$$-(-(\mu+\varepsilon)(\mu+\gamma)A_1-\frac{A_0A_2}{\varepsilon\gamma\delta}(\mu+\varepsilon)-A_2A_1(\mu+\varepsilon)+(\mu+\delta)(\mu+\varepsilon)(\mu+\gamma+A_1)+A_2(\mu+\delta)(\mu+\varepsilon)$$

$$-(\mu+\delta)(\mu+\gamma)A_1-\frac{A_0A_2}{\varepsilon\gamma\delta}(\mu+\delta)-(\mu+\delta)A_2A_1)$$

$$\lambda-(-(\mu+\delta)(\mu+\varepsilon)(\mu+\gamma)A_1-\frac{A_0A_2}{\varepsilon\gamma\delta}(\mu+\varepsilon)(\mu+\delta)-A_2A_1(\mu+\varepsilon)(\mu+\delta)+A_0)=0 \quad (38)$$

By Descartes rule of signs, Let,

$$A_3=\gamma+A_1-\varepsilon+A_2-(\mu+\delta)<0$$

$$A_4=\left((\mu+\varepsilon)(\mu+\gamma+A_1)+A_2(\mu+\varepsilon)-(\mu+\gamma)A_1-\frac{A_0A_2}{\varepsilon\gamma\delta}-A_2A_1+(\mu+\delta)(\gamma+A_1-\varepsilon+A_2)\right)<0$$

$$A_5=\left(-(\mu+\varepsilon)(\mu+\gamma)A_1-\frac{A_0A_2}{\varepsilon\gamma\delta}-(\mu+\varepsilon)-A_2A_1(\mu+\varepsilon)(\mu+\delta)(\mu+\varepsilon)(\mu+\gamma+A_1)+A_2)(\mu+\delta)(\mu+\varepsilon)\right.$$

$$\left.-(\mu+\delta)(\mu+\gamma)A_1-\frac{A_0A_2}{\varepsilon\gamma\delta}-(\mu+\varepsilon)-(\mu+\delta)A_2A_1\right)<0$$

$$A6=\left(-(\mu+\delta)(\mu+\varepsilon)(\mu+\gamma)A_1-\frac{A_0A_2}{\varepsilon\gamma\delta}(\mu+\varepsilon)(\mu+\delta)-A_2A_1(\mu+\varepsilon)(\mu+\delta)+A_0\right)<0 \quad (39)$$

Then equation (39) becomes

$$f(\lambda)=\lambda^4-A_3\lambda^3-A_4\lambda^2-A_5\lambda-A_6=0 \quad (40)$$

Let $A_3<0,A_4<0,A_5<0,A_6<0$ in Equation (40) then $f(\lambda)$ have no change in sign meaning there are no positive roots of $f(\lambda)$.

Also if λ is replaced by $-\lambda$ in Equation (40), then

$$f(-\lambda)=\lambda^4+A_3\lambda^3-A_4\lambda^2-A_5\lambda-A_6=0 \quad (41)$$

So if $A_3<0,A_4<0,A_5<0,A_6<0$ *in Equation* (41) $f(-\lambda)$, have four sign changes which implies, that there are exactly four negative roots of $f(-\lambda)$. Since there is no positive roots for $A_3<0,A_4<0,A_5<0,A_6<0$.

That is all eigenvalues are negatives, then the endemic or disease equilibrium is locally asymptotically stable if $A_3<0,A_4<0,A_5<0,A_6<0$.

