

DERIVATIVE APPLICATION IN NAVIGATION INTO OUTERSPACE

Mihai Ș. Neamțu¹, Mihaela Neamțu²

¹National College Constantin Diaconovici Loga Timișoara - Romania, Department of Mathematics

²West University of Timișoara - Romania, Department of Economics and Modelling

Corresponding Author: Mihaela Neamțu, mihaela.neamtu@e-uvt.ro

ABSTRACT: The aim of the present paper is to use the first order mathematical derivative to explain the return trajectory of an object, in our particular case it is about a spaceship belonging to NASA Apollo 13 Space Program. Firstly, the mission overview is presented followed by the mathematical modelling pertaining to this case study. We focus on the events that could follow an unforeseen accident. More precisely, we take into consideration three types of the return trajectory that the spaceship could have followed: a ricochet in outer space, crashing on the Earth's surface and safe landing on Earth. The numerical simulations are done using GeoGebra software. Finally, conclusions are drawn.

KEYWORDS: first order derivative, mathematical modelling, applications, object trajectory, numerical simulations.

1. INTRODUCTION

Mathematical modelling is an important tool used to describe, analyze and predict complex natural phenomenon and processes. Therefore, the environment can be simplified, emulated and understood in an efficient way. Based on ([MG14]), [MSC07]), many countries apply the approach of modelling in teaching and learning processes.

In the teaching process it is important to help pupils to discover and apply knowledge flexibly for studying more effectively ([Tinh17]). Nowadays, the trends in education highlight the strong connection between mathematical knowledge and the real world applications ([CQ17]).

In the present paper, we use the mathematical derivative of the first order to model, explain and describe the return trajectory of a spaceship.

In an attempt to set foot on the Moon again, NASA embarked on a new space mission named Apollo 13. An unfortunate accident took place on the way to the Moon that changed completely the entire flight plan and scope of the mission. During the attempt to stir the oxygen tanks an explosion occurred that put in danger the human crew of Apollo 13. Following this accident there were three types of the return trajectory that the spaceship could follow: a ricochet in outer space, crashing on the Earth's surface and safe landing on Earth. These orbits of the spaceship

can be interpreted using the mathematical modelling.

The structure of the paper is as follows. Section 2 describes Apollo 13 mission overview. Throughout Section 3 the mathematical modelling pertaining to the case study is addressed. GeoGebra software is used to illustrate numerical simulations. Finally, conclusions are drawn and presented in Section 4.

2. APOLLO 13 MISSION

Apollo 13 was the seventh mission in the Apollo space program and the third intended to land on the Moon. The craft was launched on April 11, 1970, at 14:13 EST (19:13 UTC) from the Kennedy Space Center, Florida, but the lunar landing was aborted after an oxygen tank exploded two days later.

A lot of problems were caused by limited power, like loss of cabin heat, shortage of potable water, and the critical need to repair the carbon dioxide removal system. After the explosion, they were faced with two dangerous options for the return trajectory to Earth: one being to turn around immediately and the other one was to continue they journey toward the Moon using its gravity to put them on return trajectory to Earth. They took the decision not to turn around immediately, but to use the Moon gravity. They shut down all the electrical power consuming systems (including the guiding system) with the exception of the communication system.

On their journey back in one point in time they had to manually orient the spaceship on the right trajectory that gave them the optimal gravitational force for a safe landing on Earth. There are three possibilities:

1. Low gravitational force will result into a ricochet in outer space. The spaceship and its crew will be lost in space (see Fig. 1).
2. High gravitational force will result into a descending speed that generates heat above the heat shield resistance limit. The spaceship will melt because of excessive heat generated due to the friction with the atmosphere (see Fig. 2).

3. Optimal gravitational force will result into a descending speed that generates heat below the heat shield resistance limit. This option is the safe one (see Fig 3).

