TEACHING MATHEMATICAL MODELLING: CONNECTING TO CLASSROOM AND PRACTICE

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ABSTRACT: Teaching through the application of mathematical modelling has been concerned in recent years in Vietnam high schools. In which it clearly shows the application of the mathematical modelling with parts of life. This paper mentions to teaching and learning the AM-GM inequality and its application from theory and practice.
KEYWORDS: Mathematical modelling, mathematics teaching methods, the AM-GM Inequality, theoretical and practical.

1. INTRODUCTION

Mathematical education has been changing rapidly. The trends of educational reform have been encouraged by educators in order to innovate teaching methods towards the development of students' problem-solving skills. Likewise, changing is the preparation for an educated and skilled workforce outside the school. Educational innovation requires students to master their knowledge and build knowledge as they interact with the dynamic environment ([Eri09], [K+04]). Currently, a common goal in the mathematics education reform strategy is that students need to gain the capacity and ability to apply math in their daily lives. According to the International Congress on Mathematical Education in Monterrey, Mexico, July 6-13, 2008, the main idea of the educational perspective is to integrate models in teaching mathematics as a means to learn math and it is an important ability. The pedagogical issues for teaching and learning models as well as research in teaching and practice has attracted a strong interest. Teachers have discussed and been introduced a number of different approaches in teaching and learning mathematics in Vietnam high schools. Alternative approaches to classical methods include changing teaching methods in math class, such as the positive learning approach, etc.

Based on research by ([MG14]), [MSC07]), many countries which has been considering applying the model and approach to teaching and learning in two decades; ([S+07]), asserted that modeling activity develops and promotes effectively mathematical thinking and learning. They further explained that in order to develop mathematical competence it is necessary to set up a strong connection between mathematical knowledge and the practical context in which knowledge can be used. They maintained that model to promote and support the development of other mathematical abilities; ([WFH93]) argued that different research methods should be used in mathematics education, especially practical approach to solve problem. For example, mathematical exercises or difficult life situations are given, and students must do research to complete tasks.

- *Purpose of the study*

The main objective of the study was to teach the AM-GM Inequality and applications of theoretical and practical for the students.

- *Research questions*

This study to answer the following questions

1. The views of mathematical modelling in solving real-world problems?
2. How to teach the AM-GM Inequality and applications of practical for the students?

- *Research method*

This study use following methods: theoretical studies (analysis, synthesis, systematized, generalized, ...); Practical observation; Interviewing experts, education management, teachers and pupils.

2. RESULTS AND DISCUSSION

2.1. The perspective of the mathematical modelling in solving problems in the real world

As the author Eric ([Eri09]) has said, recent mathematical models are being intensively researched for solving mathematical problems, while Lesh & Zawojewski ([LZ07]) argued that mathematical modeling is the most important goal of mathematical education; ([LeH16], [LL03]) asserted that the difference between the model and the optical world is not simply the problem of identifying symbols; Rather, it depends on the accumulation of experience and the corresponding symbol patterns and phenomena.

In this paper, the author would like to express that modeling is one of the practical learning activities that represent and solve problems in the real world. Students learn how to select and use mathematical models, applying appropriate mathematical methods and tools to solve real world problems. We find that many fields of mathematics derive from attempts to describe and solve real world phenomena - through mathematical models. So, the question is: What is the connection between math and life? Why are these connections important? Why are they considered to be part of the mathematical modeling capacity? And how to encourage students to engage more in the mathematical connection and life?

We can say that the mathematical connection to solve real-world problems is the ability to recognize and use connections between mathematical ideas from life to create a mathematical model. According to [LeH16], if there is a good agreement between what is observed and the predicted model, it should be believed that the Maths system really capture important aspects of real-world situations.

According to Lingefjard ([Lin06]), we should support for students to show their mathematical ability and further develop their competence. According to him, there are some competencies that students should develop while modeling. These are abilities in solving problems, using daily knowledge in modeling, validating mathematical models, reflecting and refining models, explaining, describing and communicating ideas from Mathematical models.

In fact, the main goal of the teacher in teaching is not just to solve the problem, but more importantly to teach students how to think about problem solving, try to find out why the problem has arisen. Once the students do that, the students will have a clear understanding of the problem in their mind. And then, students can think easily about the different measures that can be applied to solve the problem and they can try to choose the best measure among the proposed measures.

Problem-solving strategies are the steps students take to address the problem they are facing to achieve their goals.

The modeling process continues with the construction of real world problems in mathematical terms. When this process is completed, the math problems can be solved by applying mathematical concepts and solutions.

According to Reeff, Zabal & Blech ([RZB06]), problem solving capacity is thinking and acting in situations where there is no common pattern. The understanding of problematic situations and the gradual shift based on planning and reasoning form the problem-solving process. According to Kaiser ([Kai04]), awareness of the goals of mathematics teaching influences the structure of mathematical lessons. It is not the imposition of mathematical knowledge but rather the search for knowledge in the content of teaching. Therefore, mathematical instruction needs to be addressed through many examples.