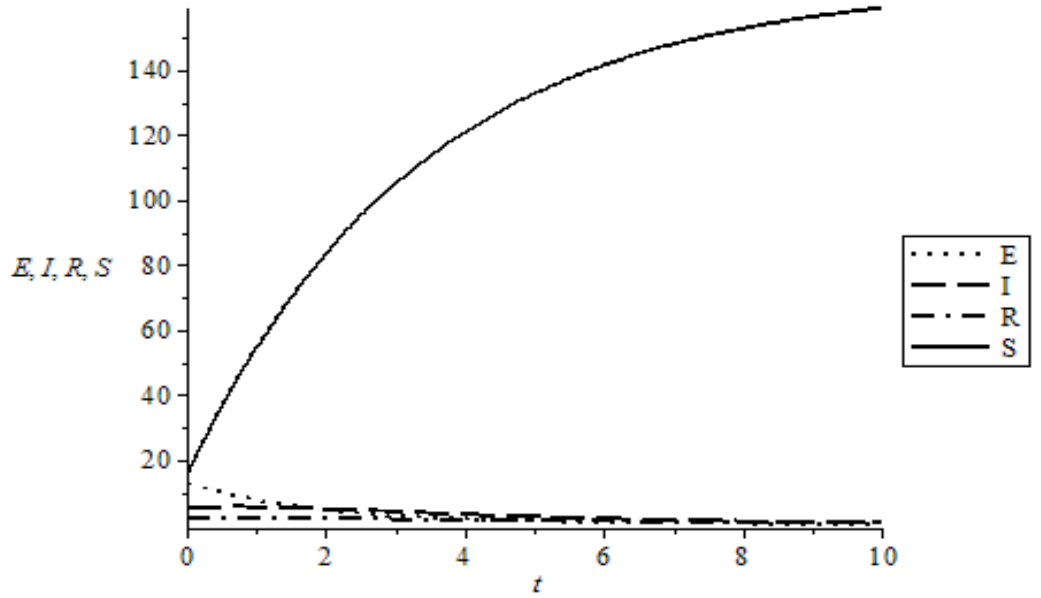

Fig 1 : Graph of Susceptible (S), Infected (I), Exposed (E) and Recovered (R) class against Time (t) with β=0.19, μ=0.3, Λ=50, γ=0.1, δ=0.05,ε=0.25, $m_1 = 0.1, R_0 < 1 \; with \; m_2 = 0$

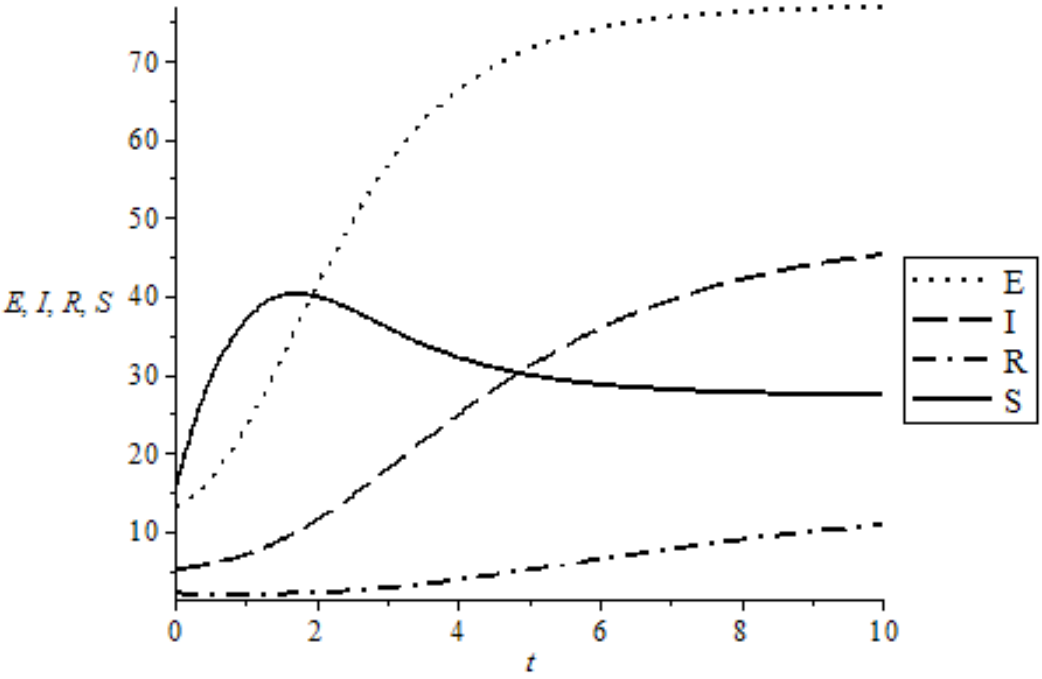

Fig 2: Graph of Susceptible (S), Infected (I), Exposed (E) and Recovered (R) class against Time (t) with β=0.19, μ=0.3, Λ=50, γ=0.1, δ=0.05,ε=0.25, $m_1 = 0.2, R_0 > 1 \; with \; m_2 = 0$

