STEADY FLOW OF A REACTIVE MHD FLUID THROUGH A PERMEABLE PIPE UNDER OPTICALLY THICK LIMIT RADIATION

[1] Usman M. A, [2] Olayiwola M. O, [3] Sikiru A. B.

[1,3] Department of Mathematical Sciences, Olabisi Onabanjo University, Ago-Iwoye, Nigeria
[2] Department of Mathematical Sciences, Faculty of Basic and Applied Sciences, Osun State University, Osogbo, Nigeria

Corresponding Author: Olayiwola M. O., olayiwola.oyedunsi@uniosun.edu.ng

ABSTRACT: This research work investigates the analytical solution of the temperature profile distribution of a one-dimensional fluid under the influence of magnetic fluid strength of a reactive hydromagnetic fluid flow through porous media between permeable beds under optically thick limit radiation. The fluid is considered to be incompressible and electrically conducting fluid flowing steadily through porous media with the effect of magnetic strength. The analytical solutions of the non-linear dimensionless energy equations governing the fluid flow are obtained using integration and series solution of Adomian Decomposition Method (ADM) and the effects of all important flow properties on the fluid flow are presented graphically and discussed.

KEYWORDS: ADM, Temperature, Permeable Pipe, Optically Thick Limit, Radiation, Porous Media, Hydromagnetic Fluid.

1. INTRODUCTION

The study of flow through porous medium is important for a wide range of technical problems such as flow through packed beds, sedimentation, environmental pollution, centrifugal separation of particles. The study of flow and heat transfer in fluid past a porous surface has attracted many researchers in view of its wide range of applications in engineering and industrial practices.

After a deep survey of literature, it was observed that studies relating to the second law analysis of reactive internal heat generating hydromagnetic fluid flow, through a porous media, between permeable beds under optically thick limit radiation with isothermal wall temperature have not been fully investigated.

In spite of this, researchers have recently shown that for fluid flows involving exothermic or endothermic reactions, fire and combustion, the effect of internal heat generation and many more cannot be neglected. Therefore, the purpose of this research work is to examine the temperature profile of a steady state of a reactive hydromagnetic fluid flow and to investigate numerically, the effect of the internal heat generation under optically thick limit radiation.

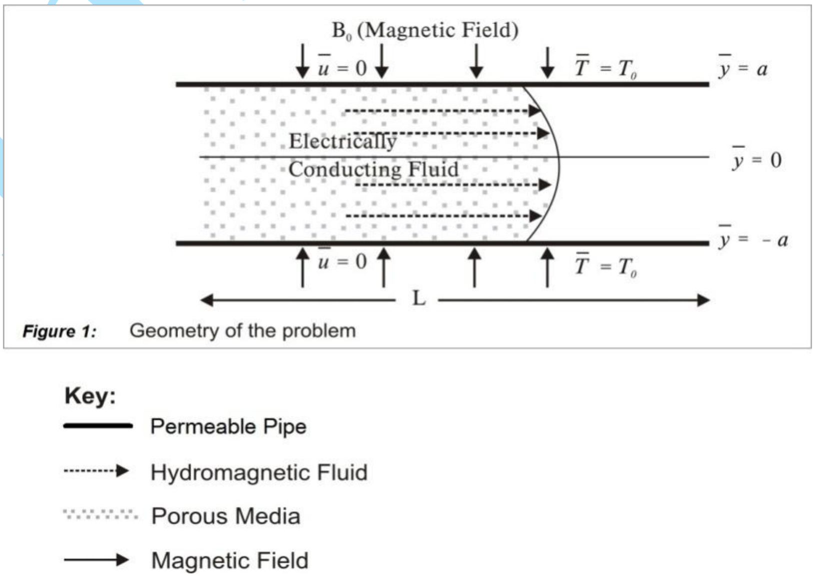

Figure 1: The geometry of the flow regime

The problem is strongly non-linear involving non-linear equation. Hence, analytical solution will be obtained using direct integration and a series solution of Adomian Decomposition Method (ADM). ADM has been showed to be reliable to solve differential equations with sizeable number of iteration that converges rapidly. As evident in the literatures, the geometry of the flow regime is showed in Figure 1 above, where L is the characteristics length of the channel.

Considerable effort has been devoted to the study of a reactive hydromagnetic fluid flow which finds numerous and wide-range applications in many engineering processes, such as polymer extrusion, nuclear reactor design, geophysics and underground storage of nuclear waste and energy storage systems amongst others ([HMG17]).

