BALANCED INCOMPLETE SEQUENCE CROSSOVER DESIGN FOR FIRST ORDER RESIDUAL EFFECT

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ABSTRACT: Crossover designs are of the type in which sequence of different treatments are administered one at a time over a certain period of time, during which the presence of residual effect or rather carryover effect can longer be ignored. This paper therefore presents two first order residual effect, that is effect on the immediate next period after the period of treatment application. The first method constructs designs for any number of treatment, v for any prime number that has x=2 as primitive root of the associated Galois field using the two algorithms $I_1 = (x^0, x^m , x^{2m},. . .,x^{2k-x})$ and $I_2 = (x^1, x^{m+1} , x^{2m+1},. . ., x^{2k-x+1})$. The second method is also for the construction of designs for any number treatment, v for any prime number that has x=3 as the primitive root of the corresponding Galois field with two algorithms $I_1 =(x^0, x^m , x^{2m},. . .,x^{2k+m-x-1})$ and $I_2 = (x^1, x^{m+1} , x^{2m+1},. . ., x^{2k+m-x})$ By exploiting cyclic development of v-1 initial sequences of treatment order, universally optimal balanced crossover design of the first order for the two methods were constructed in this paper and the two methods generate every non-zero elements of any prime number.

KEYWORD: Crossover Design; Galois Field; Balanced Design; Initial Sequence; Cyclic Development.

1. INTRODUCTION

Crossover design is a design in which different sequence of treatments are administered into a subject over several periods of time.

There are considerable earlier works on crossover designs by several authors, Jones and Kenward ([JK03]), Hinkelmann and Kempthorne ([HK05]) have shed more light on usefulness of crossover designs. Grizzle ([Gri65]), Brown ([Bro80]), Andrew and Joseph ([AJ86]) discussed methods of construction of two treatments two periods balanced crossover designs. Williams ([Wil49]) discussed how latin square can be used to obtain balanced crossover designs for more than two treatments when the number of treatment t is even and odd by using column and row method approach. Durso ([Dur84]) and Mausumi ([Mau02]) presented a method of construction for counterbalancing for immediate sequencial effects crossover design.

Sprott ([Spr54]) presented a method for construction of balanced incomplete sequence crossover designs using series of balanced incomplete block design in which the number of period is less than the number of treatment v, where v is a prime or prime power. Patterson and Lucas ([PL62]) developed a method of construction of balanced incomplete sequence crossover designs by constructing a balanced design for each set of k treatments that form a block of a balanced incomplete block design. Kanchan and Rumana ([KR13]) also developed a method of construction for an incomplete change-over design balanced for first and second-order residual effect and; Mithilesh and Archana ([MA15]) developed balanced incomplete sequence crossover design of first order residual effect. Different methods of balanced incomplete sequence crossover design have been presented for various prime number of treatment with different primitive roots for a single algorithm. Mihilesh and Archana ([MA15]) presented method of balanced incomplete sequence crossover design of first order residual effect that contain two algorithms using different primitive root to generate non-zero element for prime number of treatments therefore the purpose of this research is to present two methods of balanced incomplete sequence crossover design of first order residual effect with two algorithms along with a particular primitive root for any prime number of treatment which primitive root generates their non-zero elements. Research methodology used in this work is $I_1 = (x^0, x^m , x^{2m},. . .,x^{2k-x})$ and $I_2 = (x^1, x^{m+1} , x^{2m+1} ,. . ., x^{2k-x+1})$ for primitive root x=2; $I_1 = (x^0, x^m , x^{2m} ,. . ., x^{2k+m-x-1})$, $I_2 = (x^1, x^{m+1}, x^{2m+1},. . ., x^{2k+m-x})$

Residual or carryover effect is the effect of treatment from the previous time period on the response at the current time period. In effect a First Order residual effect is the carryover effect that occur in the immediate next period after the period of treatment application.

