VARIANCE COMPONENTS OF MODELS OF SUDOKU SQUARE DESIGN

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ABSTRACT: This study aimed at obtaining variance component estimators for all effects of Sudoku square models. The analysis of variance (ANOVA) method was used for the derivation of the variance components for the four Sudoku models.
KEYWORD: Sudoku Square design, Variance components, ANOVA.

1. INTRODUCTION

Variance components are different sources to the variation in an observation and are commonly estimated in the course of determining sampling design, establishing quality control procedure or in statistical genetic, estimating heritability and genetic correlation. The estimators used mostly often have been the analysis of variance (ANOVA) estimator which are obtained by equating observed and expected mean squares from analysis of variance ([Sea71, SCC92]). The ANOVA estimator have properties, especially if the data are balanced (that is, if they equal number of observation in corresponding rows, or column, or rowblock or sub-square) which are unbiased, minimum variance among all unbiased estimators. Hirotsu ([Hir61]) and Blischke (Bli66, Bli68]), and study the variance components under normality assumptions for sampling variances of the resulting estimator. Anderson ([And78]), Anderson & Crump ([AC67]), Bainbride ([Bai63]) gave some advantages of ANOVA method over other methods in the case of special designs planned for estimating variance components, as it is the easy to compute, has minimum variance and data need not to be normal. Searle ([Sea88]) studied variance components of balanced data under the properties of ANOVA method of estimation. Valencia et al. ([VDM07]) estimated variance components, for total milk production, cumulated milk production to 120 days and for lactation length in Saanen goats in Mexico using data from one herd .Ewa et al. ([E+17]) used Analysis of variance (ANOVA) method to obtained the Estimates of variance components some traits of BC2 population of Cassava.

Sudoku is the abbreviation of the japanese longer phrase “suji wa dokushin ni kagiru”which means the digit must occur only once ([Ber07, GL06]). Sudoku puzzle is a very popular game, the objective of the game is to complete a 9×9 grid with digits from 1 to 9. Each digit must appear once only in each row, each column and each of the 3×3 boxes see Bailey et al.([BCC08]).

Sudoku square design consists of treatments that are arranged in a square array such that each row, column or sub-square of the design contains each of the treatments only once. Hui-Dong and Ru-Gen, ([HR08]) defined Sudoku square design as an experimental design with p^2 experimental units that are divided into p rows, p columns, and p boxes (i.e., each box contains p experimental units with 1 through p treatments). In this design each treatment has p replications. They further explained how the design and randomization of Sudoku Square are done see Hui-Dong and Ru-Gen ([HR08]) for the details. However, the Sudoku design presented by Hui-Dong and Ru-Gen ([HR08]) does not contain row-blocks or column-blocks effects. Subramani and Ponnuswamy ([SP09]) extended the design to include row-blocks and column-blocks effects in which they called Sudoku designs-Type I and further proposed three other Sudoku square models.

This paper proposed variance component estimators for the various effects for Sudoku square design models presented by Subramani and Ponnuswamy ([SP09]) using Analysis of Variance (ANOVA) method.

2. METHODOLOGY

2.1 ANOVA Method of Variance Components

The procedure consists of equating the expected mean square to their observed value in the analysis of variance table and solving for the variance components ([Sea88]). Mean square of effects of the Sudoku models I-IV can be obtained as sum of squares divided by degrees of freedom. They are based on using means, variances and covariances of random effects of the models to all random factors.

This gave an indication that expectations of all effects of each factor are assumed to have zero mean. If assuming x is a random variable then the $E(x) = 0$, has constant variance and zero covariance with other. In addition, all effects of random factor assumed to have zero covariance with those of each other factor with the error terms. And error terms all have expected value zero and same variance and zero covariance with each other. Some other expectations are given in section 2.2.

2.2 Expectation of values

In this study, the following expected values will be of importance in the derivation of variance components for the various Sudoku models.

$$E[e_{i,j(k,l,p,q)}] = 0,$$
$$cov(\overline{T}_k e_{i,j(k,l,p,q)}) = cov(T_k e_{i,j(k,l,p,q)}) = cov(RB_i e_{i,j(k,l,p,q)}) = 0$$
$$cov(CB_j e_{i,j(k,l,p,q)}) = cov(R_l e_{i,j(k,l,p,q)}) = cov(C_p e_{i,j(k,l,p,q)}) = cov(s_q e_{i,j(k,l,p,q)}) = 0$$
$$\mathrm{E}[Y_{ij(k,l,p,q)}] = E[\bar{Y}_{...}] = E[\overline{T_k}] = E[\overline{RB_l}] = E[\overline{CB_J}] = E[\overline{R_l}] = E[\overline{C_p}] = E[\overline{s_q}] = \mu$$
$$E[Y_{i,j}^2] = var[Y_{ij}] + (E[Y_{ij}])^2$$

2.3 Sudoku square design models and Sum of squares for various effects

Model Type I

The row, column, and treatment effects are assumed as in the latin square, also in addition to the assumption, row-block, column-block and square effects.

$$Y_{ij(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + C_p + \gamma_l + s_q + e_{i,j(k,l,p,q)} \quad (1)$$

where $i,j = 1 \cdots m \quad k,l,p,q = 1 \cdots$

$\mu =$ Grand mean

$\alpha_i = ith$ Row block effect

$\beta_j = jth$ Column block effect

$\tau_k = kth$ treatment effect

$\gamma_l = l$th Row effect

$c_p =$ pth Column effect

$s_q = qth$ square effect

$e_{i,j(k,l,p,q,r)}$ = is the error component assumed to have mean zero and constant variance σ^2.