Reactive hydromagnetic fluid flows are often accompanied with heat transfer in many industrial and engineering applications. For instance, Makinde and Beg ([MA10]) devoted their study to investigate the inherent irreversibility and thermal stability in a reactive electrically conducting fluid flowing steadily through a channel with isothermal walls under the influence of a transversely imposed magnetic field. Bartella et al ([BN12]) observed that the effects of internal heat generation on a reactive hydromagnetic fluid flow have been studied with respect to various physical properties. Also, Ajibade ([Aji09]) investigated the free convective flow of heat generating or absorbing fluid between vertical parallel porous plates due to periodic heating of the porous plates. This analysis was performed by considering a fully developed flow and steady periodic regime.

It is well known that the rate of heat transfer is temperature dependent, which increases the interaction of moving fluid and thus influence the internal energy of the flow regime. This interaction according to Frank-Kamenetski ([Fra69]) and Makinde and Beg ([MA10]). Olayiwola ([Ola16]) also presented some numerical results on Whitham-Broer-Kaup shallow water wave equation. Gbadeyan ([Gba12]) presented a novel results on multiplicity of solutions for a reactive variable viscous couette flow under Arrhenius kinetics while Adesanya and Gbadeyan ([AG11]) also contributed immensely in the application of Ädomian decomposition approach to steady visco-elastic fluid flow with slip through a planer channel.

The present study aim to investigate the effects of internal heat generation, rate of heat transfer and other thermo-physical parameters present within the flow system. The analytical solution of the non-linear dimensionless equations governing the fluid flow are obtained using integration and a series solution of Adomian Decomposition Method (ADM). More importantly, our results shall be of interest to industries in improving the efficiency and effectiveness of hydromagnetic lubricants used in engineering systems.

In the rest of this paper, the problem is formulated in "Mathematical Formulation" section. The governing equations are solved using the Adomian Decomposition Method (ADM). Presentations of analytical results of the problem are shown and presented graphically in "Results and discussion" section and "Conclusion" section gives the concluding remark.

2. MATHEMATICAL FORMULATION

This study considered the steady flow of a reactive, incompressible and electrically conducting fluid flowing through a permeable pipe under optically thick limit radiation with isothermal wall temperature under the influence of a transverse magnetic field strength.

The dimensionless momentum equation has been solved using integration method by splitting the momentum equation into particular and complimentary functions using the appropriate and related boundary conditions and the result is shown as;

$$u(y) = C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{R}{RM^2 + \frac{1}{Da}} \qquad (1)$$

where;

$u(y)$ velocity of the fluid flow,

c_1 *and* c_2 are the arbitrary constants,

m_1 *and* m_2 are the roots of the equation,

R = Reynold Number,

M = Hartmann Number,

Da = Darcy Number.

Where;

$$m_1 = \frac{1}{2}\left[R \pm \sqrt{R^2 + 4\left(RM^2 + \frac{1}{Da}\right)}\right] \qquad (2)$$

$$m_2 = \frac{1}{2}\left[R \pm \sqrt{R^2 + 4\left(RM^2 + \frac{1}{Da}\right)}\right] \qquad (3)$$

$$C_1 = \frac{1}{e^{m_2} - e^{m_1}}\left[-u_{B_2} + u_{B_1} e^{m_2} - \frac{R}{RM^2 + \frac{1}{Da}}\left(e^{m_2} - 1\right)\right] \qquad (4)$$

$$C_2 = \frac{1}{e^{m_2} - e^{m_1}}\left[u_{B_2} - u_{B_1} e^{m_2} - \frac{R}{RM^2 + \frac{1}{Da}}\left(1 - e^{m_1}\right)\right] \qquad (5)$$

and the slip velocities are given as:

$$u_{B_1} = \frac{D_2E_2 - D_4E_1}{D_1D_4 - D_3D_2}$$

and $$u_{B_2} = \frac{D_3E_1 - D_1E_2}{D_1D_4 - D_3D_2} \qquad (6)$$

where;

$$D_1 = \frac{m_1 e^{m_2} - m_2 e^{m_1}}{e^{m_2} - e^{m_1}}$$

$$D_2 = \frac{m_2 - m_1}{e^{m_2} - e^{m_1}}$$

$$D_3 = (m_1 - m_2)e^{m_1+m_2}$$

And $$D_4 = \frac{m_2 e^{m_2} - m_1 e^{m_1}}{e^{m_2} - e^{m_1}} + m\sigma$$

$$E_1 = R\left[\frac{1}{m^2\,\mathrm{Re} + \frac{1}{Da}}\left\{\frac{m_2\left(e^{m_1}-1\right)}{e^{m_2}-e^{m_1}} - \frac{m_1\left(e^{m_2}-1\right)}{e^{m_2}-e^{m_1}}\right\} + \frac{m}{\sigma}\right] \qquad (7)$$

