PARAMETER ESTIMATION OF COBB DOUGLAS PRODUCTION FUNCTION WITH MULTIPLICATIVE AND ADDITIVE ERRORS USING THE FREQUENTIST AND BAYESIAN APPROACHES

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ABSTRACT: Nonlinear Models are generally classified as intrinsically nonlinear and intrinsically linear based on the specification of the errors. This study was aimed at estimating the parameters of Cobb-Douglas production function with additive and multiplicative errors using the classical and Bayesian approaches. The classical nonlinear method considered is the Gauss-Newton iterative Method while the Bayesian estimation was carried out using the Metropolis-within-Gibbs with independent normal-Gamma prior. For the classical, the results showed that the estimates of the parameters of the Cobb-Douglas function with additive errors performed better than those for the multiplicative errors. However, similar estimates were obtained for both multiplicative and additive errors for the Bayesian approach. Overall, the Bayesian method performed better than the classical approach.
KEYWORDS*:* Cobb-Douglas Production function, Gauss-Newton Method, Normal-Gamma Prior, MCMC.

1. INTRODUCTION

The Cobb-Douglas production function was first introduced by Charles W. Cobb and Paul H. Douglas ([CD28]), although Knut Wicksell (1851-1926) reported the production output determined by the amount of labor involved and the amount of capital invested in the different industries of the world as the fairly universal law of production; on the contrary Cobb and Douglas ([CD28]) tested against this proposition by revealing that coefficients of two factors (labour and capital) considered were not constant over time or between the same sectors of the economy, These two factors of Cobb-Douglas aggregate production function preferred in the specification of growth theory were mentioned in the seminal contribution of Solow ([Sol56]). Houthakker ([Hou55]) showed how to aggregate out a Cobb-Douglas production from underlying Leontief production functions when their coefficients are jointly Pareto distributed. Jones ([Jon05]) partially built on this to show that a global Cobb-Douglas production function could arise from general constant returns to scale production function. Hajkova and Hurnik ([HH07]) reported that the application of the Cobb-Douglas production function was unreliable for the Czech economy when the labour share gradually increases for a more general form of production function. The error terms are usually assumed to be uncorrelated with mean zero and constant variance. The classical procedures in nonlinear regression are assessed with long-run properties under hypothetical repeated sampling, if the objective is the parameter estimation, then the aim is to obtain an estimate whose distribution is "close" to the true value. However, confidence intervals are not so simple to interpret. Berger and Wolpert ([BW88]) introduced Bayesian approach to nonlinear model and confirmed by Royall ([Roy97]) that a true likelihood approach is difficult to calibrate since all approaches based on classical criteria invalidate this likelihood property. In contract, another appealing characteristic is that the Bayesian approach to inference and interpretability may be derived via decision theory to be appropriate in nonlinear model.

Other researchers including Hoque ([Hoq91]), Bhatti ([Bha93]), Baltagi ([Bal96]), Bhatti and Owen ([BO96]), Bhatti ([Bha97]), Bhatti et al. ([BKC98]), Ingene and Lusch ([IL99]), Mok ([Mok02]), Hossain et al. ([HBA04]), Hajkova and Hurnik ([HH07]), Prajneshu ([Pra08]), Antony ([Ant09]), and Hossain et al. ([HA10]), amongst others have used linear regression models to measure the log-linear Cobb-Douglas (C-D) type production processes.

This paper was aimed at estimating the parameters of C-DP function with multiplicative and additive errors using the classical and Bayesian approaches.

The rest of this paper is organised as follows; section 2 discusses the models of Cobb-Douglas production function with additive and multiplicative errors, the theoretical concepts of the methods employed namely, classical and Bayesian methods are briefly discussed, empirical illustrations, findings and discussions follow in Section 4 and Section 5 concludes the paper.

2. MODEL

Consider y_i as a set of outputs of a production process, i = 1, 2, …, N, X_1, X_2 are two factors of production, capital and labour input respectively. β_0 is constant, β_1 and β_2 are the outputs of elasticity of capital and labour, coefficients of these factors, where u_i is the error terms (multiplicative error and additive error). Cobb Douglas Model in the choice of error is as follows:
Cobb-Douglas Regression Model with Multiplicative Error (C-DME) term is given as,

$$y_i = \beta_0 X_1^{\beta_1} X_2^{\beta_2} e^{u_i} \qquad (1)$$

Equation (1) is nearly always treated as a linear relationship by making a logarithmic transformation, which yields:

$$Y^* = \beta^*_0 + \beta_1 X^*_1 + \beta_2 X^*_2 + u_i \qquad (2)$$

Where β^*_0, is transformed intercept Y^*, X^*_1, X^*_2 are the transformed variables in eqn (1)
Cobb-Douglas Regression Model with Additive Error (C-DAE) term.

