

## ESTIMATING THE HETEROGENEITY EFFECTS IN A PANEL DATA REGRESSION MODEL

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**ABSTRACT:** Violation of homoscedasticity assumption in a Panel Data Regression Model (PDRM) implies unequal variability of error terms, and this creates heterogeneity problem in estimation. This research thus attempts to investigate the presence and effect of heteroscedasticity in panel data through the estimation of a specified audit fees PDRM using Pooled ordinary least square (POLS, Least square dummy variable (LSDV) technique where all coefficients vary across individual and Random Effect estimator (REM). A conditional Lagrange multiplier test was developed via a two-way error components model, to examine the presence of heteroscedasticity in the fitted POLS model while Hausman test was used to ascertain the suitability of the LSDV Model over Random effect model and vice-versa. The conditional LM test gave a value of 7.1462 with P-value of 0.000000000000446 which shows that there is presence of unequal variance of MA(1) errors among the residuals of the fitted Pooled OLS model, thereby rendered the estimator inconsistent. Both LSDV and RE models were fitted to take care of the challenges posed by the presence of heteroscedasticity and both models captured the goodness of fit better when compared to the Pooled OLS model. However, the Hausman test revealed that random effect model will not be preferable since p-value of the former is less than 0.05.

**KEYWORDS:** Heterogeneity, Heteroscedasticity, Conditional Lagrange Multiplier, Panel Data, Audit Fees Model, Panel Data Regression Model.

### 1. INTRODUCTION

A panel is a cross-section or a kind of data in which observations are obtained on the same set of entities at several periods of time [Fre95, GP09, Hsi03, Ken08, Gre03]. Panel data models examine individual-specific effect, time effect or both in order to deal with heterogeneity/serial correlation of individual effects that may or may not be observed. In this paper, our focus shall only reflect on the problems which affect the cross sectional aspect of panel data, which is the problem imposed by heteroscedasticity. This shall be looked into via a panel data regression model of audit fees.

Heteroscedasticity is one of the associated problems with the Pooled Ordinary Least Squares (POLS)

[GP09]. By heteroscedasticity, we meant the existence of some non-constant variance function in a Panel data regression model (PDRM). [LE00] and [BFN83] confirmed that in the presence of heteroscedasticity, Ordinary least square (OLS) estimates are unbiased, but the usual tests of significance are generally inappropriate and their use can lead to incorrect inferences. Among other things, they suggest that data analysts should correct for heteroscedasticity using Heteroscedasticity Consistent Covariance Matrix (HCCM) whenever there is reason to suspect its presence. The pioneering work of [8] has given rise to further researches on the estimation of heteroscedasticity effects in panel data. Prominent among these works are those of [RWH81, Mag82, HR94, Bal88, BG88, Ran88, Wan89, LS94, Lej06, HG00, Roy02, Phi03, BBP06]. However, these works were concerned with regression models that have to do with one-way error component model while this work is based on two-way error components model. For instance, both [BGT90] and [BG88] were concerned with the estimation of a model allowing for heteroscedasticity on the individual-specific error term i.e., assuming that  $\mu_i \sim (0, \sigma_{u_i}^2)$  while  $v_{it} \sim IID(0, \sigma_v^2)$ . In contrast, [RWH81], [Mag82], [Bal88] and [Wan89] adopted a symmetrically opposite specification allowing for heteroscedasticity on the remainder error term, i.e., assuming that  $\mu_i \sim IID(0, \sigma_{\mu}^2)$  while  $v_{it} \sim (0, \sigma_{v_{it}}^2)$ . [HR94] studies the external debt repayments problem using a panel of 79 developing countries over 13 years period. These countries differ in terms of their colonial history, financial institutions, religious affiliations and political regimes. All of these country-specific variables affect the attitudes that these countries have with regards to borrowing and defaulting and the way they are treated by the lenders. Not accounting for these countries heterogeneity causes serious misspecification. [Ran88] allowed for heteroscedasticity of both the individual and remainder error component, i.e.,  $\mu_i \sim (0, \sigma_{\mu_i}^2)$  and  $v_{it} \sim (0, \sigma_{v_{it}}^2)$ , with the latter varying with every observations over time and individuals. With regards

to estimation, [LS94] proposed an adaptive estimation procedure for a one-way error component model allowing for heteroscedasticity of unknown form on the remainder error term, i.e., assuming that  $\mu_i \sim IID(0, \sigma^2_{\mu})$  while  $v_{it} \sim N(0, \sigma^2_{v_{it}})$ , where  $\sigma^2_{v_{it}}$  is a nonparametric function  $f(z'_{it})$  of a vector of exogenous variables. They also suggest a robust version of [BP80] Lagrange Multiplier (LM) test for no random individual effects, by allowing for adaptive heteroscedasticity of unknown form on the remainder error term. [Lej06] on the other hand, used Maximum Likelihood (ML) Estimation and LM to test for general heteroscedasticity in a one-way error components model while [HG00] proposed a Rao score test for homoscedasticity assuming the existence of individual effects. [Phi03] follows [MT78] in considering a one-way stratified error component model. As unobserved heterogeneity occurs through individual-specific variances changing across strata, [Phi03] provides an algorithm for estimating this model and suggests a bootstrap test for identifying the number of strata. [BBP06] derived an LM test for the null hypothesis of homoscedasticity of the individual random effects assuming homoscedasticity of the remainder error term. In relation to the general heteroscedastic model of [Ran88, Lej06], [BBP06] also derived a joint test for homoscedasticity. Under the null hypothesis, the model is a homoscedastic one-way error component regression model and is estimated by restricted MLE. This is different from [Lej06], where under the null hypothesis,  $\sigma^2_{\mu} = 0$ , so that the restricted MLE is OLS and not MLE on a one-way homoscedastic error component model. The validity of this model under the null hypothesis is exactly that of [HG00] but it is more general under the alternative hypothesis since it does not assume a homoscedastic remainder error term. This research therefore, intends to examine this opinion on a PDRM tagged Audit Fees Model by deriving a conditional LM test for the presence of heteroscedasticity via a two-way error components model, where zero serial correlation is assumed.