$G = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}$ and $N = m^4$ $\qquad \mu = G/N$

$$TSS = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - m^4\,\bar{Y}_{...}^2$$

$$SStreat = \sum_{k=1}^{m^2} m^2\overline{T_k}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSrowblock = \sum_{i=1}^{m} m^3\overline{RB_\iota}^2 - m^4\,\bar{Y}_{...}^2$$

$$SScolblock = \sum_{j=1}^{m} m^3\overline{CB_J}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSrow = \sum_{l=1}^{m^2} m^2\overline{R_l}^2 - m^4\,\bar{Y}_{...}^2$$

$$SScol = \sum_{p=1}^{m^2} m^2\overline{C_p}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSsquare = \sum_{q=1}^{m^2} m^2\overline{s_q}^2 - m^4\,\bar{Y}_{...}^2$$

$$SS_{error} = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - \sum_{i=1}^{m} m^3\overline{RB_\iota}^2 - \sum_{j=1}^{m} m^3\overline{CB_J}^2 - \sum_{k=1}^{m^2} m^2\overline{T_k}^2 - \sum_{l=1}^{m^2} m^2\overline{R_l}^2 - \sum_{p=1}^{m^2} m^2\overline{C_p}^2 - \sum_{q=1}^{m^2} m^2\overline{s_q}^2 + 5m^4E[\bar{Y}_{...}^2]$$

([SP09])

Model Type II

The model assumed that row effects are nested in the row block effect and the column effects are nested in the column block effects.

$$Y_{ij(k,l,p,q)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + C(\beta)_{p(j)} + s_q + e_{i,j(k,l,p,q)} \quad (2)$$

where $i,j\,l,p = 1 \cdots m \quad k,q = 1 \cdots m^2$

$\mu =$ Grand mean

$\alpha_i = ith$ Row block effect

$\beta_j = jth$ Column block effect

$\tau_k = kth$ treatment effect

s_q =qth square effect

$\gamma(\alpha)_{l(i)}$ = lth Row effect nested in ith row block effect

$c(\beta)_{p(j)}$= pth Column effect nested in jth column block effect

$e_{i,j(k,l,p,q,r)}$ = is the error component assumed to have mean zero and constant variance σ^2.

$G = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}$ and $N = m^4$ $\qquad \mu = G/N$

$$TSS = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - m^4\,\bar{Y}_{...}^2$$

$$SStreat = \sum_{k=1}^{m^2} m^2\overline{T_k}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSrowblock = \sum_{i=1}^{m} m^3\overline{RB_\iota}^2 - m^4\,\bar{Y}_{...}^2$$

$$SScolblock = \sum_{j=1}^{m} m^3\overline{CB_j}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSrowwithinRb = \sum_{l=1}^{m^2} m^2\bar{R_l}^2 - \sum_{i=1}^{m} m^3\overline{RB_\iota}^2$$

$$SScolwithinCb = \sum_{p=1}^{m^2} m^2\bar{c_p}^2 - \sum_{j=1}^{m} m^3\overline{CB_j}^2$$

$$SSsquare = \sum_{q=1}^{m^2} m^2\bar{s_q}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSerror = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - \sum_{l=1}^{m}\sum_{i=1}^{m} m^2\bar{R}_{l(i)}^2 - \sum_{l=1}^{m}\sum_{i=1}^{m} m^2\bar{c}_{p(j)}^2 - \sum_{k=1}^{m^2} m^2\bar{T_k}^2 - \sum_{q=1}^{m^2} m^2\bar{s_q}^2 + 3m^4\,\bar{Y}_{...}^2$$

([SP09])

Model Type III

The model assumes that the horizontal square effects are nested in the row block and vertical Square effects are nested in the column block effects.

$$Y_{ij(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma_l + c_p + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + e_{i,j(k,l,p,q,r)} \quad (3)$$

where $i,j,q,r = 1 \cdots m \quad k,l,p = 1 \cdots m^2$

$\mu =$ Grand mean effect

$\alpha_i = ith$ Row block effect

$\beta_j = jth$ Column block effect

$\tau_k = kth$ treatment effect

s_q =qth square effect

$\gamma_l = l$th Row effect

$c_p =$ pth Column effect

$s(\alpha)_{q(i)} = qth$ Horizontal square effect nested in ith Row block effect

$\pi(\beta)_{r(j)} = rth$ vertical square effect nested in the jth column block effect

$e_{i,j(k,l,p,q,r)}$ = is the error component assumed to have mean zero and constant variance σ^2.

