

DYNAMIC MATHEMATICS SOFTWARE USE CASES TO INTRODUCTORY CALCULUS IN SECONDARY SCHOOL

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ABSTRACT: The dynamical feature of calculus makes a good reason to use dynamic and interactive methods of representation in teaching and learning calculus. GeoGebra, as a completely dynamic, easy-to-use, and fully connected mix of dynamic geometry software and computer algebra system, can help students to discover and understand the connections between mathematical concepts and their representation. The goal of this paper is to present how Dynamic Geometry Software (DGS) like GeoGebra is useful in teaching calculus concepts at high school and college level.

KEYWORDS: Dynamic Geometry Software, Discovery learning, GeoGebra, eLearning.

1. INTRODUCTION

The education system has changed a lot in the last years due to the large-scale use of computers as a teaching tool. To integrate technology into education, teachers' acceptance of computers in the classes is essential ([KRD08]) and this has speeded up because more educators are recognizing the ways computers can be used in the class and help them in reaching their teaching goals. Proper use of the educational software is enhancing student learning, and at the same time increase teachers productivity, therefore, it has become an accelerator in changing the way how teaching and learning is done, especially for math courses.

Computer algebra systems such as Wolfram's Mathematica, and MathWorks' Matlab together with dynamic geometry software such as Cabri Geometry, Geometer's Sketchpad, and Autograph are dynamic and usable methods for teaching maths in schools and colleges. Research results from a wide range of studies suggest that the usage of such software tools strengthen discovery and encourage experimentation if they are used by teachers in the learning process in classrooms while students can benefit from the software's rich visualization features in making conjectures ([Lav06]). Computer algebra systems (CAS) require a considerable time amount and effort to be familiar with and use them in teaching and their sophistication level makes them more suitable for usage in upper-level education. On the other hand, teachers can use dynamic geometry

software in primary and secondary schools due to its easy-to-use user interface.

2. GEOGEBRA

GeoGebra, developed by Markus Hohenwarter from 2001, with subsequent work by many others including Yves Kreis since 2005, and Michael Borchers since 2007, is a free, open-source, dynamic mathematics software (DMS) for all levels of education, from middle school and college up to University level which combines Computer Algebra Systems (CAS) features with Dynamic Geometry System (DGS) features into a single, easy-to-use, interface. Hence, it offers algebra, calculus and geometry features in a fully integrated environment that extends the dynamic geometry concepts to mathematical analysis ([HP07]). Designed as an educational tool, GeoGebra can help learners to experiment and develop critical thinking and problem-solving abilities in mathematics, independently from the environment (classroom or at home).

3. GEOGEBRA USES CASES IN TEACHING DIFFERENTIAL CALCULUS.

Calculus is a key topic in mathematics where ideas introduced in the final years of schooling are elaborated as students take a full course in Analysis at University. Hence, the use of geometry is necessary for better understanding of calculus concepts, therefore, the use of DGS is appropriate and recommended in teaching analysis. Students' difficulties in understanding the concept of calculus are generated by a poor understanding of the algebraic and geometrical aspects of the concept ([HAP10], [Zim91]). Many calculus concepts originate in the necessity to solve geometric problems (e.g. planetary motion, fluid dynamics) and together with this dynamical feature of calculus, it makes a good reason to use dynamic geometry software for teaching and learning calculus.

The few examples presented in this paper as applets are completely dynamic, meaning that the users can

interact easily by moving points along function graphs and by changing parameters using sliders. All examples can be exported very easily into dynamic worksheets along with instructions and tasks for the learner and are accessible online after they are uploaded online, without having to download the GeoGebra software.

3.1 Derivative as a Slope Function

Within calculus, there are a number of important concepts that students will use in other fields such as engineering, therefore, the understanding of these concepts is crucial. The main concepts of calculus are “limits and continuity, differentiation, integration, series, and differential equations”

([S+96]). Among the central topics and common applications of calculus, the concept of derivative plays a fundamental role. Authors of textbooks in introductory calculus might define “the derivative of a function [as] the function with value at x given by: provided the limit exists” ([SHE03]). Derivatives can be thought of in a number of different ways, such as symbolically as a formula, formally using the epsilon-delta definition, geometrically as the slope of a line tangent to the graph of the function, numerically as a gradient, or visuospatially as velocity ([Tal97]). Because of students’ difficulty in visualizing rates of change with two different quantities, they struggle to solve related rates problems ([HK08]).