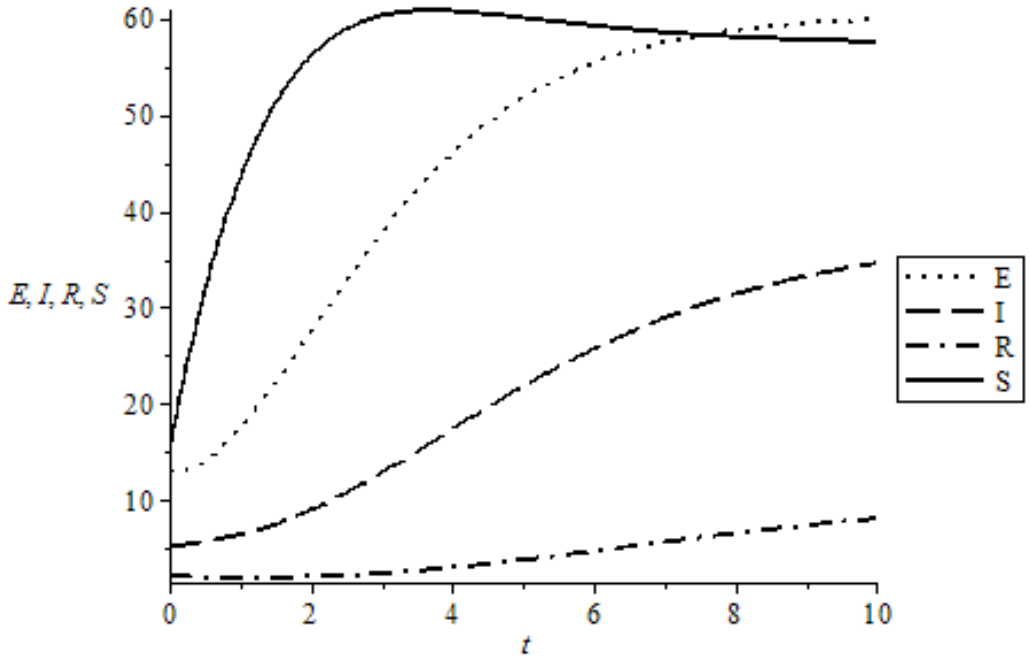

Fig 3: Graph of Susceptible (S), Infected (I), Exposed (E) and Recovered (R) class against Time (t) with β=0.19, μ=0.3, Λ=50, γ=0.1, δ=0.05,ε=0.25, $m_1 = 0.3, R_0 > 1$ *with* $m_2 = 0$

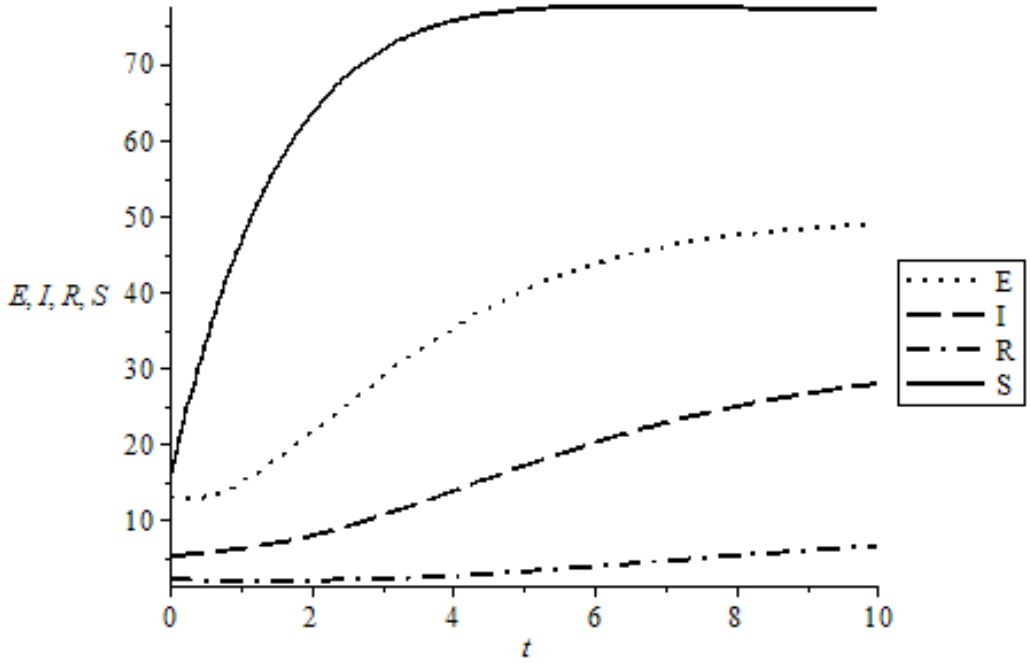

Fig 4: Graph of Susceptible (S), Infected (I), Exposed (E) and Recovered (R) class against Time (t) with β=0.19, μ=0.3, Λ=50, γ=0.1, δ=0.05,ε=0.25, $m_1 = 0.5, R_0 > 1$ *with* $m_2 = 0$