$$G = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j} \quad and\ N = m^4 \qquad \mu = G/N$$

$$TSS = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - m^4\,\bar{Y}_{...}^2$$

$$SStreat = \sum_{k=1}^{m^2} m^2\overline{T_k}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSrowblock = \sum_{i=1}^{m} m^3\overline{RB_\iota}^2 - m^4\,\bar{Y}_{...}^2$$

$$SScolblock = \sum_{j=1}^{m} m^3\overline{CB_j}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSrow = \sum_{l=1}^{m^2} m^2\bar{R_l}^2 - m^4\,\bar{Y}_{...}^2$$

$$SScol = \sum_{p=1}^{m^2} m^2\overline{C_p}^2 - m^4\,\bar{Y}_{...}^2$$

$$SShwrowblock = \sum_{i=1}^{m}\sum_{q=1}^{m} m^2\bar{s}_{q(i)}^2 - \sum_{i=1}^{m} m^3\overline{RB_\iota}^2$$

$$SSvwrowblock = \sum_{j=1}^{m}\sum_{r=1}^{m} m^2\bar{\delta}_{r(j)}^2 - \sum_{j=1}^{m} m^3\overline{cB_j}^2$$

$$SSerror = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - \sum_{l=1}^{m^2} m^2\bar{R_l}^2 - \sum_{p=1}^{m^2} m^2\bar{c}_p^2 - \sum_{k=1}^{m^2} m^2\bar{T}_k^2 - \sum_{i=1}^{m}\sum_{q=1}^{m} m^2\bar{s}_{q(i)}^2 - \sum_{j=1}^{m}\sum_{r=1}^{m} m^2\bar{\delta}_{r(j)}^2 + 4m^4\bar{Y}_{...}^2$$

([SP09])

Model Type IV

In the model below, it is assumed that the row effects and horizontal square effects are nested in the row block and the column effects and the vertical square effects are nested in the column block effects

$$Y_{ij(k,l,p,q,r)} = \mu + \alpha_i + \beta_j + \tau_k + \gamma(\alpha)_{l(i)} + c(\beta)_{p(j)} + s(\alpha)_{q(i)} + \pi(\beta)_{r(j)} + e_{i,j(k,l,p,q,r)}$$

Where

$i,j,l,p,q,r = 1 \cdots m \ \ and \ \ k = 1 \cdots m^2$ (4)

$\mu =$ Grand mean effect

$\alpha_i = ith$ Row block effect

$\beta_j = jth$ Column block effect

$\tau_k = kth$ treatment effect

s_q =qth square effect

$\gamma(\alpha)_{l(i)} = l$th Row effect nested in ith row block effect

$c(\beta)_{p(j)} =$ pth Column effect nested in jth column block effect

$s(\alpha)_{q(i)} = qth$ Horizontal square effect nested in ith Row block effect

$\pi(\beta)_{r(j)} = rth$ vertical square effect nested in the jth column block effect

$e_{i,j(k,l,p,q,r)}$ = is the error component assumed to have mean zero and constant variance σ^2.

$$G = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j} \quad and\ N = m^4 \qquad \mu = G/N$$

$$TSS = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - m^4\,\bar{Y}_{...}^2$$

$$SStreat = \sum_{k=1}^{m^2} m^2\overline{T_k}^2 - m^4\,\bar{Y}_{...}^2$$

$$SSrowblock = \sum_{i=1}^{m} m^3 \overline{RB_\iota}^2 - m^4\, \bar{Y}_{\cdots}^2$$
$$SScolblock = \sum_{j=1}^{m} m^3 \overline{CB_J}^2 - m^4\, \bar{Y}_{\cdots}^2$$
$$SShwrowblock = \sum_{i=1}^{m}\sum_{q=1}^{m} m^2 \bar{s}_{q(i)}^2 - \sum_{i=1}^{m} m^3 \overline{RB_\iota}^2$$
$$SSvwrowblock = \sum_{j=1}^{m}\sum_{r=1}^{m} m^2 \bar{\delta}_{r(j)}^2 - \sum_{j=1}^{m} m^3 \overline{cB_j}^2$$
$$SSrowwithinRb = \sum_{l=1}^{m^2} m^2 \bar{R_l}^2 - \sum_{i=1}^{m} m^3 \overline{RB_\iota}^2,$$
$$SScolwithinCb = \sum_{p=1}^{m^2} m^2 \bar{c_p}^2 - \sum_{j=1}^{m} m^3 \overline{CB_J}^2$$
$$SSerror = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 + \sum_{i=1}^{m} m^3 \overline{RB_\iota}^2 + \sum_{j=1}^{m} m^3 \overline{CB_J}^2 - \sum_{l=1}^{m^2} m^2 \bar{R_l}^2 - \sum_{p=1}^{m^2} m^2 \bar{c}_p^2 - \sum_{k=1}^{m^2} m^2 \bar{T}_k^2 - \sum_{i=1}^{m}\sum_{q=1}^{m} m^2 \bar{s}_{q(i)}^2 - \sum_{j=1}^{m}\sum_{r=1}^{m} m^2 \bar{\delta}_{r(j)}^2 + 2m^4 \bar{Y}_{\cdots}^2$$

([SP09])

3.3 Test of hypothesis of variance component

The hypothesis that we are interested in testing are
$H_0\colon \sigma_{effect}^2 = 0$
Testing of hypothesis about the variance components
Hence the test statistic is of the form
$$F_0 = \frac{MSeffect}{MSerror}$$
H_0 is rejected if $F_0 > F_{dfeffect,\ dferror,1-\propto}$