$$y_i = \beta_0 X_1^{\beta_1} X_2^{\beta_2} + u_i \qquad (3)$$

In the case of equation (3), the minimization of, $\sum_{i=1} u_i^2$is no longer a simple linear estimation problem. To estimate the production function, we need to know different types of non-linear estimation. In non-linear model it is not possible to give a closed form expression for the estimates as a function of the sample values, i.e., the likelihood function or sum of squares cannot be transformed so that the normal equations are linear. The idea of using estimates that minimize the sum squared errors is a data-analytic idea, not a statistical idea; it does not depend on the statistical properties of the observations (see [Chr01]). In most situation non-linear estimation problem can be solved by minimizing the error sum square estimation method using any of the optimization method (see [GQ978]). Gauss-Newton method is one of the methods which is used to estimate the parameters of model (3) ([HM15]).
By factoring out $\beta_0 X_1^{\beta_1} X_2^{\beta_2}$ and take the natural log of equation (3), gives

$$Y^{**} = \beta^{**}_0 + \beta_1 X^{**}_1 + \beta_2 X^{**}_2 + u^* \qquad (4)$$

where β^{**}_0 is transformed intercept, Y^{**}, $X^{**}_1$and X^{**}_2 are also the transformed variables in equation (3).

3. METHODOLOGY

3.1. Gauss Newton Method

The Gauss Newton Method is one of the classical ways of estimating the parameters of nonlinear models, therefore, the procedures stated below are the universal steps of estimating any form of nonlinear model either intrinsically nonlinear or intrinsically linear.
Suppose, we want to estimate a nonlinear model of the form

$$y_i = f(X_i, \beta) + u_i \qquad i = 1,2,\dots,N \qquad (5)$$

Where; y_i is response variable, $f(X_i, \beta)$ is the nonlinear form comprising of the explanatory variable and the coefficient of the model, and, the error component with mean 0 and variance $\sigma^2, u_i \sim N(0, \sigma^2)$
The matrix form of the model in equation (4) can be expressed as;

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \vdots \\ y_N \end{bmatrix}, \; X = \begin{bmatrix} X_{11} & X_{21} & X_{31} \\ X_{12} & X_{22} & X_{23} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ X_{1N} & X_{2N} & X_{3N} \end{bmatrix}, \; \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}, \text{ and}$$

$$U = \begin{bmatrix} u_1 \\ \vdots \\ \vdots \\ u_N \end{bmatrix}$$

The Gauss Newton method begins by expanding $f(X_i, \beta) = f_i(\beta)$ using Taylor's series up to the first derivative around a set of initial values, $\beta_j^{0i} = (\beta_0^{\ 0}$, $\beta_1^{\ 0}, \beta_2^{\ 0})$ and representing the required parameters appropriately. Set $\lambda_j = \beta_j - \beta_j^{\ 0}$, $Y_i^{\ 0} = f_i^{\ 0} = f(x_i, \lambda_j^{\ 0})$, and set initial values $\lambda_j^{\ 0} = \beta_0^{\ 0}$, $\beta_1^{\ 0}, \beta_2^{\ 0}$.
Using the OLS method, we obtain the OLS estimates by

$$\hat{\lambda} = \left(Z^{0\prime} Z^0\right)^{-1} Z^{0\prime}(D),$$

Where; D = Y - f^0, since, $\lambda^0{}_j = \beta^1{}_j - \beta_j^{\ 0}$, the revised estimate of β_j is $\beta^1{}_j$.
Hence, $\beta^1{}_j = \hat{\lambda}_j + \beta_j^{\ 0}$, the process is repeated to obtained desired estimates as a general rule.

3.2. Bayesian Approach

The Markov Chain Monte Carlo technique (MCMC) is employed by Metropolis-Hasting algorithm.

Procedures for analyzing the "Cobb-Douglas with Multiplicative and additive error term are the same.