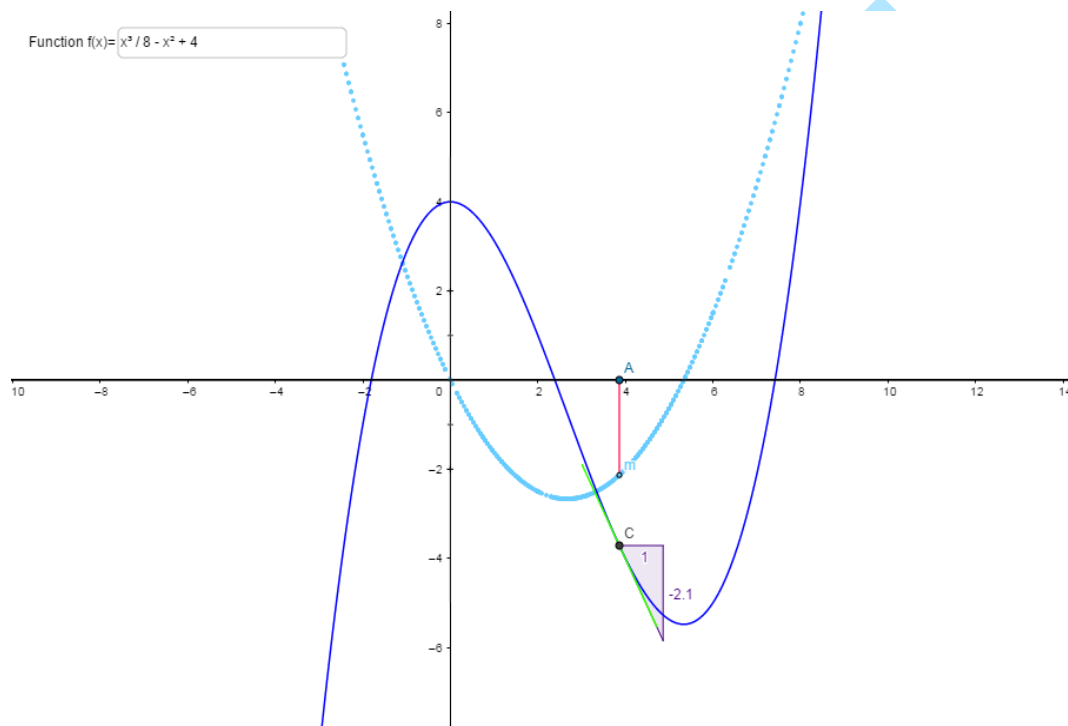


Fig. 1. The trace of the slope function and the graph of the original function $f(x)$

Therefore, we can use GeoGebra to make the connection between the slope of the tangent and the derivative of a function $f(x)$. Hence, in the same applet, we can have both the graph of the function $f(x)$ along with the slope function (see Fig. 1). A new point m is created using the slope of the tangent of function $f(x)$ over a point A that belongs to the function graph. This way the slope function of function $f(x)$ is obtained by sliding the point A along the graph of $f(x)$, hence the point m is drawing this slope function. After this, learners are able to verify their assumption about the derivative by typing in the derivative function and comparing it with the slope function created by point m . The original function $f(x)$ can be changed to investigate any standard function and its derivatives.

3.2 Roots, extremal point, and derivatives for a second order polynomial

The next example explores the relations between the derivative of a function and features like monotony or convexity. For simplicity, a second order polynomial is used and with the help of this applet, students are able to find the roots of the function and its first order derivative, identify the local extreme point, and observe the connection between monotony intervals are the sign of the first derivative. If we consider a second order polynomial in form $f(x) = ax^2 + bx + c$, and we denote its roots with R_1 and R_2 , the extremal point with E and the first order derivative with f' , we can use the applet to represent the second order polynomial with these points and its derivatives and use the figure’s sliders

to investigate how parameters influence the graph of the function (e.g. how parameter c drags the graph of the function upward or downward without changing the graph's shape), and to explore the connections

between the original function and its first order derivative (e.g. the function f has a local extreme point where the first order derivative has a root).