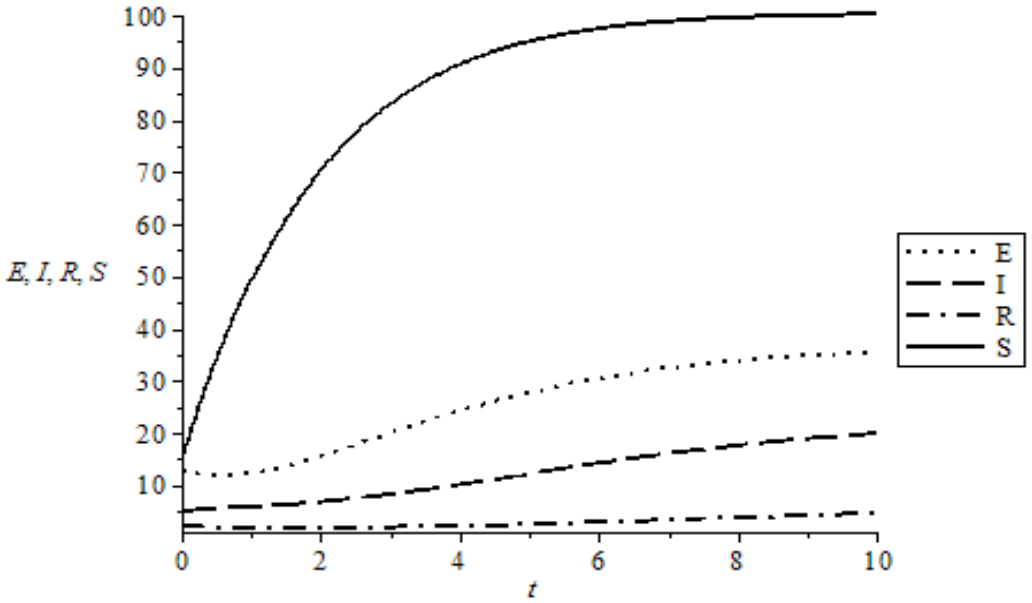

Fig 5: Graph of Susceptible (S), Infected (I), Exposed (E) and Recovered (R) class against Time (t) with β=0.19, μ=0.3, Λ=50, γ=0.1, δ=0.05,ε=0.25, $m_1 = 0.9, R_0 > 1$ *with* $m_2 = 0$

7. DISCUSSION OF SIMULATED RESULTS AND CONCLUSION

Fig 1: reveals the asymptotic stability of the disease free equilibrium if $R_0 < 1, m_1 = 0.1$ *and* $m_2 = 0$ $\beta = 0.19, \mu = 0.3, \Lambda = 50, \gamma = 0.1, \delta = 0.05, \varepsilon = 0.25.$ It is also observed that susceptible class increases while the exposed and the infected classes reduce with few individuals recovered. This shows the effect of m_1 provided $R_0 < 1$ *and* $\beta < 0.00528.$

Fig 2: The simulation result reveals the unstable nature of disease free equilibrium at $R_0 > 1, m_1 = 0.2. m_2 = 0$ $\beta = 0.19, \mu = 0.3, \Lambda = 50, \gamma = 0.1, \delta = 0.05, \varepsilon = 0.25$. It is observed that the susceptible class reduces while the exposed and the infected classes increase with few individuals recovered. This shows the effect of m_1 since $R_0 > 1$ *and* β *is* $> 0.00528.$

Fig 3: The simulation result reveals the a slight stability nature of disease free equilibrium at $R_0 > 1, m_1 = 0..3 m_2 = 0$ $\beta = 0.19, \mu = 0.3, \Lambda = 50, \gamma = 0.1, \delta = 0.05, \varepsilon = 0.25.$ It is observed that the susceptible class begins to increase gradually more than other classes because m_1 has increased to 0.3. This shows the effect of m_1 since $R_0 > 1$ *and* β *is* $> 0.00528.$

Fig 4: Parameters set at $m_1 = 0.5$ *and* $R_0 > 1$ and $m_1 = 0$ *and* $\beta = 0.19, \mu = 0.3, \Lambda = 50, \gamma = 0.1, \delta = 0.05, \varepsilon = 0.25$ to test the effect of m_2 with other parameters fixed. We observed that the exposed and the infected classes are decreasing while susceptible class is increasing and better number of individual recovered. This also shown little stability nature of the disease free equilibrium when m_1 increases to 0.5 because the susceptible class increases more than other classes.