3.3. The Likelihood Function and Prior (C-DME)

The log normal regression mean $(X^*\beta^*)$ with error precision $(h = \frac{1}{\sigma^2})$, random variable ln(y) as the data information using the multivariate Normal density, the log likelihood can be given as:

$$P(y^*|\beta,h) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \, exp\left[-\frac{h}{2}(y^* - X^*\beta)'(y^* - X^*\beta)\right] \quad (6)$$

The previous information about a study before seeing the data denoted by independent prior $P(\beta,h)$. Given $P(\beta,h) = P(\beta).P(h)$ with $P(\beta)$ being Normal and $P(h)$ being Gamma

$$P(\beta) = \frac{1}{(2\pi)^{\frac{N}{2}}} |\underline{V}|^{\frac{1}{2}} \, exp\left[-\frac{1}{2}\left(\beta - \underline{\beta}\right)' \underline{V}^{-1}(\beta - \underline{\beta})\right]$$

and

$$P(h) = C^{-1}{}_G h^{\frac{\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right)$$

Where, C_G is the integrating constant for Gamma, It is deduced that: $E[\beta|y^*] = \underline{\beta}$ is the prior mean of β and $Var(\beta|h) = \underline{V}$ is the prior covariance matrix of β with the mean of h as $\underline{s}^{-2}$and $\underline{v}$ degree of freedom.

3.4. The Posterior (C-DME)

The posterior is proportional to the prior and the likelihood, which is also the information obtained after seeing the data and some mathematical techniques being applied, usually denoted

$$P(\beta,h|y) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \, exp\left[-\frac{h}{2}(y^* - X^*\beta)'(y^* - X^*\beta)\right] \, x \, \frac{1}{(2\pi)^{\frac{N}{2}}} |\underline{V}|^{\frac{1}{2}} \, exp\left[-\frac{1}{2}\left(\beta - \underline{\beta}\right)' \underline{V}^{-1}(\beta - \underline{\beta})\right] \, x \, C^{-1}{}_G h^{\frac{\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right)$$

$$P(\beta,h|y) \propto exp\left[-\frac{1}{2}\{h(y^* - X^*\beta)'(y^* - X^*\beta)\right] + \left[\left(\beta - \underline{\beta}\right)' \underline{V}^{-1}(\beta - \underline{\beta})\}\right] \, x \, h^{\frac{N+\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) \quad (7)$$

this joint posterior density for β and h does not take any well-known distributional form; so it cannot be solved analytically but only through a posterior simulation method.

By ignoring the terms that do not involve β in equation (7) we obtain,

$$P(\beta|y^*,h) \propto exp\left[-\frac{1}{2}\left\{\left(\beta - \bar{\beta}\right)'\bar{V}^{-1}\left(\beta - \bar{\beta}\right)\right\}\right] \quad (8)$$

Which implies that $\beta|y^*,h \sim N(\bar{\beta},\bar{V})$ a Multivariate Normal density,

where, $\bar{V} = (\underline{V}^{-1} + hX^{*\prime}X^*)^{-1}$ and $\bar{\beta} = \bar{V}(hX^{*\prime}Y^* + \underline{V}^{-1}\underline{\beta})$

Similarly, by treating equation (7) as a function of h ignoring terms that do not involve h we can obtain

$$P(h|y^*,\beta) \propto h^{\frac{N+\underline{v}-2}{2}} \, exp\left[-\frac{h}{2}\{(y^* - X^*\beta)'(y^* - X^*\beta) + \underline{vs}^2\}\right] \quad (9)$$

This also implies that $h|y^*,\beta \sim G(\bar{s}^{-2},\bar{v})$, a Gamma density,

where, $\bar{v} = N + \underline{v}$ and $s^{-2} = \frac{(y^* - X^*\beta)'(y^* - X^*\beta) + \underline{vs}^2}{\bar{v}}$

The formulae of equation (8) and (9) look familiar to those of the conjugate normal-gamma priors now but it does not relate directly to the posterior of interest, since

$$P(\beta,h|y^*) \neq P(\beta|y^*,h) \, x \, P(h|y^*,\beta).$$

Therefore, the conditional posteriors in equation (8) and (9) do not directly tells everything about the posterior, $P(\beta,h|y^*)$. Nevertheless, there is a posterior simulator called the Metropolis- Within-Gibbs which makes use of the conditional posteriors like (8) and (9) to produce random draws $\beta^{(s)}$ and $h^{(s)}$ for $s = 1,2,\ldots,S$ which can be averaged to produce estimates of the posterior properties just as the Monte Carlo integration.