([Sea88])

4. RESULTS

4.1 Derivation of variance components of Sudoku square design models

Sudoku Square design model Type I
From equation (1) we have
$\alpha_i = (\alpha_1 \cdots \alpha_m)' \sim N(0, \sigma_\alpha^2 I)$
$\beta_j = (\beta_1 \cdots \beta_m)' \sim N(0, \sigma_\beta^2 I)$
$\tau_k = (\tau_1 \cdots \tau_{m^2})' \sim N(0, \sigma_\tau^2 I)$
$\gamma_l = (\gamma_1 \cdots \gamma_{m^2})' \sim N(0, \sigma_\gamma^2 I)$
$\gamma_l = (\gamma_1 \cdots \gamma_{m^2})' \sim N(0, \sigma_\gamma^2 I)$
$C_p = (C_1 \cdots C_{m^2})' \sim N(0, \sigma_C^2 I)$
$s_q = (s_1 \cdots s_{m^2})' \sim N(0, \sigma_s^2 I)$
$e_{i,j(k,l,p,q)} = (e_1 \cdots e_{m^4})' \sim N(0, \sigma^2 I)$

Hence the variance of response value ($Y_{ij(k,l,p,q)}$) is $\mathrm{Var}(Y_{ij(k,l,p,q)}) = \sigma_\alpha^2 + \sigma_\beta^2 + \sigma_\tau^2 + \sigma_\gamma^2 + \sigma_C^2 + \sigma_s^2 + \sigma^2$
$\sigma_\alpha^2, \sigma_\beta^2, \sigma_\tau^2, \sigma_\gamma^2, \sigma_C^2, \sigma_s^2$, and σ^2 are called variance components of Sudoku square design model Type I. The derivation of the variance components of the Sudoku square design model Type I begins by taking the expectation of mean sum of squares of each of the sources of variation.
$$var[Y_{ij(k,l,p,q)}] = E[Y_{i,j}^2] - (E[Y_{ij}])^2$$
$$E[Y_{i,j}^2] = \sigma_\alpha^2 + \sigma_\beta^2 + \sigma_\tau^2 + \sigma_\gamma^2 + \sigma_C^2 + \sigma_s^2 + \sigma^2 + \mu^2$$
$$E[\bar{Y}_{\cdots}^2] = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^4} + \mu^2$$
$$E\left[\bar{T_k}^2\right] = var[\bar{T_k}\,] + (E[\bar{T_k}\,])^2$$
$$var[\bar{T_k}\,] = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \sigma_\tau^2 + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^4}$$
$$E\left[\bar{T_k}^2\right] = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \sigma_\tau^2 + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2$$
$$E[SStreatmt] = E\left[\sum_{k=1}^{m^2} m^2 \bar{T_k}^2 - m^4\, \bar{Y}_{\cdots}^2\right]$$
$$= m^2 \sum_{k=1}^{m^2} E[\,\bar{T_k}^2] - m^4 E[\bar{Y}_{\cdots}^2]$$
$$= m^4 \left[\frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \sigma_\tau^2 + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2\right] - [m^4[\frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^4} + \mu^2]]$$
$$E[SStreatmt] = m^2(m^2-1)\sigma_\tau^2 + (m^2-1)\sigma^2$$
$MSStreatmt$ is obtained by dividing $[SStreatmt]$ by degree of freedom of treatments
$$E[MStreatmt] = \frac{E[SStreatmt]}{(m^2-1)}$$
$$E[MSStreatmt] = m^2\sigma_\tau^2 + \sigma^2 \qquad (5)$$
Similarly
$$E[MSSrow] = m^2\sigma_\gamma^2 + \sigma^2 \qquad (6)$$
$$E[MSScol] = m^2\sigma_C^2 + \sigma^2 \qquad (7)$$
$$E[MSSsquare] = m^2\sigma_s^2 + \sigma^2 \qquad (8)$$
$$E[SSrowblock] = E\left[\sum_{i=1}^{m} m^3 \overline{RB_\iota}^2 - m^4\, \bar{Y}_{\cdots}^2\right]$$
$$= \sum_{i=1}^{m} m^3 E[\overline{RB_\iota}^2] - m^4\, E[\bar{Y}_{\cdots}^2]]$$
$$E\left[\overline{RB_\iota}^2\right] = var[\overline{RB_\iota}\,] + (E[\overline{RB_\iota}])^2$$
$$= \sigma_\alpha^2 + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2$$
$$E[SSrowblock] = m^4\left[\sigma_\alpha^2 + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \right.$$
σs2m2+σ 2m2+μ2−[m4[σα2m +σβ 2m+στ2m2+σγ2m2+σC2m2+σs2m2+σ 2m4 $+\mu^2]$
$$E[SSrowblock] = m^3(m-1)\sigma_\alpha^2 + (m-1)\sigma^2$$
Dividing the result of $E[SSrowblock]$ by its degree of freedom i.e $(m-1)$ we have
$$E[MSSrowblock] = m^3\sigma_\alpha^2 + \sigma^2 \qquad (9)$$
Similarly,