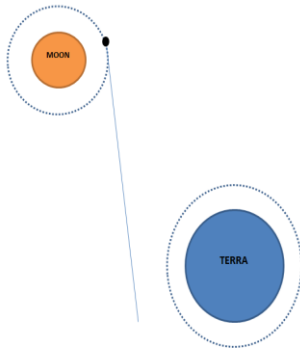


Fig. 1. Low gravitational force leads to a ricochet in outer space

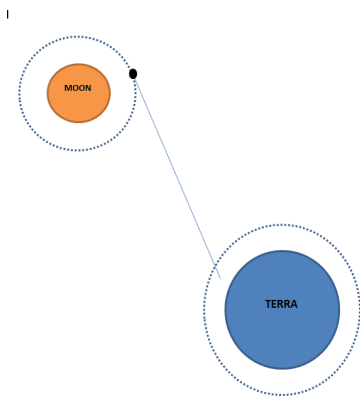


Fig. 2. High gravitational force leads to the destruction of the spaceship

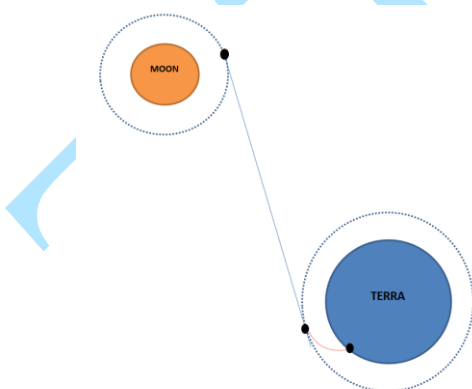


Fig. 3. Optimal gravitational force leads to a safe landing of the spaceship

Optimal gravitational force was obtained having a flight path defined by the right Earth orbit tangent. In mathematical terms the equation of the tangent is based on the first degree of a derivative function. The crew returned safely to Earth on April 17, 1970, six days after launch.

3. MATHEMATICAL APPROACH

3.1. General considerations

For guiding an object on a trajectory to intersect a given point in space one option is to use the mathematical derivative of a real function. The derivative is a way to show rate of change: that is, the amount by which a function is changing at one given point. For functions that act on the real numbers, it is the slope of the tangent line at a point on a graph.

Let $f : D \rightarrow \mathbf{R}$ be a real function, where D is an interval or an union of intervals within \mathbf{R} .

Definition 1 ([Gan97]). We say that function f admits a derivative in $x_0 \in D$ if the limit

$$\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists in $\overline{\mathbf{R}}$. In this case the limit is denoted with $f'(x_0)$ and it is called the derivative of the function f with respect to x_0 .

Therefore,

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

Definition 2 ([Gan97]). Function f is said to be differentiable at $x_0 \in D$ if the limit

$$\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists in \mathbf{R} (exists and it is finite). In this case the limit is denoted by $f'(x_0)$ meaning

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

Observation 1 ([Gan97]) If function f admits a finite derivative in $x_0 \in D$, then it can be interpreted as the slope of the tangent line at $A = (x_0, f(x_0))$ on the graph of the function f . In this case the equation of the tangent is given by:

$$y - f(x_0) = f'(x_0)(x - x_0).$$

Observation 2 ([Gan97]) If the derivative of function f is infinity the equation of the tangent is given by:

$$x = x_0.$$

3.2. Numerical examples

We suppose the equation of the Earth's atmospheric shape is given by the function

$$f: [-5,5] \rightarrow \mathbb{R}, f(x) = \sqrt{7 - \frac{x^2}{3}}.$$

Based on the above function, in what follows, using GeoGebra we want to visualize three cases of the spaceship's trajectory, where the gravitational force is low, high and optimal, respectively.

Let us have the spaceship's position defined by S having the coordinates as (5,5).

In the first the case we do not have a safe landing due to a low gravitational force that will result into a ricochet in outer space of the spaceship. The trajectory of the spaceship will intersect the point with the coordinates A=(-4,2) and the equation is given by:

$$x - 3y + 10 = 0.$$

In the second case, we do not have a safe landing due to high gravitational force that will result into a descending speed that generates heat above the heat shield resistance limit.

The trajectory of the spaceship will intersect the point with the coordinates A=(-2, 1) and the equation is given by:

$$4x - 7y + 15 = 0.$$

In the third case, in order to assure a safe landing the option is to follow a trajectory which is tangent to Earth's atmosphere. If we choose to reach the point with the coordinates A=(-2.5, 2.2), the equation of the tangent of f at the point A is given by:

$$y - f(-2.5) = f'(-2.5)(x + 2.5)$$

or equivalently

$$y = 0.38x + 3.16.$$

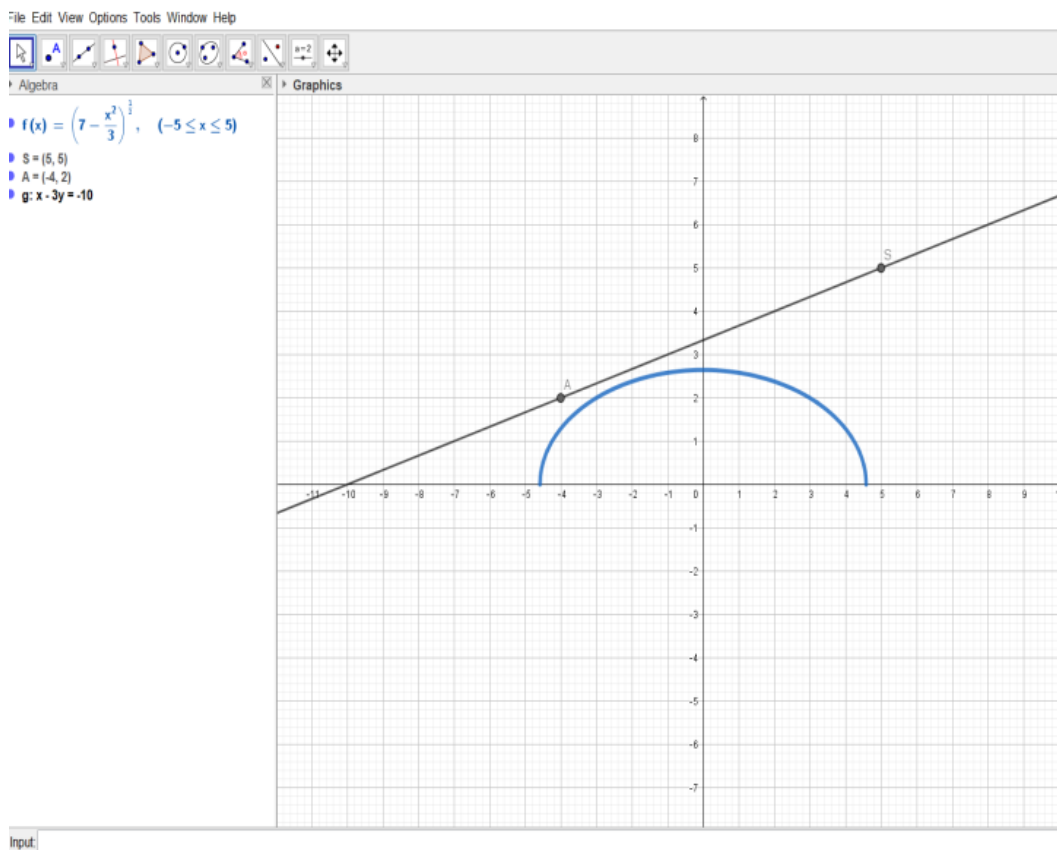


Fig. 4. Outerspace trajectory

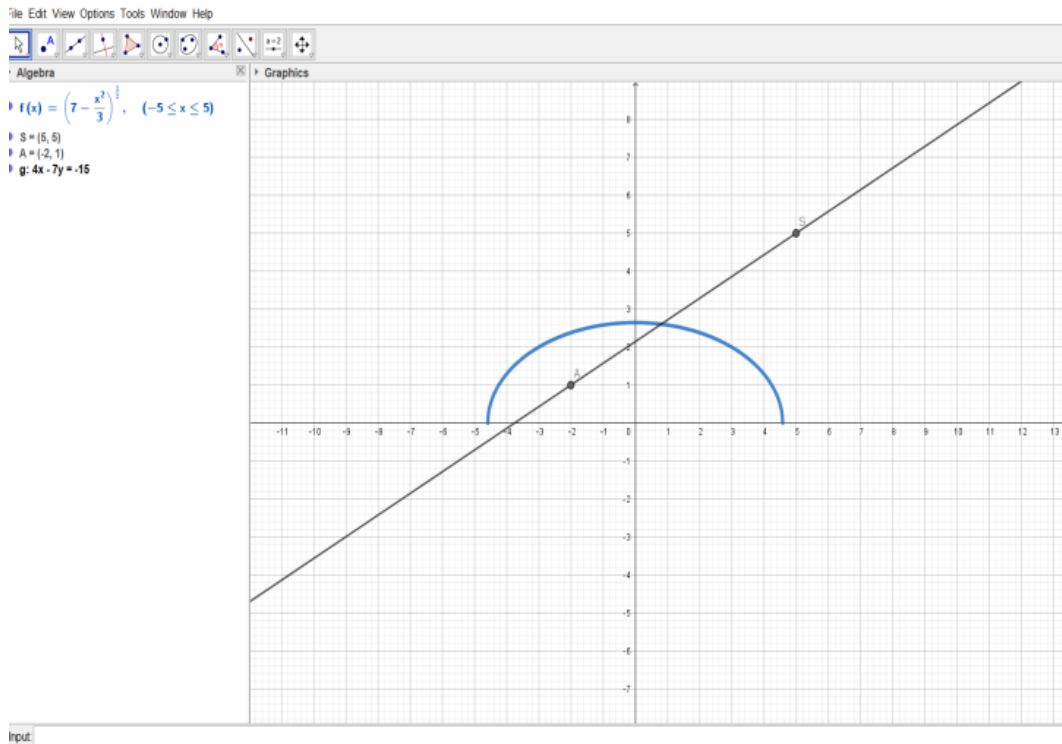


Fig. 5. Crash trajectory

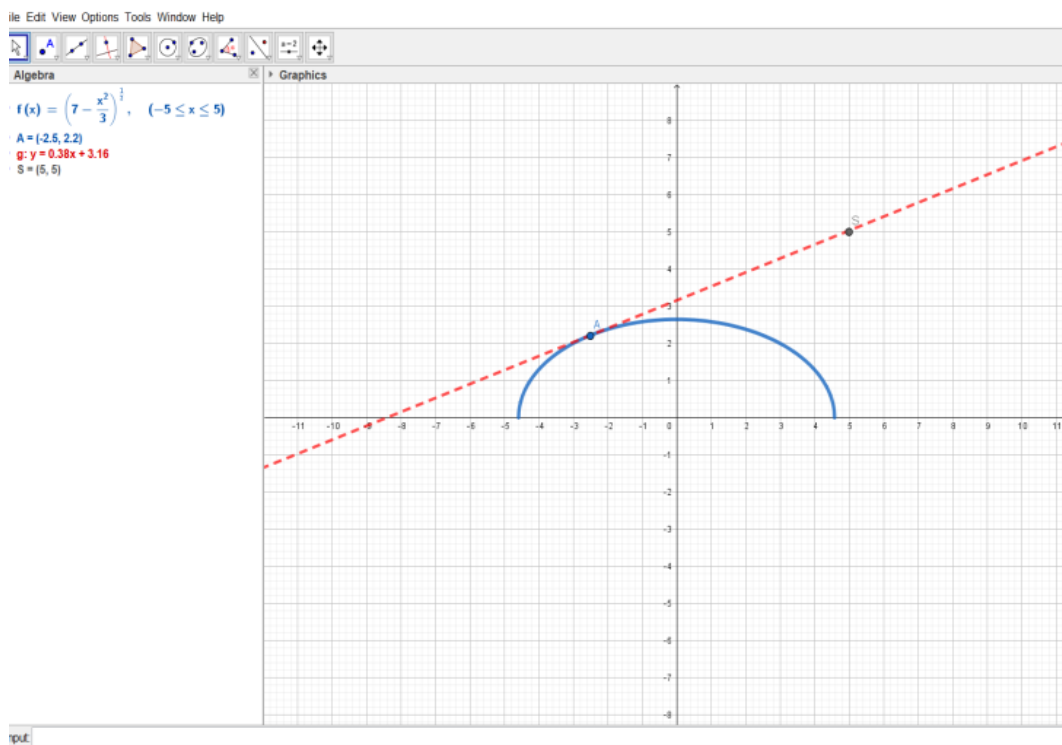


Fig. 6. Optimal trajectory

CONCLUSIONS

Nowdays, in the educational process it is of paramount importance the link between the theoretical concepts and the real life situations. In the present paper, the mathematical modelling that was used to describe and analyze the trajectory of an object, in close connection with a real life case, was based on the geometrial interpretation of the first order derivative for a real

function with a single real variable.

The case study is related to the trajectory of the spaceship belonging to NASA Apollo 13 Space Program, after unplanned accident occurred. We analyzed three possible return trajectories that the spaceship could have followed: a ricochet in outer space, crashing on the Earth's surface and safe landing on Earth. In order to visualize them the GeoGebra software is used.

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