+ Students will understand mathematical relationships with everyday life, our society and science.

+ Students will gain the ability to solve math problems in their daily lives, in their learning environment and in science.

According to ([MG14]) about levels for mathematical modelling competency. In vietnam, we see students at Vietnam high schools, their modeling capacity in levels following (see Table 1).

Table 1. Mathematical model capacity

Level	Description
Level 1	Students understand only a part of the actual situation, but they can not structure and simplify that situation or can not find any link it to any mathematical ideas.
Level 2	After exploring the real situation, students find a logical model through structure and simplification, but they can not transform it into a math problem.
Level 3	Students can find not only a rational model, but also transform it into a mathematical problem, but they have not been yet able to work with it explicitly in mathematics.
Level 4	Students can find a math problem from real-world situations, work with math problems, and have mathematical results.
Level 5	Students may experience the mathematical modeling process and propose solutions to math problems related to the given situation.

2.2. AM-GM inequality and its application in the real world

In addition to the application of mathematics (such as the application of algorithms) in real-world contexts to describe concepts, mathematical models of issues such as real-world examples are increasingly important. In this modeling approach, complex mathematical problems are made based on model perception of real world and mathematical relationships ([Kai04]).

According to Eric ([Eri09]), the problems raised in research to solve mathematical problems are the concerns about the student's willingness to cooperate in solving problems in the increasingly complex world.... Mousoulides, Sriraman and Christou ([MSC07]) and Lesh & Doerr ([LD03]) argued that the need of students to encounter unfamiliar situations leads to the development of problem solving capacity and behavior. Teachers are encouraged to let their students seek out their knowledge, and the teacher may be surprised to discover how mathematics comes to students naturally. The issues in the real world will be described and developed into a number of learning projects.

- ***Framework theory of The AM-GM inequality***

+ For nonnegative real numbers a and b. We have $\frac{a+b}{2} \geq \sqrt{ab}$ and that equality holds if and only if $a = b$.

+ For 3 nonnegative real numbers a, b and c. We have $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ and that equality holds if and only if $a = b = c$.

+ For a set of nonnegative real numbers $a_1, a_2, ..., a_n$ the arithmetic mean is $AM = \frac{a_1 + a_2 + ... + a_n}{n}$

For a set of nonnegative real numbers $a_1, a_2, ..., a_n$ the Geometric mean is $GM = \sqrt[n]{a_1 a_2 ... a_n}$

The AM-GM Inequality

For n nonnegative real numbers. Then we have

$$\frac{a_1 + a_2 + ... + a_n}{n} \geq \sqrt[n]{a_1 a_2 ... a_n}$$

and that equality holds if and only if $a_1 = a_2 = ... = a_n$.

Remark 1. The AM-GM for two positive numbers can be a useful tool in examining some optimization problems.

+ We are well known that for rectangles with a fixed perimeter, the maximum area is given by a square having that perimeter.

+ If rectangles with a fixed area, the minimum perimeter is given by a square having that area.

The full inequality is an extension of this idea to n dimensions.

Remark 2. The AM–GM inequality states that only the n-cube has the smallest sum of lengths of edges connected to each vertex amongst all n-dimensional boxes with the same volume

- ***Some examples of real life problems***

According to ([HT17]) state that, the competency model for a student's mathematical to be set is: their profound readiness to perform through all parts of a mathematical modeling process in real-world situations life. Hence, we suggest that three problems are adaptive for teaching AM-GM inequality and its application in the real world. The examples is several angles of looking at the AM-GM Inequality of introducing the teaching of mathematical modelling to the students at the high schools in Vietnam. As well, the benefits of teaching and learning mathematical modelling are discussed.

Problem 1. ([Sta10]). In New York City, as Broad way crisscrosses major avenues, the resulting blocks are cut like triangles, and buildings have been built on these shapes; one such building is the triangularly shaped Flatiron Building. This structure becomes a landmark icon. As Jodidio ([Jod09]), Designers have made houses in Norway using triangular themes.

+ *Modeling Real World Problem*

They have real life problems is a triangle land in New York City, this land has fixed perimeter and they want to build a Flatiron building is the maximum area!

+ *Mathematical Model*

Maximun area of a triangle with fixed perimeter

Given a triangle ABC with side of length a, b and c with a fixed perimeter $P = \frac{a+b+c}{2}$. We using Heron's formula is $A = \sqrt{p(p-a)(p-b)(p-c)}$.