$$E[MSScolblock] = m^3\sigma_\beta^2 + \sigma^2 \quad (10)$$

To obtain $E[MSE]$ we take the expectation of SS_{error} divided by error degree of freedom

$$E[SSerror] = E\left[\sum_{i=1}^{m^2}\sum_{j=1}^{m^2} Y_{i,j}^2 - \sum_{i=1}^{m} m^3\overline{RB_i}^2 - \sum_{j=1}^{m} m^3\overline{CB_j}^2 - \sum_{k=1}^{m^2} m^2\overline{T_k}^2 - \sum_{l=1}^{m^2} m^2\bar{R_l}^2 - \sum_{p=1}^{m^2} m^2\overline{C_p}^2 - \sum_{q=1}^{m^2} m^2\bar{s_q}^2 + 5m^4E[\bar{Y}_{..}^2]\right]$$

$$= \sum_{i=1}^{m^2}\sum_{j=1}^{m^2} E[Y_{i,j}^2] - \sum_{i=1}^{m} m^3E[\overline{RB_i}^2] - \sum_{j=1}^{m} m^3E[\overline{CB_j}^2] - \sum_{k=1}^{m^2} m^2E\left[\overline{T_k}^2\right] - \sum_{l=1}^{m^2} m^2E[\bar{R_l}^2] - \sum_{p=1}^{m^2} m^2E\left[\overline{C_p}^2\right] - \sum_{q=1}^{m^2} m^2E[\bar{s_q}^2] + 5m^4\,E[\bar{Y}_{..}^2]$$

We obtain the expectation of the each of the squares in the SSerror of model I

$$E\left[\overline{RB_i}^2\right] = var[\overline{RB_i}\,] + (E[\overline{RB_i}])^2 = \sigma_\alpha^2 + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^3} + \mu^2$$

$$E\left[\overline{CB_j}^2\right] = var[\overline{CB_j}\,] + (E[\overline{CB_j}])^2 = \frac{\sigma_\alpha^2}{m} + \sigma_\beta^2 + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^3} + \mu^2$$

$$E\left[\overline{T_k}^2\right] = var[\overline{T_k}\,] + (E[\overline{T_k}\,])^2 = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \sigma_\tau^2 + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2$$

$$E\left[\bar{R_l}^2\right] = var[\bar{R_l}\,] + (E[\bar{R_l}\,])^2 = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \sigma_\gamma^2 + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2$$

$$E[\bar{c_p}^2] = var[\bar{c_p}\,] + (E[\bar{c_p}\,])^2 = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \sigma_C^2 + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2$$

$$E[\bar{s_q}^2] = var[\bar{s_q}\,] + (E[\bar{s_q}\,])^2 = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \sigma_s^2 + \frac{\sigma^2}{m^2} + \mu^2$$

$$E[Y_{i,j}^2] = \sigma_\alpha^2 + \sigma_\beta^2 + \sigma_\tau^2 + \sigma_\gamma^2 + \sigma_C^2 + \sigma_s^2 + \sigma^2 + \mu^2$$

$$E[\bar{Y}_{..}^2] = \frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^4} + \mu^2$$

Substituting the expected values of the squares into SSerror of model I, then we have

$$= m^4[\sigma_\alpha^2 + \sigma_\beta^2 + \sigma_\tau^2 + \sigma_\gamma^2 + \sigma_C^2 + \sigma_s^2 + \sigma^2 + \mu^2]$$
$$-m^4[\sigma_\alpha^2 + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^3} + \mu^2]$$
$$-m^4[\frac{\sigma_\alpha^2}{m} + \sigma_\beta^2 + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^3} + \mu^2]$$
$$-m^4[\frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \sigma_\tau^2 + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2]$$
$$-m^4[\frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \sigma_\gamma^2 + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2]$$
$$-m^4[\frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \sigma_C^2 + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2]$$
$$-m^4[\frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \sigma_s^2 + \frac{\sigma^2}{m^2} + \mu^2]$$
$$+5m^4[\frac{\sigma_\alpha^2}{m} + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_C^2}{m^2} + \frac{\sigma_s^2}{m^2} + \frac{\sigma^2}{m^4} + \mu^2]$$

$$E[SSerror] = [m^4 - 4m^2 - 2m + 5]\sigma^2$$

We divide $E[SSerror]$ by degree of freedom of error to obtain $E[MSerror]$

$$E[MSerror] = \frac{[m^4-4m^2-2m+5]\sigma^2}{(m-1)[(m+1)(m^2-3)-2]}$$

$(m-1)[(m+1)(m^2-3)-2]$ obtained by Subramani and Ponnuswamy ([SP09])

$$E[MSerror] = \sigma^2 \quad (11)$$

The estimates $\hat{\sigma}^2, \hat{\sigma}_\alpha^2, \hat{\sigma}_\beta^2, \hat{\sigma}_\tau^2, \hat{\sigma}_\gamma^2, \hat{\sigma}_C^2$ and $\hat{\sigma}_s^2$ of the variance components $\sigma^2, \sigma_\alpha^2, \sigma_\beta^2, \sigma_\tau^2, \sigma_\gamma^2, \sigma_c^2$ and σ_s^2 are computed from the equations (5) – (10).