3.5. The Likelihood Function and Prior (C-DAE)

$$P(y^{**}|\beta,h) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \, exp\left[-\frac{h}{2}(y^{**} - X^{**}\beta)'(y^{**} - X^{**}\beta)\right] \quad (10)$$

The information at hand about a particular study before seeing the data, denoted by the independent prior by $P(\beta,h)$

$$P(\beta) = \frac{1}{(2\pi)^{\frac{N}{2}}} |\underline{V}|^{\frac{1}{2}} \, exp\left[-\frac{1}{2}\left(\beta - \underline{\beta}\right)' \underline{V}^{-1}(\beta - \underline{\beta})\right]$$

and

$$P(h) = C^{-1}{}_G h^{\frac{\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right)$$

$$P(\beta, h|y^{**}) \propto exp\left[-\frac{1}{2}\{h(y^{**} - X^{**}\beta)'(y^{**} - X^{**}\beta) + \left(\beta - \underline{\beta}\right)' \underline{V}^{-1}\left(\beta - \underline{\beta}\right)\}\right] x\ h^{\frac{N+\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right)$$

3.6. Posterior (C-DAE)

$$P(\beta, h|y^{**}) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} exp\left[-\frac{h}{2}(y^{**} - X^{**}\beta)'(y^{**} - X^{**}\beta)\right] x\ \frac{1}{(2\pi)^{\frac{N}{2}}} |\underline{V}|^{\frac{1}{2}} exp\left[-\frac{1}{2}\left(\beta - \underline{\beta}\right)' \underline{V}^{-1}(\beta - \underline{\beta})\right] x\ C^{-1}{}_G h^{\frac{\underline{v}-2}{2}} \exp\left(\frac{-h\underline{v}}{2\underline{s}^{-2}}\right) \quad (11)$$

By ignoring the terms that do not involve β we obtain,

$$P(\beta|y^{**}, h) \propto exp\left[-\frac{1}{2}\left\{\left(\beta - \bar{\beta}\right)' \bar{V}^{-1}(\beta - \bar{\beta})\right\}\right] \quad (12)$$

Which implies that $\beta|y^{**}, h \sim N(\bar{\beta}, \bar{V})$ a Multivariate Normal density,
where, $\bar{V} = (\underline{V}^{-1} + hX^{**\prime}X^{**})^{-1}$ and $\bar{\beta} = \bar{V}(hX^{**\prime}Y^{**} + \underline{V}^{-1}\underline{\beta})$
Similarly, by treating equation (11) as a function of h ignoring terms that do not involve h we can obtain

$$P(h|y^{**}, \beta) \propto h^{\frac{N+\underline{v}-2}{2}} exp\left[-\frac{h}{2}\{(y^{**} - X^{**}\beta)'(y^{**} - X^{**}\beta) + \underline{vs}^2\}\right] \quad (13)$$

This also implies that $h|y^{**}, \beta \sim G(\bar{s}^{-2}, \bar{v})$, a Gamma density,
where, $\bar{v} = N + \underline{v}$ and

$$s^{-2} = \frac{(y^{**} - X^{**}\beta)'(y^{**} - X^{**}\beta) + \underline{vs}^2}{\bar{v}}$$

The formulae of equation (12) and (13) look familiar to those of the conjugate normal-gamma priors now but it does not relate directly to the posterior of interest, since

$$P(\beta, h|y^{**}) \neq P(\beta|y^{**}, h)\ x\ P(h|y^{**}, \beta).$$

Therefore, the conditional posteriors in equation (12) and (13) do not directly tells everything about the posterior, $P(\beta, h|y^{**})$ .Nevertheless, there is a posterior simulator called the Metropolis- Within-Gibbs which makes use of the conditional posteriors like (12) and (13) to produce random draws $\beta^{(s)}$ and $h^{(s)}$for $s = 1, 2, \ldots, S$ which can be averaged to produce estimates of the posterior properties just as the Monte Carlo integration.

4. EMPIRICAL ILLUSTRATIONS

The data used for analysis under this study were simulated using Monte Carlo simulation technique. The explanatory variables is drawn independently from a uniform [0,1] distribution, $X_{ij} \sim U[0,1], i = 0,1,2\ and\ j = 1,2, \ldots, N$. Values fixed for the regression coefficients $\beta_i, i = 0,1,2$ i.e $\beta_0 = 15, \beta_1 = 0.85, \beta_2 = 0.15$. The error component drawn from a standard normal distribution, $\varepsilon_j \sim N[0,1]$, incorporated into non- linear Cobb-Douglas model to obtain the response variable (data of interest) using four (4) different sample sizes N=30, N=50, N=100, N=150 setting up a 10,000 iterations and a burning of 1000 to attain convergence of the posterior estimates.