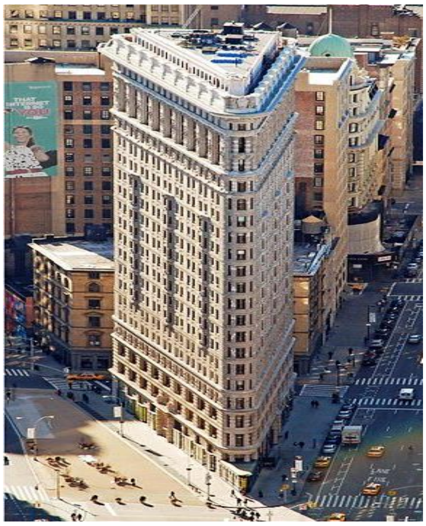
So p is a constan and the issue of maximum are is in the product $(p-a)(p-b)(p-c)$.

The application of the AM-GM inequality for three positive number $p-a, p-b, p-c$. We have

$$p(p-a)(p-b)(p-c)$$

$$\leq p\left(\frac{p-a+p-b+p-c}{3}\right)^3 = \frac{p^4}{27}$$

Hence, The maximum area is $A = \frac{p^2}{3\sqrt{3}}$. The maximum occurs if and only if $p-a = p-b = p-c$. This means $a = b = c$.



Practical model : Flatiron Building (New York)

Figure 1. The practical model

Problem 2. In packaging a Thai Nguyen tea gift box in a can the shape of cylinder. What if the design was based on minimizing the art paper used to make the can? This would mean that for a fixed volume V the shape of the can (e.g. the radius and the height) would be determined by the minimum surface area for the can. What is the relationship between the radius and the height in order to minimize the surface area for a fixed volume? and If art paper costs VND 60 per square cm, What is the minimum expense?

+ Modeling Real World Problem

Thai Nguyen tea gift box is shape of cylinder, it is a fixed volume V. The design was based on minimizing the art paper used to make the can? From that, what is the minimum expense?

Practical model : Thai Nguyen tea gift box

Figure 2. The practical model

+ Mathematical Model

We have $V=\pi r^2 h$, deduce $A=2\pi r^2+2\pi rh$. What is the minimun A? Where, fixed volume V.

So $h=\dfrac{V}{\pi r^2}$, deduce we have

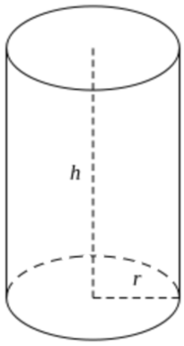


Figure 3. The mathematical model

$\mathrm{A}=2\pi \mathrm{r}^2+2\pi r\left(\dfrac{V}{\pi r^2}\right)$. Equivalently we can write $\mathrm{A}=2\pi \mathrm{r}^2+\dfrac{V}{r}+\dfrac{V}{r}$. Where, we noticed that $2\pi \mathrm{r}^2,\dfrac{V}{r},\dfrac{V}{r}$ are three positive numbers. Hence, by the AM–GM inequality we have

$$A\geq 3\sqrt[3]{2\pi r^2\left(\frac{V}{r}\right)\left(\frac{V}{r}\right)}=3\sqrt[3]{2\pi V^2}.$$

Deduce $\min\{A\}=3\sqrt[3]{2\pi V^2}$ if and only if $2\pi \mathrm{r}^2=\dfrac{V}{r}=\dfrac{V}{r}$ or $2\pi \mathrm{r}^3=\pi \mathrm{r}^2 h$ or that is $2r=h$.

Then, we deduce the minimum expense on production of Thai Nguyen tea gift box.

$$\cos\{\mathrm{A}\}=\mathrm{VND}\,60.3\sqrt[3]{2\pi V^2}$$

Problem 3. The Nguyen Ninh shop at 11 Hang Than street in Ha Noi capital in Vietnam. The shop sells traditional sheet cake popular at wedding in Vietnam.

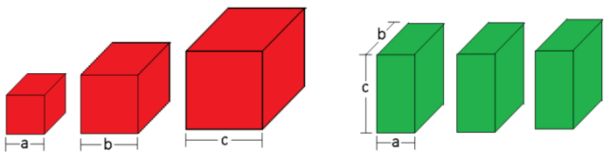


Figure 4. The cake problem

+ Modeling Real World Problem

Here, the shop sells its products in two different bags:

The first bag of the cake consists of the 3 red cake cubes, where they are of side length $a<b<c$.

The second bag of the cake consists of the 3 green cake cuboids are identical with dimensions a,b,c as shown above.

Which option would give you more cake? Note that, *the two bags are the same price.*

+ Mathematical Model

Firt Bag: $V\{\text{red cake}\}=\mathrm{a}^3+b^3+c^3$

Second Bag: $V\{\text{green cake}\}=\text{abc+abc+abc=3abc}$

The AM–GM inequality we have

$$V\{\text{red cake}\} \geq 3\sqrt[3]{a^3b^3c^3} = 3abc = V\{\text{green cake}\}$$

Hence, red bag is more cake than green bag.

3. CONCLUSIONS

It is clear to show that an important way to engage students in their studies is to ensure the problems that teachers teach them. The teaching strategies that are adjusted through the teaching process will link the lessons with the real experiences of the students. The effectiveness of teaching is not confirmed through what students learn by passing on but by what students can do in their life.

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