Audit fees represent fees a company pays an external auditor in exchange for performing an audit. Prominent among authors who have worked on modelling of audit fees are [AA12a, AA12b, A+13, Gam12, SO13, Has15], but they all conjectured differently from the background knowledge of the audit fees model specified in this research, which is in line with [\*\*\*90].

## 2. MATERIAL AND METHODS

### 2.1. Specification of Audit Fees Model

This model employed the use of four (4) Pre-determined variables namely Profit before Tax (PBT), Total Assets

(TA), Total Liability (TL) and Shareholders Fund (SHF) which shall be originated from panel data of published annual reports of sixteen (16) Nigerian Commercial Banks for periods of ten (10) years. The model as implied by the scope of auditor's work in CAMA ([\*\*\*90]) is thus presented as:

$$AF = f(PBT, TA, TL, SHF) + \varepsilon \quad (1)$$

When the model is expressed in an explicit format, we have

$$AF_{it} = \beta_1 + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \varepsilon_{it} \quad (2)$$

where  $i = 1, 2, 3, \dots, N$  and  $t = 1, 2, 3, \dots, T$

$\beta_1, \beta_2, \beta_3, \beta_4$  and  $\beta_5$  are parameters to be estimated and  $\varepsilon_{it}$  is a composite error term. Within the context of this research,  $i = 1, 2, 3, \dots, 16$  and  $t = 1, 2, 3, \dots, 10$

In the course of this study, we hope to demonstrate that the conditional variance of  $AF_{it}$  increases as each of  $PBT_{it}, TA_{it}, TL_{it}$  and  $SHF_{it}$  increases. Within the context of PDRM, both the parameters and error terms of equation (2) shall be varied based on space that will result into the following equations:

$$AF_{it} = \beta_{1i} + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \varepsilon_{it} \quad (3)$$

In estimation, we employ the dummy variable technique (i.e. the differential intercept dummies) to account for the individual effect. Thus the model becomes

$$AF_{it} = \alpha_1 + \alpha_2 D_{2i} + \alpha_3 D_{3i} + \dots + \alpha_{16} D_{16i} + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \varepsilon_{it} \quad (4)$$

Where  $\alpha_1$  represents the intercept of the first individual (i.e. bank) and  $\alpha_2, \alpha_3, \dots, \alpha_{16}$  are the differential intercept coefficients which tell us by how much the intercept of the remaining banks differ from the intercept of the first while  $D_{2i}, \dots, D_{16i}$  are the dummy variables which was not created for the first individual to avoid falling into dummy variables trap. In a situation where all the coefficients are allowed to vary across individuals, we extend equation (4) with the introduction of individual dummies in additive manner. Thus we have

$$AF_{it} = \alpha_1 + \alpha_2 D_{2i} + \alpha_3 D_{3i} + \dots + \alpha_{16} D_{16i} + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \gamma_1 (D_{2i} PBT_{it}) + \gamma_2 (D_{2i} TA_{it}) + \gamma_3 (D_{2i} TL_{it}) + \gamma_4 (D_{2i} SHF_{it}) + \dots + \gamma_{57} (D_{16i} PBT_{it}) + \gamma_{58} (D_{16i} TA_{it}) + \gamma_{59} (D_{16i} TL_{it}) + \gamma_{60} (D_{16i} SHF_{it}) + \varepsilon_{it} \quad (5)$$

where  $\gamma_1, \gamma_2, \dots, \gamma_{60}$  are the differential slope coefficients, just as  $\alpha_2, \alpha_3, \dots, \alpha_{16}$  are the differential intercepts.

Similarly, for the REM, we recalled equation (4) and instead of treating as fixed, we assume that it is a random variable with a mean value of  $\beta_1$ . Thus, the intercept value for the individual bank can be expressed as

$$\beta_{1i} = \beta_1 + u_i, \quad i = 1, 2, \dots, N \quad (6)$$

The individual differences in the intercept values of each bank are reflected in the error term  $u_i$ . if we substitute equation (7) in (4), we have

$$AF_{it} = \beta_1 + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + u_i + \varepsilon_{it} \quad (7)$$

Equation (8) implies

$$AF_{it} = \beta_1 + \beta_2 PBT_{it} + \beta_3 TA_{it} + \beta_4 TL_{it} + \beta_5 SHF_{it} + \omega_{it} \quad (8)$$

$$\text{where } \omega_{it} = u_i + \varepsilon_{it} \quad (9)$$

Thus, the composite error term  $\omega_{it}$  consists of two components  $u_i$  (cross section error component) and  $\varepsilon_{it}$  which is the combined time series and cross-section error component.

## 2.2. Model Estimation Techniques

Here, we provide brief theoretical overview of the three (3) techniques considered in this study.