$$E_2 = R\left[\frac{1}{m^2 R + \frac{1}{Da}}\left\{\frac{m_2 e^{m_2}\left(e^{m_1}-1\right)}{e^{m_2}-e^{m_1}} - \frac{m_1 e^{m_1}\left(e^{m_2}-1\right)}{e^{m_2}-e^{m_1}}\right\} + \frac{m}{\sigma}\right] \qquad (8)$$

Introducing non-dimensionalized variables and parameters;

$$\alpha = \frac{16\sigma T_\infty^3}{3kk*}, \varepsilon = \frac{RT_0}{E},$$

$$\gamma = \frac{QEAh^2C_0}{KRT_0^2} e^{-\frac{E}{RT_0}},$$

$$M^2 = \frac{\sigma_0 B_0 h^2}{\rho\mu}, Da = \frac{k}{h^2}$$

$$\delta = \frac{Q_0 RT_0^2}{QAEC_0} e^{\frac{E}{RT_0}}, \qquad (9)$$

$$\gamma = \frac{\mu U^2}{QA_aC_0} e^{\frac{E}{RT_0}},$$

$$\theta = \frac{E\left(\overline{T} - T_0\right)}{RT_0^2}$$

The approximate solution of the energy equation is obtained by substituting the solution of the momentum equation into the energy equation using series solution of Adomian Decomposition method (ADM). The energy equation of the fluid flow regime in non-dimensionless form is given as:

$$K\frac{\partial^2\overline{T}}{\partial\overline{y}^2} + \mu\left(\frac{\partial\overline{u}}{\partial\overline{y}}\right)^2 + \frac{\sigma B_0^2}{\rho}\overline{u}^2$$

$$+ QC_0Ae^{\frac{E}{R\overline{T}}} + \frac{\mu}{k}\overline{u}^2 + Q_0\left(\overline{T} - T_0\right) \qquad (10)$$

$$-\frac{\partial q_r}{\partial\overline{y}} = 0$$

Where:

$$\frac{\partial q_r}{\partial\overline{y}} = \frac{-16}{3\alpha}\sigma T^3\frac{\partial^2\overline{T}}{\partial\overline{y}^2} \qquad (11)$$

with the following boundary conditions:

$$u = u_{B_1}, \frac{du}{dy} = \frac{\alpha}{\sqrt{k_1}}\left(u_{B_1} - Q_1\right),$$

and T = 0 at y = 0 (12)

$$u = u_{B_2}, \frac{du}{dy} = \frac{\alpha}{\sqrt{k_2}}\left(u_{B_2} - Q_2\right) \qquad (13)$$

and T = 0 at y = h

where;
k = Thermal Conductivity

T = Temperature of the fluid
Q_0 = Heat of the reaction term
C_o= Reactant species of initial concentration,
A = Reaction ratio constant
E = Activation energy
R = Universal gas constant
T_o = Wall temperature of the lower plate
Re = Reynold number
q_r = radioactive flux coefficient
B_0 = Magnetic field strength
μ =Fluid viscosity and
σ = Electrical conductivity

Substituting equation (11) into equation (10), the equation gives:

$$K\frac{\partial^2 \overline{T}}{\partial \overline{y}^2}+\mu\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)^2+\frac{\sigma_0 B_0^2}{\rho}\overline{u}^2$$
$$+QC_0Ae^{\frac{E}{R\overline{T}}}+\frac{\mu}{k}\overline{u}^2+Q_0\left(\overline{T}-T\right) \tag{14}$$
$$+\frac{16}{3\alpha}\partial T^3\frac{\partial^2 \overline{T}}{\partial y^2}=0$$