Table 1: The Additive and Multiplicative Error Based Cobb-Douglas Production Functions

		ADDITIVE-ERROR-BASED			MULTIPLICATIVE-ERROR-BASED		
	Prior (SD)	GNM (S.E)	Posterior (S.D)	NSE	GNM (S.E)	Posterior (S.D)	NSE
N=50							
β_0 (15)	0.001 (500.40)	15.445690 (0.446410)	15.815700 (0.058995)	0.000622	10.879500 (0.221240)	10.882991 (0.056498)	0.000595
β_1 (0.85)	0.005 (90.7)	0.845610 (0.046720)	0.934454 (0.035239)	0.000372	0.829240 (0.119940)	0.829921 (0.030103)	0.000317
β_2 (0.15)	0.01 (449.097)	0.186740 (0.026600)	0.160182 (0.045231)	0.000477	0.18113 (0.091720)	0.180948 (0.023506)	0.000247
N=100							
β_0 (15)	0.001 (500.40)	14.749440 (0.289740)	15.69809 (0.042246)	0.000445	12.723619 (0.170030)	12.726243 (0.040442)	0.000426
β_1 (0.85)	0.005 (90.7)	0.832800 (0.030560)	1.027199 (0.027920)	0.000294	0.791700 (0.108620)	0.792139 (0.025392)	0.000267
β_2 (0.15)	0.01 (449.097)	0.128510 (0.015610)	0.243364 (0.028505)	0.000301	0.174650 (0.084730)	0.174478 (0.020215)	0.000213
N=250							
β_0 (15)	0.001 (500.40)	15.038910 (0.221834)	15.67601 (0.027615)	0.000291	13.243674 (0.107760)	13.245939 (0.025597)	0.000269
β_1 (0.85)	0.005 (90.7)	0.841602 (0.020145)	0.837894 (0.018495)	0.000195	0.806600 (0.068750)	0.806481 (0.016405)	0.000173
β_2 (0.15)	0.01 (449.097)	0.159087 (0.009644)	0.159936 (0.014683)	0.000155	0.157030 (0.062110)	0.157264 (0.014474)	0.000152
N=500							
β_0 (15)	0.001 (500.40)	14.976400 (0.137190)	13.573850 (0.018704)	0.000197	15.170155 (0.084730)	15.17211 (0.017879)	0.000186
β_1 (0.85)	0.005 (90.7)	0.841800 (0.014130)	0.759519 (0.011394)	0.000120	0.845710 (0.04892)	0.845861 (0.010812)	0.000114
β_2 (0.15)	0.01 (449.097)	0.148060 (0.007060)	0.132379 (0.011079)	0.000117	0.176550 (0.049910)	0.176499 (0.009809)	0.000103

5. FINDINGS AND DISCUSSIONS

Table shows the estimates obtained using the Cobb-Douglas production function with additive error and multiplicative error term. The priors used are 0.001 (500.40), 0.005 (90.7), 0.01 (449.097) under the various sample sizes of study with the standard errors (SD) in bracket, a metropolis Hasting Within Gibbs algorithm was used with the Normal-Gamma prior to obtain the posterior estimates as recorded above using the additive and multiplicative error models. The result shows that the multiplicative error model behaves better than the additive error model.

Furthermore, the nuisance parameter β_0 shows a fluctuated and unsteady behaviour by producing values that are close to the true values for additive error model using the Gauss-Newton method while the estimates are far from the true values as sample size increased for the multiplicative error model. The estimates obtained by the Bayesian method are also closer to the true parameter values for the additive model than for the multiplicative model. In general, the parameter estimates from additive model are better than those produced by the multiplicative model for both classical and Bayesian approaches.

Lastly, the standard error of the GNM and the Numerical Standard Error of the posterior decrease consistently as sample size increases, however, the standard errors of the Bayesian approach are generally better than those of the classical approach.

6. CONCLUSION

This paper has been able to achieve the objectives by checking which model is more appropriate between the Cobb-Douglas production function with additive error term and the Cobb-Douglas production function with multiplicative error term, the error model in the additive behaved better than the multiplicative error model. Parameters estimated for the two scenarios of Cobb-Douglas production function (i.e. multiplicative error and additive error term) using the Bayesian approach and Frequentist approach, the results made it obvious that the

Bayesian approached is preferred in using the Cobb-Douglas production function based on the minimal numerical standard errors produced in between the two approaches under investigation. The level of efficiency in the Bayesian estimation as sample size increases is shown as the numerical standard errors decreased with increase in sample sizes.

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