- (i) **Pooled OLS:** This technique pool the data over  $i$  and  $t$  into one  $nT$  observations, and estimates of the parameters are obtained by OLS using the model

$$y = X'\beta + \omega \quad (10)$$

where  $y$  is an  $nT \times 1$  column vector of response variables,  $X$  is an  $nT \times k$  matrix of regressors,  $\beta$  is a  $(k+1) \times 1$  column vector of regression coefficients,  $\omega$  is an  $nT \times 1$  column vector of the combined error terms (i.e  $\varepsilon_i + u_{it}$ ). The Pooled estimator is given as

$$\hat{\beta}_{pooled} = (X'X)^{-1} X' y \quad (11)$$

- (ii) **Fixed Effect Least Square Dummy Variable:** Let  $Y_i$  and  $X_i$  be the  $T$  observations for the  $i_{th}$  unit,  $i$  be a  $T \times 1$  column of ones, and let  $e_i$  be associated  $T \times 1$  vector of disturbances. Then

$$Y_i = X_i\beta + \mathbf{i}\alpha_i + e_i \quad (12)$$

Connecting these terms in matrix form gives

$$Y = [X \quad d_1 \quad d_2 \quad \dots \quad d_N] \begin{bmatrix} \beta \\ \alpha \end{bmatrix} + e_i \quad (13)$$

where  $d_i$  is a dummy variable indicating the  $i^{th}$  unit. Let the  $NT \times N$  matrix  $D = [d_1 \quad d_2 \quad \dots \quad d_n]$  then, assembling all  $NT$  rows gives;

$$Y = X\beta + D\alpha + e_i \quad (14)$$

Estimating the equation using OLS gives an estimator

$$\hat{\beta} = [X' M_D X]^{-1} [X' M_D y] \quad (15)$$

where  $D$  is the  $i^{th}$  dummy variable,  $M_D = -D(D'D)^{-1}D'$ , the transformed data  $X = M_D X$  and  $Y = M_D y$ .

The OLS referred in equation (15) shall be a Within-Group (WG) estimator. According to [Woo12], the within transformation implements the LSDV model better because the regression on de-meanded data yields the same results as estimating the model from the original data and a set of  $(N-1)$  indicator variables for all but one of the panel units. It is often not workable to estimate that LSDV model directly because we may have hundreds or thousands of individual panel units in our dataset.

- (iii) **Random Effect Estimator:** Consider a random effect model

$$Y_{it} = \beta_0 + \beta_i X_{it} + a_i + u_{it} \quad (16)$$

we employ GLS estimator by transforming model (16) into

$$\bar{Y}_{it} = \beta_0 + \beta_1 \bar{X}_{it} + \bar{V}_{it} \quad (17)$$

We then multiply equation (17) by  $\lambda$  and takes its difference from equation (16) to have

$$Y_{it} - \lambda \bar{Y}_{it} = \beta_0(1 - \lambda) + \beta_1(X_{it} - \lambda \bar{X}_{it}) + v_{it} - \lambda \bar{V}_{it} \quad (18)$$

Thus, the GLS estimator for the slope parameter of (18) becomes

$$\hat{\beta}_{RE} = (X' \Omega^{-1} X)^{-1} (X' \Omega^{-1} y) \quad (19)$$

where

$$\Omega^{1/2} = \sigma_u^{-1} (I - T^{-1} \lambda i_T i_T') \quad (20)$$

And  $\lambda$  (the key transformation parameter) is given as

$$\hat{\lambda} = 1 - \left( \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\varepsilon^2} \right)^{1/2} \quad (21)$$

Thus, equation (19) is the specific GLS estimator called Random effect estimator.

### 2.3. Model Testing

Here, we shall employ a two-way error component model as earlier emphasized, to test for the violation of homoscedasticity assumption in our researched model.

Considering a two-way error component model stated as:

$$y_{it} = x_{it}\beta + u_{it}, \quad i = 1, 2, \dots, N \quad t = 1, 2, \dots, T \quad (22)$$

Within the context of two-way error component, the regression disturbances term  $u_{it}$  can be described by the equation

$$u_{it} = \mu_i + \lambda_t + v_{it} \quad (23)$$

With  $\mu_i$  representing individual-specific effect,  $\lambda_t$  representing time-specific effect and  $v_{it}$  the idiosyncratic remainder disturbance term, which is usually assumed to be well-behaved and independent from both the regressors  $x_{it}$  and  $\mu_i$ . The two-way error component model can be written in matrix form as

$$y = X\beta + u \quad (24)$$

The disturbance term  $u$  in equation (24) can be written in vector form as

$$u = (I_{NT} \otimes I_{NT})v + (I_N \otimes I_T)\mu + (I_T \otimes I_N)\lambda + V \quad (25)$$

Where  $I_{NT}$  is an identity matrix of dimension  $NT$ ,  $I_N$  is an identity matrix of dimension  $N$ ,  $I_T$  is an identity matrix of dimension  $T$ ,  $I_{NT}$  is a vector of ones of dimension  $NT$ ,  $I_T$  is a vector of ones of dimension  $T$ ,  $I_N$  is a vector of ones of dimension  $N$ ,  $\mu' = (\mu_1, \dots, \mu_N)$ ,  $\lambda' = (\lambda_1, \dots, \lambda_T)$ ,  $V$  is the AR(1) covariance matrix of dimension  $T$ ,  $\otimes$  denotes the kronecker product and

$$Var(\mu_i) = \sigma_{\mu i}^2 = h(f_i'(\alpha)) \quad , i = 1, \dots, N \quad (26)$$

According to [BP80], the function  $h(\cdot)$  is an arbitrary strictly positive twice continuously differentiable function,  $\alpha$  is a  $P \times 1$  vector of unrestricted parameters and  $f_i$  is a  $P \times 1$  vector of strictly exogenous regressors which determine the

heteroscedasticity of the individual specific effects and the first element of  $f_i$  is one, and without loss of generality,  $h(\alpha_1) = \sigma_\mu^2$ .