Using the following non-dimensionalize variables and parameters

$$\frac{\partial \overline{T}}{\partial \overline{y}}=\frac{\partial}{\partial y}\left(\frac{dy}{d\overline{y}}\right)=\frac{1}{h}\frac{\partial}{\partial y} \tag{15}$$

$$\frac{\partial}{\partial \overline{y}}\left(\frac{\theta RT_0^2}{E}+T_0\right)=\frac{\partial \overline{T}}{\partial \overline{y}}$$
$$=\frac{1}{h}\frac{\partial}{\partial y}\left(\frac{\theta RT_0^2}{E}+T_0\right) \tag{16}$$

$$\frac{\partial \overline{T}}{\partial \overline{y}}=\frac{RT_0^2}{Eh^2}\frac{\partial \theta}{\partial y}\ \text{and}\ \frac{\partial^2 \overline{T}}{\partial \overline{y}^2}=\frac{RT_0^2}{Eh^2}\frac{\partial^2 \theta}{\partial y^2} \tag{17}$$

Substituting equations (17) into equation (14), the result is;

$$\frac{KRT_0^2}{Eh^2}\frac{d^2\theta}{dy^2}+\mu\left(\frac{A_1}{V}\frac{du}{dy}\right)^2$$
$$+\frac{\sigma B_0^2}{\rho}\left(\frac{uA_1h}{V}\right)^2+ \tag{18}$$
$$QC_0Ae^{\frac{E}{R\left(\frac{QRT_0^2+T_0}{E}\right)}}+\frac{\mu}{k}\left(\frac{uA_1h}{V}\right)^2$$
$$+Q_0\left(\frac{QRT_0^2}{E}+T_0-T_0\right)$$
$$+\frac{16\sigma T_\infty^3}{3k*}\frac{RT_0^2}{Eh^2}\frac{\partial^2\theta}{\partial y^2}=0$$

Multiplying each term in equation (18) by $\frac{Eh^2}{kRT_o^2}$, it gives:

$$\frac{d^2\theta}{dy^2}+\frac{Eh^2}{KRT_0^2}\mu\frac{A_1^2}{V^2}\left(\frac{du}{dy}\right)^2$$
$$+\frac{Eh^2}{KRT_0^2}\frac{\sigma B_0^2}{\rho}\frac{A_1^2h^2}{V^2}u^2+ \tag{19}$$
$$\frac{Eh^2}{KRT_0^2}QC_0Ae^{\frac{E}{RT_0\left(\frac{RT_0}{E}+1\right)}}+\frac{Eh^2}{KRT_0^2}$$
$$\frac{\mu}{k}\frac{A_1^2h^2}{V^2}u^2+\frac{Eh^2}{KRT_0^2}\frac{QRT_0^2}{E}\theta$$
$$+\frac{Eh^2}{KRT_0^2}\frac{16\sigma}{3k*}T_\infty^3\frac{RT_0^2}{Eh^2}\frac{d^2\theta}{dy^2}=0$$

Re-arranging equation (19) with equation (9), the equation becomes:

$$\frac{d^2\theta}{dy^2}+\frac{Eh^2}{KRT_0^2}\frac{\mu A_1^2}{V^2}\left[\left(\frac{du}{dy}\right)^2+\frac{\sigma B_0^2h^2}{\rho\mu}u^2+\frac{h^2}{k}u^2+\frac{Eh^2}{KRT_0^2}QC_0Ae^{\frac{E}{RT_0}\frac{\theta}{1+\varepsilon\theta}}\right]+ \qquad (20)$$

$$\frac{Q_0h^2}{k}\theta+\frac{16\sigma T_\infty^3}{3k*k}\frac{d^2\theta}{dy^2}=0$$

Re-arranging equation (20), the result is:

$$\frac{d^2\theta}{dy^2}+\frac{Eh^2}{KRT_0^2}\frac{\mu A_1^2}{V^2}\left[\left(\frac{du}{dy}\right)^2+M^2u^2+\frac{1}{Da}u^2+\frac{Eh^2}{KRT_0^2}QC_0Ae^{\frac{E}{RT_0}\frac{\theta}{1+\varepsilon\theta}}\right] \qquad (21)$$

$$+\frac{Q_0h^2}{k}\theta+\alpha\frac{d^2\theta}{dy^2}=0$$

Likewise, collecting like terms in equation (21), the result gives:

$$\frac{d^2\theta}{dy^2}+\alpha\frac{d^2\theta}{dy^2}+\left[\frac{Eh^2}{KRT_0^2}QC_0Ae^{\frac{E}{RT_0}\frac{\theta}{1+\varepsilon\theta}}+\frac{Eh^2}{KRT_0^2}\mu\frac{A_1^2}{V^2}\left\{\left(\frac{du}{dy}\right)^2+M^2u^2+\frac{1}{Da}u^2\right\}+\frac{Q_0h^2}{k}\theta\right]=0 \qquad (22)$$