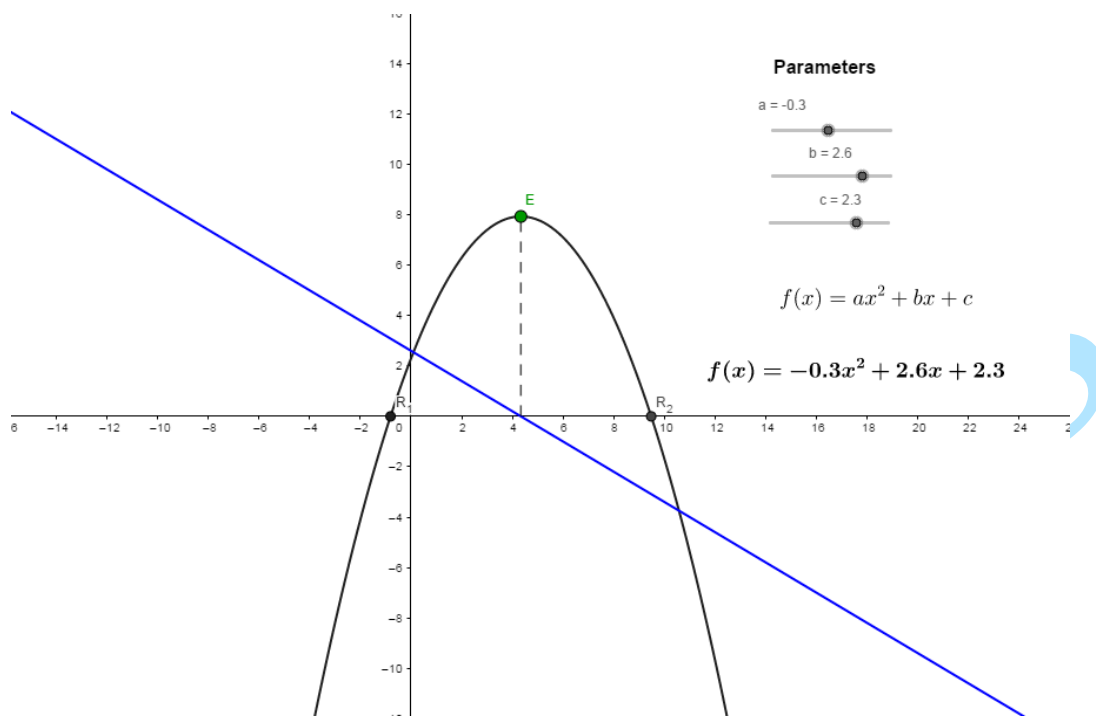


Fig. 2. The roots and the extreme point of a second order polynomial along with its first order derivative

An analysis of these characteristic aspects of functions (roots, points of local extreme) along with the function derivatives could help students to better understand and visualize characteristics of functions like extremal points and monotony.

3.3 Upper and Lower Sums and the Area below the graph of a function

Students have visualization and understanding problems when they encounter Riemann sums in calculus. The effort involved in drawing rectangles and doing sums can be handed over to the computer that can easily generate large numbers of rectangles even for very complicated functions. This way it becomes more feasible to do many more examples using larger numbers and varying limits of intervals on more complex functions and student time would be spent on understanding the concepts, instead of wasting time doing calculations.

The following example visualizes in an interactive

way the Riemann integral concept with the help of lower and upper Riemann sums (see Fig. 3). The graph of function f is represented along with two points a and b that are introduced by the user and the corresponding investigation interval $[a,b]$. With the help of a slider the number of divisions, n , (i.e. the number of rectangles used to calculate the Riemann sums) can be changed, in respect with function f and the $[a,b]$ interval. The values for the upper and lower Riemann sums are dynamically displayed along with the surface area value. Different components of the applet can be selected for appearance through the use of checkboxes. With the help of this applet, users can also find the width of the rectangles as an expression of interval limits a and b , and the number of the divisions, the height of the rectangles as an expression of function f , and observe the convergence of the lower and upper Riemann sums as the number of intervals increases.