Following [BJS10], the variance-covariance matrix of  $u$  can be written as

$$\begin{aligned} E(uu') &= \Sigma = \sigma_u^2(I_N \otimes I_T I_T') + (I_T \otimes I_N I_N')\sigma_\lambda^2 + \\ &\sigma_v^2 I_{NT} \otimes V \\ &= (I_N \otimes I_T)diag[h(f_i' \alpha)](I_N \otimes I_T)' + \\ &(I_T \otimes I_N I_N')\sigma_\lambda^2 + \sigma_v^2 I_{NT} \otimes V \\ &= diag[h(f_i' \alpha)] \otimes J_T + (I_T \otimes I_N I_N')\sigma_\lambda^2 + \sigma_v^2 I_{NT} \otimes V \end{aligned} \quad (27)$$

Where  $J_T$  is a matrix of ones of dimension  $T$ ,  $diag[h(f_i' \alpha)]$  is a diagonal matrix of dimension  $N \times N$  and  $V$  can be expressed as

$$V = E(VV') = \sigma_v^2 \left( \frac{1}{1-\rho^2} \right) V_1 \quad (28)$$

where  $V_1$  is a symmetric matrix of order  $\rho^{T-N}$

#### 2.3.1. Conditional LM Test for $H_0: \sigma_{\mu i}^2 \neq \sigma_\mu^2, \forall i$ and $\sigma_{\lambda t}^2 = 0$ but $\sigma_{v_{it}}^2 \neq 0, \rho = 0$

In this section, we derive a conditional LM test for presence of individual heteroscedasticity in the absence of serial correlation.

Under normality of the disturbances, the log-likelihood function,  $L$  of a Lagrange multiplier follows that of a multivariate normal distribution. Thus,

$$L(\beta, \theta) = \frac{-NT}{2} \ln(2\pi) - \frac{1}{2} \ln|\Sigma| \quad (29)$$

Where  $\theta' = (\sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho, \alpha')$  and  $y = X\beta$ . In this case, we set  $\tilde{\theta}' = (\sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho)$ . Thus,  $L(\beta, \theta)$  becomes  $(\beta, \tilde{\theta}')$  and we set  $\eta_1 = (\beta', \sigma_v^2, \sigma_\mu^2, \sigma_\lambda^2, \rho)$ . In order to obtain the conditional LM statistic, we need to obtain the score statistic  $D(\theta) = \frac{\partial L}{\partial \theta}$  and the Information matrix  $I(\theta) = -E\left[\frac{\partial^2 L}{\partial \theta \partial \theta'}\right]$ . Following [32], we obtain  $D(\theta)$  and  $I(\theta)$  as

$$\frac{\partial L}{\partial \theta} - \frac{1}{2} tr \left[ \Sigma^{-1} \left( \frac{\partial \Sigma}{\partial \theta} \right) \right] + \frac{1}{2} \left[ u' \Sigma^{-1} \left( \frac{\partial \Sigma}{\partial \theta} \right) \Sigma^{-1} u \right] \quad (30)$$

$$-E \left[ \frac{\partial^2 L}{\partial \theta \partial \theta'} \right] = \frac{1}{2} tr \left[ \Sigma^{-1} \left( \frac{\partial \Sigma}{\partial \theta} \right) \Sigma^{-1} \frac{\partial \Sigma}{\partial \theta'} \right] \quad (31)$$

Under  $H_0$ , the variance covariance matrix of the disturbances as given by equation (27) becomes

$$\Sigma = diag[h(f_i' \alpha)] \otimes J_T + \sigma_v^2 I_{NT} \otimes I_T + \sigma_\lambda^2 I_N \quad (32)$$

And according to [WK82], the spectral decomposition and inverse of  $\Sigma$  respectively becomes

$$\Sigma = \text{diag}[Th(f'_i \alpha) + \sigma_v^2] \otimes \bar{J}_T + \sigma_v^2 I_{NT} \otimes I_T + \sigma_\lambda^2 I_N \quad (33)$$

$$\Sigma^{-1} = \text{diag} \left[ \frac{1}{\hat{\Omega}_i^2} \right] \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N$$

$$\text{Where } \Omega_i^2 = Th(f'_i \alpha) + \sigma_v^2 \quad (34)$$

Therefore,

$$\frac{\partial L}{\partial \rho} = D(\hat{\rho}) = -\frac{1}{2} \text{tr} \left[ \Sigma^{-1} \left( \frac{\partial \Sigma}{\partial \rho} \right) \right] + \frac{1}{2} \left[ \hat{u}' \Sigma^{-1} \left( \frac{\partial \Sigma}{\partial \rho} \right) \Sigma^{-1} \hat{u} \right]$$

$$= -\frac{1}{2} \text{tr} \left[ \text{diag} \left[ \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right] \otimes \bar{J}_T Z + I_{NT} \otimes E_T Z + \right.$$