Factorising and substituting equations (10) into equation (22), the result gives:

$$(1+\alpha)\frac{d^2\theta}{dy^2}+\lambda\left[e^{\frac{\theta}{1+\varepsilon\theta}}+\gamma\left\{\left(\frac{du}{dy}\right)^2+M^2u^2+\frac{1}{Da}u^2\right\}\delta\theta\right]=0 \qquad (23)$$

So, the energy equation becomes;

$$\frac{d^2\theta}{dy^2}+\frac{\lambda}{1+\alpha}\left[e^{\frac{\theta}{1+\varepsilon\theta}}+\gamma\left\{\left(\frac{du}{dy}\right)^2+\left(M^2+\frac{1}{Da}\right)u^2\right\}+\delta\theta\right]=0 \qquad (24)$$

Subject to the following boundary conditions:

$$\theta(0)=0,\quad \theta(1)=0 \qquad (25)$$

where
α = Thermal radiation term
$\lambda = Frank - Kamenectts\,ki$ parameter
ε = Activation energy
γ = Viscous dissipation term
M = Hartmann number
δ = Internal heat generation term
Da = Darcy permeability temperature
θ = Non-dimensionless temperature

Substituting the result obtained from the momentum equation (1) into the energy equation (24) and integrate the equation (24) to obtain:

$$\frac{d\theta}{dy}(y)=\frac{d\theta}{dy}(0)-\frac{\lambda}{1+\alpha}\int_0^y\left[\begin{array}{l} e^{\frac{\theta}{1+\varepsilon\theta}}+ \\ \gamma\left\{\begin{array}{l}\left(\frac{du}{dy}\right)^2 \\ +\left(M^2+Da\right)u^2\end{array}\right\}+\delta\theta\end{array}\right]dy \tag{26}$$

If $\frac{d\theta}{dy}(0)=a_0$, then, equation (26) becomes:

$$\frac{d\theta}{dy}(0)=a_0-\frac{\lambda}{1+\alpha}\int_0^y\left[\begin{array}{l} e^{\frac{\theta}{1+\varepsilon\theta}}+ \\ \gamma\left\{\begin{array}{l}\left(\frac{du}{dy}\right)^2 \\ +\left(M^2+Da\right)u^2\end{array}\right\} \\ +\delta\theta\end{array}\right]dy \tag{27}$$

Integrating equation (27) again, the result gives:

$$\theta(y)=a_0 y\frac{\lambda}{1+\alpha}\int_0^y\int_0^y\left[\begin{array}{l} e^{\frac{\theta}{1+\varepsilon\theta}}+ \\ \gamma\left\{\begin{array}{l}\left(\frac{du}{dy}\right)^2+ \\ \left(M^2+Da\right)u^2\end{array}\right\} \\ +\delta\theta\end{array}\right]dydy \tag{28}$$

Introducing a series solution of Adomian Decomposition Method, where;

$$\theta(y)=\sum_{n=0}^{\infty}\theta_n(y) \tag{29}$$

Substituting equation (29) into equation (28), the equation (28) becomes:

$$\theta(y)=a_0 y-\frac{\lambda}{1+\alpha}\int_0^y\int_0^y\left[\begin{array}{l} e^{\frac{\sum_{n=0}^{\infty}\theta_n}{1+\varepsilon\sum_{n=0}^{\infty}\theta_n}} \\ +\gamma\left\{\begin{array}{l}\left[\begin{array}{l}\left(\frac{du}{dy}\right)^2 \\ +\left(\begin{array}{l}M^2+ \\ Da\end{array}\right)u^2\end{array}\right] \\ +\delta\sum_{n=0}^{\infty}\theta_n(y)\end{array}\right\}\end{array}\right]dydy \tag{30}$$

From (30), we let the non-linear term in equation (30) is written as:

$$\sum_{n=0}^{\infty}A_n(y)=e^{\frac{\sum_{n=0}^{\infty}\theta_n(y)}{1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)}} \tag{31}$$

where A_0, A_1, A_2 are called Adomian Polynomials such that;

$$A_0=e^{\frac{\theta_0(y)}{1+\varepsilon\theta_0(y)}} \tag{32}$$

$$A_1=\frac{e^{\frac{\theta_0(y)}{1+\varepsilon\theta_0(y)}}}{(1+\varepsilon\theta_0(y))^2}\theta_1(y) \tag{33}$$

$$A_2=\frac{e^{\frac{\theta_0}{1+\varepsilon\theta_0}}}{(1+\varepsilon\theta_0)^4}\left(\begin{array}{l}\theta_1^2(y) \\ -2\varepsilon\theta_1^2 \\ -2\varepsilon^2\theta_0\theta_1^2 \\ +2\theta_2+4\varepsilon\theta_0\theta_2 \\ +2\varepsilon^2\theta_0\theta_2\end{array}\right) \tag{34}$$