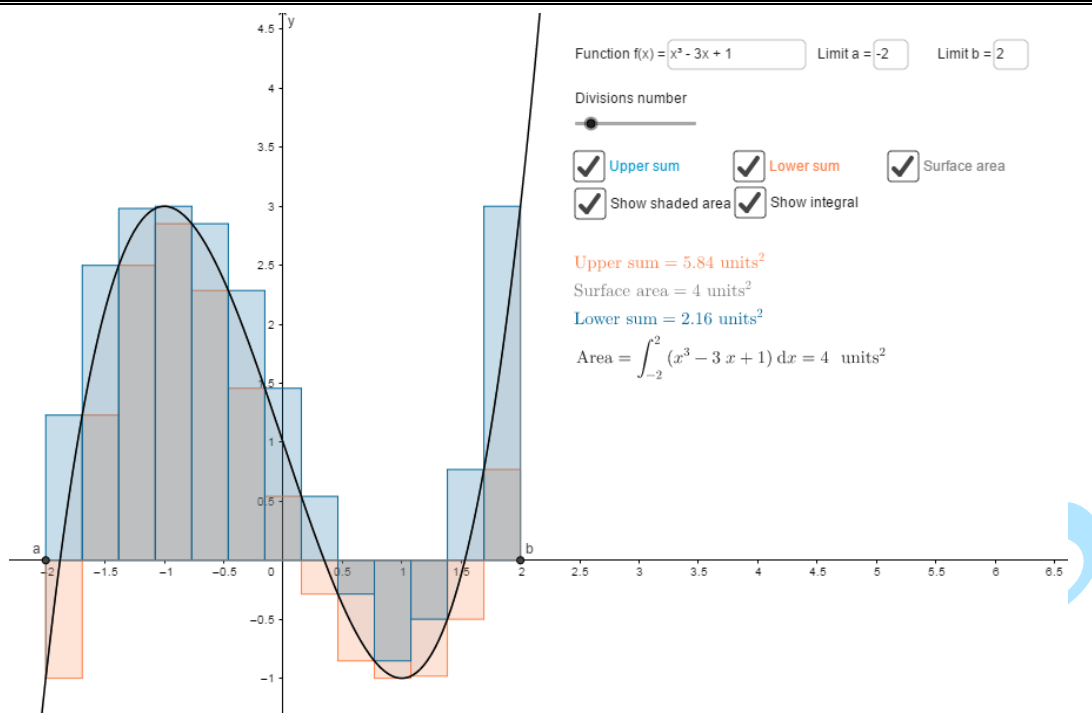


Fig. 3. Applet showing the lower and upper Riemann sums for a polynomial function

CONCLUSIONS

The static aspect of graphics used in books and in classes on boards require a well-trained imagination. Because Calculus involves the study of change and movement, the fundamental calculus concepts are better represented and understood together with the use of free programs with interactive representation and versatile capabilities that allows dynamic visualization of these concepts.

REFERENCES

- [HK08] **A. O. Hausknecht, R. E. Kowalczyk** - *Exploring calculus using innovative technology*, in: Proceedings of the 19th Annual International Conference on Technology in Collegiate Mathematics, Boston, MA, 2008.
- [HP07] **M. Hohenwarter, J. Preiner** - *Dynamic mathematics with GeoGebra*, in: The Journal of Online Mathematics and its Applications, Volume 7. Article ID:1448, 2007.
- [HAP10] **E. S. Haciomeroglu, L. Aspinwall, N. Presmeg** - *Contrasting Cases of Calculus Students' Understanding of Derivative Graphs*, in: Mathematical Thinking and Learning, 12(2), 152-176, 2010.
- [KRD08] **N. Kumar, R. Rose, J. D'Silva** - *Teachers' readiness to use technology*

in the classroom: An empirical study, in: European Journal of Scientific Research, 21(4), 603-616, 2008.

- [Lav06] **Z. Lavicza** - *Factors influencing the integration of Computer Algebra Systems into university-level mathematics education*, in: International Journal for Technology in Mathematics Education. 14(3), 2006.
- [SHE03] **S. L. Salas, E. Hille, G. J. Etgen** - *Calculus: One and several variables*. New York, NY: Wiley, 2003.
- [S+96] **S. Seltzer, S. Hilbert, J. Maceli, E. Robinson, D. Schwartz** - *An active approach to calculus*, in: New Directions for Teaching and Learning, 68, 83-90, 1996.
- [Tal97] **D. Tall** - *Functions and calculus*, in A. J. Bishop et al (Eds.), International Handbook of Mathematics Education, Dordrecht, the Netherlands: Kluwer, 1997.
- [Zim91] **W. Zimmerman** - *Visual Thinking in Calculus*, in: W. Zimmermann and S. Cunningham (Editors), Visualization in Teaching and Learning Mathematics, Washington, DC: Mathematical Association of America, 1991.