$$\left. \sigma v 2 \sigma \lambda 2 I N T \otimes Z + 1 2 u' \text{diag} \sigma v 2 \Omega i 4 \otimes J T Z + 1 \sigma v 2 I N T \otimes E T Z + \sigma v 2 \sigma \lambda 4 I N T Z u \right]$$

$$= -\frac{1}{2} \left[ \frac{2(T-1)}{T} \sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} - \frac{2(T-1)}{T} - \right.$$

$$\left. \text{tri}=1 N t=1 T \sigma v 2 \sigma \lambda 2 + \sigma v 2 2 u' \text{diag} 1 \Omega i 4 \otimes J T Z + 1 \sigma v 4 I N T \otimes E T Z + 1 \sigma \lambda 4 I N T Z u \right]$$

$$\text{since } \text{tr}(Z) = 0 \text{ and } \text{tr}(\bar{J}_T Z) = \text{tr}(E_T) Z$$

$$= \frac{2(T-1)}{T}$$

$$= -\frac{1}{2} \left[ \frac{2(T-1)}{T} \sum_{i=1}^N \sum_{t=1}^T \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} - \frac{2(T-1)}{T} - 0 \right]$$

$$+ \frac{\hat{\sigma}_v^2}{2} \left[ \hat{u}' \left( \text{diag} \left[ \frac{1}{\hat{\Omega}_i^4} \right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + \frac{1}{\hat{\sigma}_\lambda^4} I_{NT} Z \right) \hat{u} \right]$$

$$E \left[ -\frac{\partial L^2}{\partial \eta_1 \partial \eta_1'} \right] = \frac{1}{2} \text{tr} \left[ \Sigma^{-1} \left( \frac{\partial \Sigma}{\partial \eta_1} \right) \Sigma^{-1} \frac{\partial \Sigma}{\partial \eta_1'} \right]$$

$$E \left[ -\frac{\partial^2 L}{\partial \beta \partial \beta'} \right] = \frac{1}{2} \text{tr} [X' \Sigma^{-1} X]^2 = \frac{1}{2} \text{tr} [X' \Sigma^{-1} X X' \Sigma^{-1} X] = N \xrightarrow{\text{lim}} \infty \left[ \frac{X' \Sigma^{-1} X}{NT} \right] = I_{\beta\beta}(\hat{\eta}_1)$$

$$E \left[ -\frac{\partial^2 L}{\partial \beta \partial \sigma_v^2} \right] = \frac{1}{2} \text{tr} \left[ X' \Sigma^{-1} X \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \right] = 0$$

$$\left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \beta \partial \sigma_v^2} \right] \xrightarrow{N, T \rightarrow \infty} 0 \right)$$

$$E \left[ -\frac{\partial^2 L}{\partial \beta \partial \sigma_\mu^2} \right] = \frac{1}{2} \text{tr} \left[ X' \Sigma^{-1} X \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \right] = 0 \left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \beta \partial \sigma_\mu^2} \right] \xrightarrow{N, T \rightarrow \infty} 0 \right)$$

$$E \left[ -\frac{\partial^2 L}{\partial \beta \partial \sigma_\lambda^2} \right] = \frac{1}{2} \text{tr} \left[ X' \Sigma^{-1} X \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I' N \right) \right] = 0$$

$$\left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \beta \partial \sigma_\lambda^2} \right] \xrightarrow{N, T \rightarrow \infty} 0 \right)$$

$$E \left[ -\frac{\partial^2 L}{\partial \beta \partial \rho} \right] = \frac{1}{2} \text{tr} \left[ X' \Sigma^{-1} X \left( \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z \right) \right]$$

$$= \frac{T-1}{T} \text{tr} \left( (X' \Sigma^{-1} X) \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) + I_{NT} + \frac{1}{\hat{\sigma}_\lambda^2} I_{NT} \right) = 0 \left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \beta \partial \rho} \right] \xrightarrow{N, T \rightarrow \infty} 0 \right)$$

$$\left( \text{since } \text{tr}(Z) = 0, \text{tr}(\bar{J}_T Z) = \text{tr}(E_T Z) = 2((T-1)/T) \right)$$

$$E \left[ -\frac{\partial^2 L}{\partial \sigma_v^4} \right] = \frac{1}{2} \text{tr} \left[ \text{diag} \left[ \frac{1}{\hat{\Omega}_i^4} \right] \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^4} I_N \right]^2 = \left[ \text{diag} \left[ \frac{1}{\hat{\Omega}_i^4} \right] \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^4} I_N \right]$$

Since there's no serial correlation of which its variance has been expressed as

$$\hat{\sigma}_\lambda^2 = \frac{(T-1)}{T} \sum_{i=1}^N \sum_{t=1}^T \left( \frac{\hat{\Omega}_i^2 - \hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right)$$

$$+ \frac{\hat{\sigma}_v^2}{2} \left[ \hat{u}' \left( \text{diag} \left[ \frac{1}{\hat{\Omega}_i^4} \right] \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T Z + 1 \sigma \lambda 4 I N T Z u \right) \right]$$

$$(35)$$

Equation (35) is the solution obtained after maximization of the first order condition, where  $\hat{u} = y - x\hat{\beta}_{GLS}$  is the generalized least square residuals under  $H_0$ ,  $\hat{\Omega}_i^2 = Th(f'_i \hat{\alpha}) + \hat{\sigma}_v^2$ , where  $\hat{\alpha}$  is the ML estimator of  $\alpha$  under  $H_0$ , and  $h'(f'_i \hat{\alpha})$  is the evaluated value of  $\partial h(f'_i \hat{\alpha}) / \partial f'_i \alpha$ . All the components of the score test statistic  $\frac{\partial L}{\partial \eta_1}(\cdot)$  evaluated at maximization of the first order condition are all equal to zero except  $\frac{\partial L}{\partial \rho}$  [K+14]. Thus, the partial derivatives under  $H_0$  are expressed in vector form as

$$D(\hat{\eta}_1) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ D(\hat{\rho}) \end{pmatrix} \quad (36)$$