Taking the zeroth components with special modification of Adomian Decomposition Method, the result becomes:

$$\theta_0(y)=0 \tag{35}$$

$$\theta_1(y) = a_0 y - \frac{\lambda}{1+\alpha}\int_0^y \int \left[A_0 + \gamma\left\{ \left(\frac{du}{dy}\right)^2 + \left(M^2 + Da\right)u^2 \right\} + \delta\theta_0 \right] dy \quad (36)$$

$$\theta_{n+1}(y) = -\frac{\lambda}{1+\alpha}\int_0^y \int \left[A_n + \delta\theta_n \right] dydy \quad (37)$$

Such that n ≥ 1

Finally, the solution of the energy equation is approximately given as:

$$\theta(y) = \sum_{n=0}^{k} \theta_n(y) \quad (38)$$

where k is the number of iterations done.

3. DISCUSSION OF RESULTS

The non-linear differential energy equations with the boundary conditions are solved analytically using integration and the ADM. The effect of various imperative parameters of temperature profiles of a steady state of a reactive hydromagnetic fluid flow through porous media between permeable pipe are calculated analytically and presented graphically as follow;

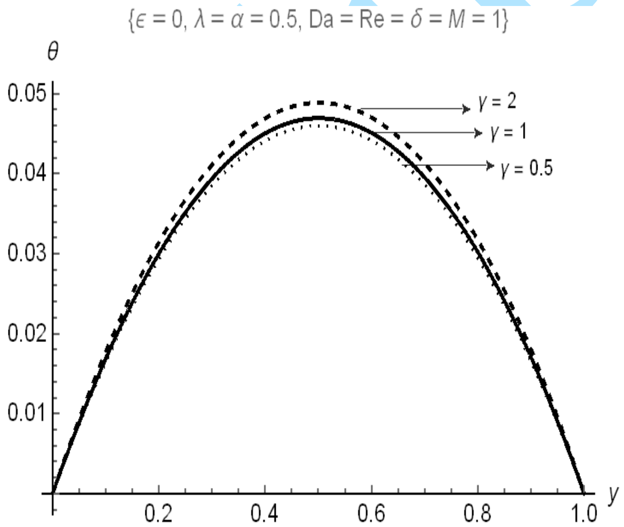

Figure 2: Effect of viscous heating parameter on temperature profile

The effects of viscous heating or dissipation on the temperature profile is illustrated in figure 2. The maximum flow is observed at the centreline of the flow channel. It is clearly seen that as the temperature increases, the viscous heating parameter increases.

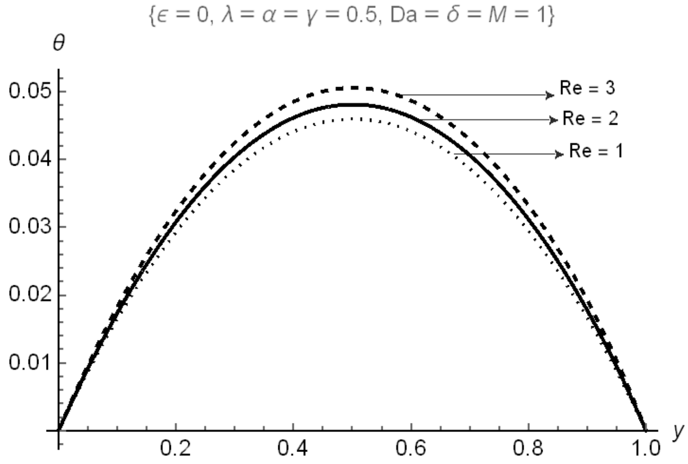

Figure 3: Effect of Reynold number on temperature profile

Figure 2 shows the effect of Reynold number R on the temperature profile of the fluid flow. It shows that maximum temperature is recorded at the centreline of the flow channel. It is clearly observed that an increase in R brings about an increase in the temperature profile of the fluid flow.