Variance components estimators for various effects for Sudoku square model I are as follow

$$\hat{\sigma}^2 = MSerror \quad (12)$$
$$\hat{\sigma}_\alpha^2 = \frac{1}{m^3}(MSSrowblock - MSerror) \quad (13)$$
$$\hat{\sigma}_\beta^2 = \frac{1}{m^3}(MSScolblock - MSerror) \quad (14)$$
$$\hat{\sigma}_\tau^2 = \frac{1}{m^2}(MStreatmet - MSerror) \quad (15)$$
$$\hat{\sigma}_\gamma^2 = \frac{1}{m^2}(MSrow - MSerror) \quad (16)$$
$$\hat{\sigma}_C^2 = \frac{1}{m^2}(MScol - MSerror) \quad (17)$$
$$\hat{\sigma}_s^2 = \frac{1}{m^2}(MSsquare - MSerror) \quad (18)$$

The hypothesis that we are interested in testing are $H_0: \sigma_{effect}^2 = 0$

Testing of hypothesis about the variance components

Hence the test statistic is of the form

$$F_0 = \frac{MSeffect}{MSerror}$$

H_0 is rejected if $F_0 > F_{dfeffect,\ dferror,1-\alpha}$

Sudoku Square design model Type II

$\gamma(\alpha)_{l(i)} = (\gamma(\alpha)_{1(1)} \cdots \gamma_{m(m)})' \sim N(0, \sigma^2_{\gamma(\alpha)} I)$

$c(\beta)_{p(j)} = (c(\beta)_{1(1)} \cdots c(\beta)_{m(m)})' \sim N(0, \sigma^2_{c(\beta)} I)$

Hence the variance of response value ($Y_{ij(k,l,p,q)}$) is Var$(Y_{ij(k,l,p,q)}) = \sigma^2_\alpha + \sigma^2_\beta + \sigma^2_\tau + \sigma^2_{\gamma(\alpha)} + \sigma^2_{c(\beta)} + \sigma^2_s + \sigma^2$

$\sigma^2_\alpha, \sigma^2_\beta, \sigma^2_\tau, \sigma^2_{\gamma(\alpha)}, \sigma^2_{c(\beta)}, \sigma^2_s$, and σ^2 are called variance components of Sudoku square design Model Type II. The derivation of the variance components $\sigma^2_\alpha, \sigma^2_\beta, \sigma^2_\tau$, σ^2_s and their respective estimates have been carried out in model Type I and are the same with through the four models.

To derive the expectation of $SSrowwithinRb$

$E[SSrowwithinRb]$

$$= E\left[\sum_{l=1}^{m^2} m^2 \bar{R}^2_{l(i)} - \sum_{i=1}^{m} m^3 \overline{RB_\iota}^2\right]$$

$$= \sum_{l=1}^{m^2} m^2 E[\bar{R}^2_{l(i)}] - \sum_{i=1}^{m} m^3 E[\overline{RB_\iota}^2]$$

$E[\bar{R}^2_{l(i)}] = var[\bar{R}_{l(i)}] + (E[\bar{R}_{l(i)}])^2$

$$= \sigma^2_\alpha + \frac{\sigma^2_\beta}{m} + \frac{\sigma^2_\tau}{m^2} + \sigma^2_{\gamma(\alpha)} + \frac{\sigma^2_{c(\beta)}}{m^2} + \frac{\sigma^2_s}{m^2} + \frac{\sigma^2}{m^2} + \mu^2$$

$E\left[\overline{RB_\iota}^2\right] = var[\overline{RB_\iota}] + (E[\overline{RB_\iota}])^2$

$$= \sigma^2_\alpha + \frac{\sigma^2_\beta}{m} + \frac{\sigma^2_\tau}{m^2} + \frac{\sigma^2_{\gamma(\alpha)}}{m^2} + \frac{\sigma^2_{c(\beta)}}{m^2} + \frac{\sigma^2_s}{m^2} + \frac{\sigma^2}{m^3} + \mu^2$$

$E[SSrowwithinRb]$

$$= m^4\left[\sigma^2_\alpha + \frac{\sigma^2_\beta}{m} + \frac{\sigma^2_\tau}{m^2} + \sigma^2_{\gamma(\alpha)} + \frac{\sigma^2_{c(\beta)}}{m^2} + \frac{\sigma^2_s}{m^2} + \frac{\sigma^2}{m^2} + \mu^2\right]$$

$$-m^4\left[\sigma^2_\alpha + \frac{\sigma^2_\beta}{m} + \frac{\sigma^2_\tau}{m^2} + \frac{\sigma^2_{\gamma(\alpha)}}{m^2} +\right.$$

σcβ2m2+σs2m2+σ 2m3+μ2

$$= (m^4 - m^2)\sigma^2_{\gamma(\alpha)} + (m^2 - m)\,\sigma^2$$

$$= m(m-1)[m(m+1)\sigma^2_{\gamma(\alpha)} + \sigma^2]$$

The above result $E[SSrowwithinRb]$ is divided by the degree of freedom which is $m(m-1)$

$E[MSSrowwithinRb]$

$$= \frac{m(m-1)[m(m+1)\sigma^2_{\gamma(\alpha)} + \sigma^2]}{m(m-1)}$$

$$= m(m+1)\sigma^2_{\gamma(\alpha)} + \sigma^2 \quad (19)$$

Similarly, the

$$E[SScolwithincb] = m(m+1)\sigma^2_{c(\beta)} + \sigma^2 \quad (20)$$

To obtain $E[MSerror]$ for model Type II, we take the expectation of $SSerror$ divided by error degree of freedom. The same procedures is followed as in model type I.