Also, we obtain information matrix under the null hypothesis as follow:

$$\begin{aligned}
 &= \frac{1}{2} \sum_{i=1}^N \sum_{t=1}^T \left( \frac{1}{\hat{\Omega}_i^4} + \frac{NT-1}{\hat{\sigma}_v^4} + \frac{1}{\hat{\sigma}_\lambda^4} \right) = \frac{1}{2} (\hat{\sigma})_v^4 \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_v^2 \partial \sigma_\mu^2} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \left( \text{diag} T \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \right] \\
 &= \frac{1}{2} \text{tr} \left( \text{diag} T \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) = 0 \quad \left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \sigma_v^2 \partial \sigma_\mu^2} \right] \xrightarrow{N,T \rightarrow \infty} 0 \right) \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_v^2 \partial \sigma_\lambda^2} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N \right) \right] \\
 &= \frac{1}{2} \text{tr} \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N \right) = 0 \quad \left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \sigma_v^2 \partial \sigma_\lambda^2} \right] \xrightarrow{N,T \rightarrow \infty} 0 \right) \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_v^2 \partial \rho} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \left( \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z \right) \right] \\
 &= \frac{1}{2} \text{tr} \left( \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes E_T Z \right) \\
 &= \frac{1}{2 \hat{\sigma}_v^2} \text{tr} \left( \text{diag} \left( \frac{\hat{\sigma}_v^4}{\hat{\Omega}_i^4} \right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^4}{\hat{\sigma}_\lambda^4} I_N \otimes E_T Z \right) = \frac{1}{2 \hat{\sigma}_v^2} N \xrightarrow{\lim} \infty \left[ \frac{I'_N Z}{N} \right] \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_\mu^4} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} T \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \right]^2 \\
 &= \frac{1}{2} \text{tr} \left[ \text{diag} T^2 \left[ \frac{1}{\hat{\Omega}_i^4} \right] \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^4} I_N \right] \\
 &= \frac{1}{2} \sum_{i=1}^N \sum_{t=1}^T T^2 \left( \frac{1}{\hat{\Omega}_i^4} + \frac{NT-1}{\hat{\sigma}_v^4} + \frac{1}{\hat{\sigma}_\lambda^4} \right) = \frac{1}{2} (\hat{\sigma})_\mu^4 \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \sigma_\lambda^2} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} T \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N \right) \right] \\
 &= \frac{1}{2} \text{tr} \left( \text{diag} T \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N \right) = 0 \quad \left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \sigma_\lambda^2} \right] \xrightarrow{N,T \rightarrow \infty} 0 \right) \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \rho} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} T \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N \right) \left( \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z \right) \right] \\
 &= \frac{1}{2} \text{tr} \left( \text{diag} T \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes E_T Z \right) = 0 \quad \left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \sigma_\mu^2 \partial \rho} \right] \xrightarrow{N,T \rightarrow \infty} 0 \right) \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_\lambda^4} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N \right) \right]^2 = \frac{1}{2} \text{tr} \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^4} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^4} I_{NT} \otimes E_T + \right. \\
 &\left. \frac{1}{\hat{\sigma}_\lambda^4} I_N I'_N \right) = \frac{1}{2} (\hat{\sigma})_\lambda^4 \\
 E \left[ -\frac{\partial^2 L}{\partial \sigma_\lambda^2 \partial \rho} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} \left( \frac{1}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{1}{\hat{\sigma}_\lambda^2} I_N I'_N \right) \left( \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z \right) \right] \\
 &= \frac{1}{2} \text{tr} \left( \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + \frac{1}{\hat{\sigma}_v^2} I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes E_T Z \right) = 0 \quad \left( \text{since } E \left[ -\frac{\partial^2 L}{\partial \sigma_\lambda^2 \partial \rho} \right] \xrightarrow{N,T \rightarrow \infty} 0 \right) \\
 E \left[ -\frac{\partial^2 L}{\partial \rho^2} \right] &= \frac{1}{2} \text{tr} \left[ \left( \text{diag} \left( \frac{\hat{\sigma}_v^2}{\hat{\Omega}_i^2} \right) \otimes \bar{J}_T Z + I_{NT} \otimes E_T + \frac{\hat{\sigma}_v^2}{\hat{\sigma}_\lambda^2} I_{NT} \otimes Z \right) \right]^2 \\
 &= \frac{1}{2} \text{tr} \left( \text{diag} \left( \frac{\hat{\sigma}_v^4}{\hat{\Omega}_i^4} \right) \otimes \bar{J}_T Z \bar{J}_T Z + I_{NT} \otimes E_T Z + \frac{\hat{\sigma}_v^4}{\hat{\sigma}_\lambda^4} I_{NT} \otimes Z' Z \right) = \frac{1}{2} N \xrightarrow{\lim} \infty \left[ \frac{Z' Z}{N} \right]
 \end{aligned}$$