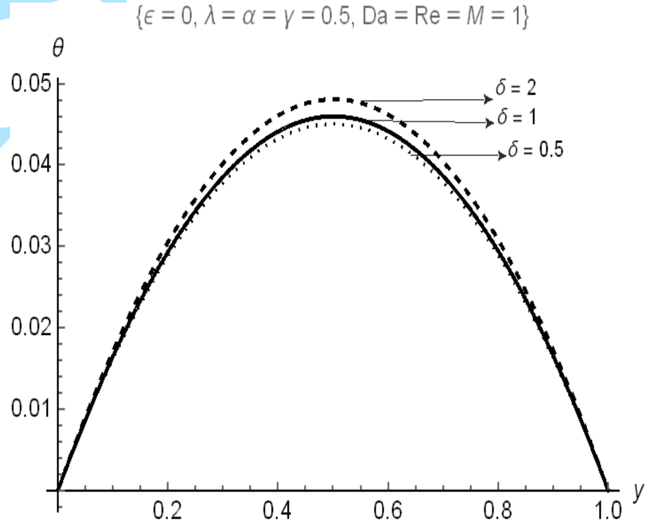

Figure 4: Figure showing the effect of heat source on temperature profile

Figure 4 also shows the effect of internal heat generation on temperature profile. The flow is observed at the centreline of the flow channel. It is clearly seen that as the internal heat generation increases, the temperature also increases.

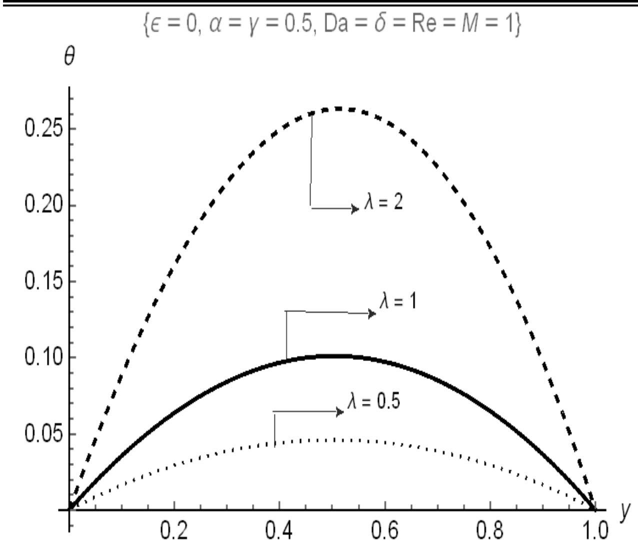

Figure 5: Effect of Frank-Kamenettski parameter on temperature profile

Figure 5 shows the effect of Frank-Kamenestski parameter on temperature profile. It shows that the effect of Frank-Kamenestski brings about an increase in the temperature of fluid flow regime.

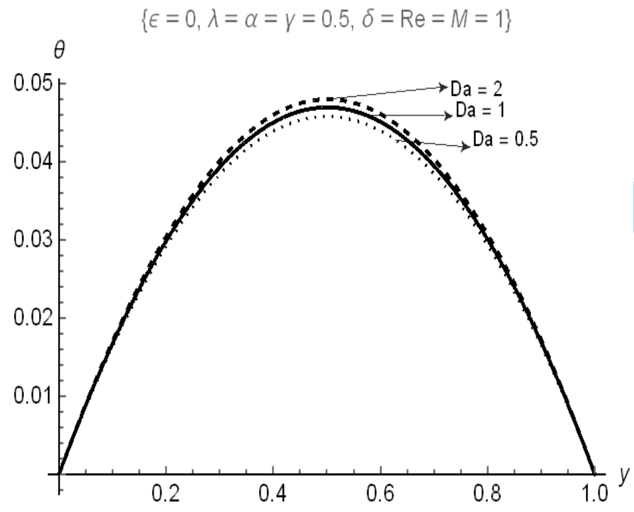

Figure 6: Effect of Darcy porous permeability parameter on temperature profile

Figure 6 shows the effect of Darcy porous permeability on temperature profile. Maximum fluid flow is also observed at the centreline of the channel. It is clearly seen that as the Darcy number increases, the temperature also increases. The reason for this is due to the inverse nature of Darcy permeability influence on the fluid flow regime.