Variance components estimators for various effects for Sudoku square model II are as follow

$\hat{\sigma}^2 = MSerror$

$$\hat{\sigma}^2_\alpha = \frac{1}{m^3}(MSSrowblock - MSerror)$$

$$\hat{\sigma}^2_\beta = \frac{1}{m^3}(MSScolblock - MSerror)$$

$$\hat{\sigma}^2_\tau = \frac{1}{m^2}(MSStreatmet - MSerror)$$

$$\hat{\sigma}^2_{\gamma(\alpha)} = \frac{1}{m(m+1)}(MSSrowwithinRb - MSerror)$$

(21)

$$\hat{\sigma}^2_{c(\beta)} = \frac{1}{m(m+1)}(MSScolwithinCb - MSerror)$$

(22)

$$\hat{\sigma}^2_s = \frac{1}{m^2}(MSsquare - MSerror)$$

The hypothesis that we are interested in testing are

$H_0: \sigma^2_{effect} = 0$

Testing of hypothesis about the variance components

Hence the test statistic is of the form

$$F_0 = \frac{MSeffect}{MSerror}$$

H_0 is rejected if $F_0 > F_{dfeffect,\ dferror, 1-\propto}$

Sudoku Square design model Type III

$s(\alpha)_{q(i)} = (s(\alpha)_{1(1)} \cdots s(\alpha)_{m(m)})' \sim N(0, \sigma^2_{s(\alpha)} I)$

$\pi(\beta)_{r(j)} = (\pi(\beta)_{1(1)} \cdots \pi(\beta)_{m(m)})' \sim N(0, \sigma^2_{\pi(\beta)} I)$

Hence the variance of response value ($Y_{ij(k,l,p,q)}$) is Var$(Y_{ij(k,l,p,q,r)}) = \sigma^2_\alpha + \sigma^2_\beta + \sigma^2_\tau + \sigma^2_\gamma + \sigma^2_c + \sigma^2_{s(\alpha)} + \sigma^2_{\pi(\beta)} + \sigma^2$

$\sigma^2_\alpha, \sigma^2_\beta, \sigma^2_\tau, \sigma^2_\gamma, \sigma^2_c, \sigma^2_{s(\alpha)}, \sigma^2_{\pi(\beta)}$ and σ^2 are called variance components of Sudoku square design model Type III. To derive the expectation of $SShwrowblock$

$E[SShwrowblock]$

$$= E\left[\sum_{i=1}^{m}\sum_{q=1}^{m} m^2 \bar{s}^2_{q(i)} - \sum_{i=1}^{m} m^3 \overline{RB_\iota}^2\right]$$

$$= \sum_{i=1}^{m}\sum_{q=1}^{m} m^2 E[\bar{s}^2_{q(i)}] - \sum_{i=1}^{m} m^3 E[\overline{RB_\iota}^2]$$

Note

$E[\pi(\beta)] = E[\bar{s}_{q(i)}] = \mu$

$[E[\bar{s}^2_{q(i)}] = var[\bar{s}_{q(i)}] + (E[\bar{s}_{q(i)}])^2$

$$= \sigma^2_\alpha + \frac{\sigma^2_\beta}{m} + \frac{\sigma^2_\tau}{m^2} + \frac{\sigma^2_\gamma}{m^2} + \frac{\sigma^2_c}{m^2} + \sigma^2_{s(\alpha)} + \frac{\sigma^2_{\pi(\beta)}}{m^2} + \frac{\sigma^2}{m^2} + \mu^2$$

$E\left[\overline{RB_\iota}^2\right] = var[\overline{RB_\iota}] + (E[\overline{RB_\iota}])^2$

$$= \sigma_\alpha^2 + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_c^2}{m^2} + \frac{\sigma_{s(\alpha)}^2}{m^2} + \frac{\sigma_{\pi(\beta)}^2}{m^2} + \frac{\sigma^2}{m^3} + \mu^2$$

$E[SShwrowblock]$

$$= m^4\left[\sigma_\alpha^2 + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_c^2}{m^2} + \sigma_{s(\alpha)}^2 + \frac{\sigma_{\pi(\beta)}^2}{m^2} + \frac{\sigma^2}{m^2} + \mu^2\right]$$

$$-m^4\left[\sigma_\alpha^2 + \frac{\sigma_\beta^2}{m} + \frac{\sigma_\tau^2}{m^2} + \frac{\sigma_\gamma^2}{m^2} + \frac{\sigma_c^2}{m^2} + \frac{\sigma_{s(\alpha)}^2}{m^2} + \frac{\sigma_{\pi(\beta)}^2}{m^2} + \frac{\sigma^2}{m^3} + \mu^2\right]$$

$$= (m^4 - m^2)\sigma_{s(\alpha)}^2 + (m^2 - m)\,\sigma^2$$

$$= m(m-1)[m(m+1)\sigma_{s(\alpha)}^2 + \sigma^2]$$

The above result $E[SShwrowblock]$ is divided by the degree of freedom which is $m(m-1)$