Thus, information matrix under the null hypothesis can be obtained as a symmetric matrix of the form

$$I(\hat{\eta}_1) = \begin{pmatrix} \beta' \beta & \beta \sigma_v^2 & \beta \sigma_\mu^2 & \beta \sigma_\lambda^2 & \beta \rho \\ \beta \sigma_v^2 & \sigma_v^4 & \sigma_v^2 \sigma_\mu^2 & \sigma_v^2 \sigma_\lambda^2 & \sigma_v^2 \rho \\ \beta \sigma_\mu^2 & \sigma_v^2 \sigma_\mu^2 & \sigma_\mu^4 & \sigma_\mu^2 \sigma_\lambda^2 & \sigma_\mu^2 \rho \\ \beta \sigma_\lambda^2 & \sigma_v^2 \sigma_\lambda^2 & \sigma_\mu^2 \sigma_\lambda^2 & \sigma_\lambda^4 & \sigma_\lambda^2 \rho \\ \beta \rho & \sigma_v^2 \rho & \sigma_\mu^2 \rho & \sigma_\lambda^2 \rho & \rho^2 \end{pmatrix} \quad (37)$$

$$= \begin{pmatrix} I_{\beta\beta}(\hat{\eta}_1) & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} (\hat{\sigma})_v^4 & 0 & 0 & \frac{1}{2 \hat{\sigma}_v^2} N \xrightarrow{\lim} \infty \left[ \frac{I'_N Z}{N} \right] \\ 0 & 0 & \frac{1}{2} (\hat{\sigma})_\mu^4 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} (\hat{\sigma})_\lambda^4 & 0 \\ 0 & \frac{1}{2 \hat{\sigma}_v^2} N \xrightarrow{\lim} \infty \left[ \frac{I'_N Z}{N} \right] & 0 & 0 & \frac{1}{2} N \xrightarrow{\lim} \infty \left[ \frac{Z' Z}{N} \right] \end{pmatrix} \quad (38)$$

Thus, a conditional  $LM$  statistic under the specified  $H_0$  is given as

$$LM_{\rho|\alpha} = D(\hat{\rho})'[(I_{NT}(\hat{\eta}_1))^{-1}|_{\rho\rho}]D(\hat{\rho}) \quad (39)$$

Setting

$$H_{NT}^{\rho} = \text{diag}\left(\frac{1}{\sqrt{NT}}I_k, \frac{1}{\sqrt{NT}}, \frac{1}{\sqrt{N}}, \frac{1}{\sqrt{T}}, \frac{1}{\sqrt{NT}}\right),$$

$LM$  statistic also becomes

$$LM_{\rho|\alpha} = \left[D(\hat{\rho})_{H_{NT}^{\rho}}\right]' [H_{NT}^{\rho}(I_{NT}(\hat{\eta}_1))H_{NT}^{\rho}]^{-1}|_{\rho\rho} \quad (40)$$

$$H_{NT}^{\rho}(I_{NT}(\hat{\eta}_1))H_{NT}^{\rho} \xrightarrow{N,T \rightarrow \infty} I(\hat{\eta}_1)$$

Thus, the  $LM$  statistic becomes

$$LM_{\rho|\alpha} = D(\hat{\rho})'[(I(\hat{\eta}_1))^{-1}|_{\rho\rho}]D(\hat{\rho}) \quad (41)$$

Where

$$(I(\hat{\eta}_1))^{-1}|_{\rho\rho} = \frac{1}{2} N \xrightarrow{\text{lim}} \infty \left[\frac{1}{N} Z' \left(I_N - \frac{I_N I_N'}{N}\right) Z\right]$$

Under  $H_0$ ,  $LM$  statistic is asymptotically distributed as  $\chi_1^2$  as  $N, T \rightarrow \infty$

### 3.RESULTS AND DISCUSSION

The results of the three models fitted from the analytical techniques discussed and that of the test carried out to showcase the heterogeneity effects, as occasioned by the presence of heteroscedasticity are presented and discussed in this section.

**Table 1: Presentation of Pooled OLS Results**

Variables	Coefficients	Standard Error	t-value	Pr(> t )
<b>Intercept</b>	120,970	10,011	12.0840	0.0000
<b>PBT</b>	0.00080588	0.00027278	2.9543	0.0036
<b>TA</b>	-0.000026096	0.000012963	-2.0131	0.0458
<b>TL</b>	0.0000080482	0.000035418	2.2724	0.0244
<b>SHF</b>	0.00025419	0.00012246	2.0757	0.0396