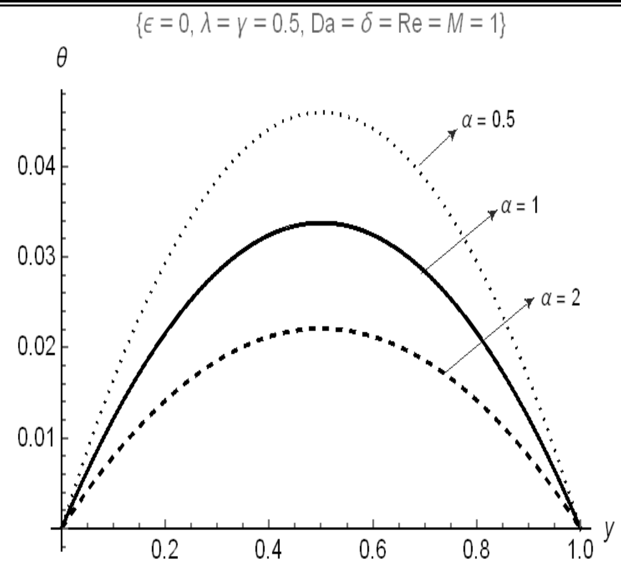

Figure 7: Effect of Radiation parameter on temperature profile

The effect of thermal radiation on temperature is shown in figure 7. The maximum temperature is seen at the centreline of fluid flow. As the thermal radiation decreases, the temperature of the fluid increases.

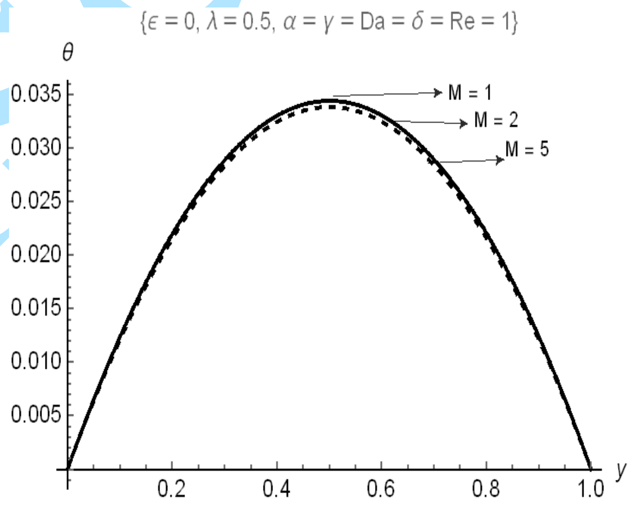

Figure 8: Effect of Magnetic field parameter on temperature profile

Finally, the maximum flow is observed at the centreline of the fluid channel. Figure 8 displays the graph of momentum equation with variation in “Hartmann M” number, it is clearly observed that as M increases, the temperature of the flow reduces due to the magnetic influence retarding the fluid flow while maximum flow is observed.

4. CONCLUSION

The analytical solution of a reactive hydromagnetic internal heat generating fluid flow through permeable pipes/beds is carried out. The ADM was used to obtain the analytical solutions of the governing energy equations.

The results revealed that as the internal heat generation parameters $(\varepsilon, \lambda, \alpha, Da, R, \delta, M)$ increases, the temperature also increases and that the effect of the heat source influenced the fluid flow by increasing the fluid temperature and that an increase in the magnetic field intensity increases the thermal critically values.

Generally, figures 2-6 show that the temperature profiles of the fluid is at the centreline of the flow channel. The reason is that it is a Pousellie flow of which the parallel plates are fixed and concentration is at the centre as shown in the flow, if the graph is rotated by 90 degree right. The fluid temperature increases as $(\varepsilon, \lambda, Da, R, \delta,)$ increases due to the fact that energy is produced when viscous heat, Reynold number, heat source, Frank-Kamenetski and Darcy porous permeability increases. This is due to the fact that energy is produced due to the reaction of the fluid and the interaction within the particles increases and hence increase the fluid temperature. In figure 7 and 8, as the thermal radiation and Hartmann number decreases, the temperature of the fluid flow increases due to the effect of the magnetic field strength.

Therefore, the result leads to the following conclusions;

An increase in the viscous dissipation increases the temperature profile of the fluid flow under optically thick limit radiation; an increase in the Reynold number also increases the temperature of the fluid; It is also seen that an increase in the internal heat generation increases the temperature of the fluid flow. The heat generated internally contributes to an increase in the temperature profile under optically thick limit radiation.

Also, as the Darcy number increases, the temperature also increases, due to the inverse nature of the Darcy permeability influence and finally, as the thermal radiation and Hartmann number decreases, the temperature of the fluid flow increases due to the effect of the magnetic field strength.

However, the study is of the interest to lubrication companies in improving the efficiency and effectiveness of hydromagnetic materials used in engineering systems, crude oil and natural gas transfer.

The results of this study are in good agreement with the existing results in the literature.

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