$E[MSShwrowblock]$

$$= \frac{m(m-1)[m(m+1)\sigma_{s(\alpha)}^2 + \sigma^2]}{m(m-1)}$$

$$= m(m+1)\sigma_{s(\alpha)}^2 + \sigma^2 \qquad (23)$$

Similarly, the

$$E[MSSvwcolblock] = m(m+1)\sigma_{r(\beta)}^2 + \sigma^2 \qquad (24)$$

To obtain $E[MSerror]$ for model Type III, we take the expectation of $SSerror$ for model III divided by error degree of freedom of model III

$$E[MSerror] = \sigma^2 \qquad (25)$$

The estimates $\hat{\sigma}_{s(\alpha)}^2$ and $\hat{\sigma}_{\delta(\pi)}^2$ of the variance components $\sigma_{s(\alpha)}^2$ and $\sigma_{\delta(\pi)}^2$ which are computed from their respective expectation (23-24) above.

Variance components estimators for various effects for Sudoku square model III are as follow

$$\hat{\sigma}^2 = MSerror$$

$$\hat{\sigma}_\alpha^2 = \frac{1}{m^3}(MSSrowblock - MSerror)$$

$$\hat{\sigma}_\beta^2 = \frac{1}{m^3}(MSScolblock - MSerror)$$

$$\hat{\sigma}_\tau^2 = \frac{1}{m^2}(MSStreatmet - MSerror)$$

$$\hat{\sigma}_\gamma^2 = \frac{1}{m^2}(MSSrow - MSerror)$$

$$\hat{\sigma}_c^2 = \frac{1}{m^2}(MSScol - MSerror)$$

$$\hat{\sigma}_{s(\alpha)}^2 = \frac{1}{m(m+1)}(MSShwrowblock - MSerror) \qquad (26)$$

$$\hat{\sigma}_{\delta(\pi)}^2 = \frac{1}{m(m+1)}(MSSvwrowblock - MSerror) \qquad (27)$$

The hypothesis that we are interested in testing are

$H_0: \sigma_{effect}^2 = 0$

Testing of hypothesis about the variance components.

Hence the test statistic is of the form

$$F_0 = \frac{MSeffect}{MSerror}$$

H_0 is rejected if $F_0 > F_{dfeffect,\ dferror,1-\propto}$

Sudoku square design model Type IV

The estimates $\hat{\sigma}^2, \hat{\sigma}_\alpha^2, \hat{\sigma}_\beta^2, \hat{\sigma}_\tau^2, \hat{\sigma}_{\gamma(\alpha)}^2, \hat{\sigma}_{c(\beta)}^2$ $\hat{\sigma}_{s(\alpha)}^2$ and $\hat{\sigma}_{\delta(\pi)}^2$ of the variance components $\sigma^2, \sigma_\alpha^2, \sigma_\beta^2, \sigma_\tau^2, \sigma_{\gamma(\alpha)}^2$, $\sigma_{c(\beta)}^2, \sigma_{s(\alpha)}^2$ and $\sigma_{\delta(\pi)}^2$ and all have been derived in the previous models above, what we need to do is to substitute their respective variance components into model IV.

This yields the unique solution of variance components for the Sudoku square design model Type IV.

Variance components estimators for various effects for Sudoku square model IV are as follow

$$\hat{\sigma}^2 = MSerror$$

$$\hat{\sigma}_\alpha^2 = \frac{1}{m^3}(MSSrowblock - MSerror)$$

$$\hat{\sigma}_\beta^2 = \frac{1}{m^3}(MSScolblock - MSerror)$$

$$\hat{\sigma}_\tau^2 = \frac{1}{m^2}(MSStreatmet - MSerror)$$

$$\hat{\sigma}_{\gamma(\alpha)}^2 = \frac{1}{m(m+1)}(MSSrowwithinRb - MSerror)$$

$$\hat{\sigma}_{c(\beta)}^2 = \frac{1}{m(m+1)}(MSScolwithinCb - MSerror)$$

$$\hat{\sigma}_{s(\alpha)}^2 = \frac{1}{m(m+1)}(MSShwrowblock - MSerror)$$

$$\hat{\sigma}_{\delta(\pi)}^2 = \frac{1}{m(m+1)}(MSSvwrowblock - MSerror)$$

The hypothesis that we are interested in testing are

$H_0: \sigma_{effect}^2 = 0$

Testing of hypothesis about the variance components.

Hence the test statistic is of the form

$$F_0 = \frac{MSeffect}{MSerror}$$

H_0 is rejected if $F_0 > F_{dfeffect,\ dferror,1-\propto}$

5. CONCLUSION

In this paper, the derivation of variance components for the Sudoku Square design models presented by Subramani and Ponnuswamy ([SP09]) have been carried out using ANOVA method and the variance component estimators for the various effects for the four models were obtained as well test of significants were given.

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