**Table 2: Conditional Lagrange Multiplier Test for Heteroscedasticity**

Z	P-value
7.1462	0.0000000000000446

**Table 3: Presentation of LSDVM Results that Accounts for Only Individual Effects**

Variables	Coefficients	Standard Error	t-value	Pr(> t )
<b>Intercept</b>	218,000	31250	6.975	0.000000
<b>PBT</b>	-0.000001940	0.00001413	-0.137	0.890995
<b>TA</b>	0.000008032	0.000003303	2.432	0.016299
<b>TL</b>	0.0005581	0.0002772	2.014	0.045967
<b>SHF</b>	0.00002475	0.0001289	0.192	0.848021
<b>BANKS -DIAMOND</b>	-117,900	43000	-2.743	0.006889
<b>ECO</b>	-166,400	46020	-3.615	0.000418
<b>FIDELITY</b>	-144,200	42920	-3.360	0.001004
<b>FIRST</b>	-20,520	43130	-0.476	0.634959
<b>FCMB</b>	-81,720	42880	-1.906	0.058699
<b>GTB</b>	-12,500	43690	-0.286	0.75219
<b>- SKYE</b>	-102,200	43070	-2.373	0.018993
<b>SIBTC</b>	-96,630	42980	-2.248	0.026119
<b>SCB</b>	-204,600	43710	-4.681	0.0000667
<b>STERLING</b>	-120,900	43270	-2.794	0.005930
<b>UNION</b>	-77,310	44620	-1.733	0.085324
<b>UBA</b>	-11,780	43400	-0.271	0.786517
<b>UNITY</b>	-66,920	43330	-1.545	0.124695
<b>WEMA</b>	-138,400	43450	-3.186	0.001782
<b>ZENITH</b>	-25,100	45210	-0.555	0.579692

**Table 4: Presentation of Random Effect Model Results that Accounts for Both Individual and Time Effects (Twoways effects Model)**

Effects	Variance	Standard Dev	Shares	Theta (Lambda)
idiosyncratic	5809000000	76210	0.746	-
individual	1894000000	43520	0.243	0.5155
time	85800000	9263	0.011	0.1006
Total	-	-	-	0.08774
Variables	Coefficients	Standard Error	t-value	Pr(> t )
Intercept	130400	130400	1.6386	0.0000
PBT	0.00061736	26438	2.3351	0.02082
TA	-0.000011315	0.000013033	0.8682	0.38664
TL	0.0000079030	0.000003179	2.4860	0.01398
SHF	0.000099058	0.00012030	0.8234	0.41152

**Table 5: Presentation Hausman Test Results**

Chi square	Df	p-value
1193.6	4	0.0000

The specified models for POLS, LSDVM and REM from tables 1-3 respectively are given as follows:

$$AF = 120,970 + 0.00080588PBT - 0.000026096TA + 0.00000482TL + 0.00025419SHF \quad (41)$$

$$(R^2 = 0.2043, \bar{R}^2 = 0.2523, F = 13.6451, DF(4,155), P - value = 0.0000)$$

$$(se(\hat{\beta}_2) = 0.00027278, se(\hat{\beta}_3) = 0.000012963, se(\hat{\beta}_4) = 0.000035418, se(\hat{\beta}_5) = 0.00012246)$$

$$AF = 131,320 - 0.0000019PBT + 0.00000803TA + 0.000558TL + 0.0000247SHF \quad (42)$$

$$(R^2 = 0.4584, \bar{R}^2 = 0.3896, F = 6.236, DF(19,140), P - value = 0.0000)$$

$$(se(\hat{\beta}_2) = 0.00001413, se(\hat{\beta}_3) = 0.000003303, se(\hat{\beta}_4) = 0.00002772, se(\hat{\beta}_5) = 0.00001289)$$

$$AF = 130,400 + 0.000062PBT - 0.000011TA + 0.0000079TL + 0.000099SHF \quad (43)$$

$$(R^2 = 0.15611, \bar{R}^2 = 0.15611, F = 7.16815, DF(4,155), P - value = 0.0000)$$

$$(se(\hat{\beta}_2) = 0.00001201, se(\hat{\beta}_3) = 0.00002691, se(\hat{\beta}_4) = 0.000003255, se(\hat{\beta}_5) = 0.00002691)$$

The three specified models are statistically significant based on their P-values which are less than 0.05 while their coefficient of determination,  $R^2$  indicates that our exogenous variables explained 20.43%, 45.85% and 15.61% variation in the audit fees of Nigerian banks for the years under review respectively for POLS, LSDVM and REM. Meanwhile, the standard errors of regression coefficients for the POLS model are a bit higher than

that of LSDV and REM models. The POLS's standard errors were due to the inefficiency of POLS estimator as induced by the presence of heteroscedasticity which has not been taken care prior to the model's fitting. The fact that heteroscedasticity is present in the POLS estimator was established through the conduct of conditional LM test. The LM result is asymptotically chi squared distributed with Z-value of 7.1462 and a P-value of 0.0000000000000446, which is far less than the critical value of 0.05. This result prompts the rejection of our null hypothesis and thereby validates the presence of heteroscedasticity in the POLS residual.

The LSDVM seems to be a better model to explain the specified audit fees model as a result of its lower standard errors and higher coefficient of determination, and this is further confirmed by its preference based on Hausman test. Thus, model presented as equation (42) shall be our chosen model for the scientific fitting of audit fees across commercial banks in Nigeria and diaspora. This model, which is non heteroscedastic, presents a superior goodness of fit.

## CONCLUSION

Various results obtained in this work generally showed that the behaviours of the three estimators investigated for modeling audit fees vary due to violation of homoscedasticity assumption. The effect heteroscedasticity has on modeling panel data using these techniques for estimating audit fees model with violation of homoscedasticity assumption has been addressed.

Failure of the homoscedasticity assumption makes the POLS estimators to be biased and imprecise. For POLS to be accurately used in estimating the

parameters of panel data models, errors have to be independent and homoscedastic. These conditions are so atypical and mostly unrealistic in many real life situations that would have warranted the use of POLS for modeling panel data efficiently, hence the needs for developing a suitable LM test to ameliorate its ugly incidence.

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