Fig 5: Parameters set at $m_2 = 0.$ *and* $R_0 > 1$ and $m_1 = 0.9$ *and* $\beta = 0.19, \mu = 0.3,$ $\Lambda = 50, \gamma = 0.1, \delta = 0.05, \varepsilon = 0.25,$ to test the effect of m_1 with other parameters fixed. It was observed that susceptible class increases while exposed and infected classes reduced with large number of individuals recovered. This shows the asymptotic stability nature of disease free equilibrium at $m_1 = 0.9$ despite $R_0 > 1.$

8. CONCLUSIONS

The simulation results show that saturation term for the susceptible individual play vital role in disease eradication for $R_0 < 1$ and $R_0 > 1$ .The higher the saturation term m_1 , the better stability of disease free equilibrium, hence the disease will be eradicated from the population. Therefore susceptible individual in a population should be given better orientation about the treatment of a particular disease in terms of seminars, conferences, public awareness and others for better eradication. The result is in line with the existing result of Kolawole and Olayiwola (KO16a).

REFERENCES

[CD96] **K. Cooke, P. Van den Driessche** - *Analysis of an SEIRS epidemic model with two delays. J. Math Biol.35*, pp 240, 1996.

[Gre92**]** **D. Greenhalgh** - *Some results for a SEIR epidemic model with density dependence in the death rate.* IMA J Math Appl Med Biol. 9, pp (67), 1992.

[Gre97] **D. Greenhalgh** - *Hopf bifurcation in epidemic models with a latent period and Non permanent immunity.* Math. Comput. Model; 25, pp 85, 1997.

[HT80] **H. W. Hethcote, D. W. Tudor** - *Integral equations models for endemic infectious disease.* J. Math. Biol.vol 9 pp 37-47, 1980.

[Kor04] **A. Korobeinikov** - *Lyapunov functions and global properties for SEIR and SEIS epidemic models.* Mathematical Medicine and Biology, 21, pp 75-83, 2004.

[KN12] **T. Kuniya, Y. Nakata** - Permanence and extinction for a nonautonomous SEIRS epidemic model. *Applied Mathematics and Computations*, vol. 218 (18), pp 9321-9331, 2012.

[KO16a] **M. K. Kolawole, M. O. Olayiwola** - *On the Numerical Effect of Saturation Terms on the Susceptible Individual in Susceptible – Exposed – Infected – Recovered - Susceptible (SEIRS) Epidemic Model.* Computing, Information Systems, Development Informatics & Allied Journal Vol 7 No 2, pp 83-90, 2016.

[KO16b] **M. K. Kolawole, M. O. Olayiwola** - *Analysis of a SEIRS Epidemic model with saturated incidence Rate, Considering the Initial State of the Disease.* Transactions of NAMP, vol.2, pp 171-180, 2016.

[LJ05] **G. Li, Z. Jin** - *Global stability of an SEIR epidemic model with infectious force in latent infected and immune period.* Chaos, Solitons & Fractals 25, pp 1177–1184, 2005.

[LM95] **M. Y. Li, J. S. Muldowney J.S**. - *Global stability for the SEIR model in epidemiology.* Math. Biosci. 125 pp 155–564, 1995.

[LHL87] **W.-M. Liu, H. W. Hethcote, S. A. Levin** - *Dynamical behavior of epidemiological models with non linear incidence rate.* J. Math Biol.;25: pp 359–380, 1987.

[L+91] **M. Y. Li, J. R. Graef, L. C. Wnag, J. Karsai , J.** - *Global dynamics of a SEIR model with a varying total population size.* Math Biosic. 160, pp 199-213, 1991.

[OK16] **M .O, Olayiwola, M. K. Kolawole** - *On the study of saturation terms on the infected individuals in a Susceptible – Exposed – Infected – Recovered - Susceptible (SEIRS) Epidemic model.* Transactions of NAMP, vol.2, pp 181-186, 2016.