This paper presents two new construction methods for balanced incomplete sequence crossover design for first order residual effect, first BISCOD where v is any prime number in which primitive element, x=2 generates its non-zero elements; second BISCOD where v is any prime number in which

primitive element, x=3 generates its non-zero elements.
For construction, if the number of treatment v is even, a suitable cyclic Latin square is required. If the number of treatment t is odd, two cyclic Latin square are required for the designs in order that the number of replicate must be a multiple number of treatment. Illustrations of column method and also row method are provided in examples below.

Example 1: Construction of balanced crossover design for even number of treatment, say v=4 column method.

Table 1. Balanced crossover design for even number of treatment, v=4

Period	Sequence			
	1	**2**	**3**	**4**
1	1	2	3	0
2	0	1	2	3
3	2	3	0	1
4	3	0	1	2

Remark on example: each treatment is preceded by each other treatment once.

Example 2: Construction of balanced crossover design for odd number of treatments, say v=5 column method.

Table 2. Balanced crossover design for odd number of treatment, v=5

Period	Sequence									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	0	2	3	4	0	1
2	0	1	2	3	4	3	4	0	1	2
3	2	3	4	0	1	1	2	3	4	0
4	4	0	1	2	3	4	0	1	2	3
5	3	4	0	1	2	0	1	2	3	4

Remark on example: each treatment is preceded by each other treatment twice.

Example 3: Construction of balanced crossover design for number of treatments, say v=4 row method.

Table 3. Balanced crossover design for odd number of treatment, v=4

Period	Sequence			
	1	**2**	**3**	**4**
1	0	1	3	2
2	1	2	0	3
3	2	3	1	0
4	3	0	2	1

Remark on example: any treatment is preceded equally often in the design by each other treatments t-1 times.

Example 4: Construction of balanced crossover design for number of treatments, say v=5.

Table 4. Balanced crossover design for odd number of treatment, v=5 row method

Period	Sequence				
	1	**2**	**3**	**4**	**5**
1	0	1	4	2	3
2	1	2	0	3	4
3	2	3	1	4	0
4	3	4	2	0	1
5	4	0	3	1	2
6	3	2	4	1	0
7	4	3	0	2	1
8	0	4	1	3	2
9	1	0	2	4	3
10	2	1	3	0	4

Remark on example: each treatment is preceded by each other treatment an unequal number of times and one of the treatments preceded itself once.

2. CONDITION TO BE SATISFIED FOR BALANCED CROSSOVER DESIGN

Conditions for the class of competing designs to search for a universally optimal crossover design was given by Dey et al ([DGS83]) as follows:

(i) No treatment symbol occurs in a given sequence more than once;

(ii) Each treatment symbol occurs in a given period an equal number of times;

(iii) Every pair of treatments occurs together in the same number of sequences; and

(iv) Every pair of treatments occurs together in the same number of curtailed sequences formed by omitting the final period.

3. CONSTRUCTION OF DESIGN

3.1 Case 1 Design Construction when x=2

Consider the balanced incomplete block design (BIBD) with v = 2k+1, b = 2(2k+1), r = 2k, k = $\frac{v-1}{2}$, λ = k-1, where v is any prime number in which primitive root, x = 2 generates its non-zero elements. The two initial sequences are

$$I_1 = (x^0, x^m, x^{2m}, \ldots, x^{2k-x}) \quad (1a)$$
$$I_2 = (x^1, x^{m+1}, x^{2m+1}, \ldots, x^{2k-x+1}) \quad (1b)$$

(1) From the above two initial sequences 1a and 1b, suppose any one is chosen
(2) Multiplying with every non-zero element of GF(v) to obtain (v-1) initial sequences
(3) A universally optimal balanced incomplete sequence crossover design of first order with parameters v = 2k+1, N = 2k(2k+1), $k = \frac{v-1}{2}$ is constructed by developing the (v-1) initial sequences cyclically.

Example 1: For v = 11, m = 2, x = 2 is a primitive element of GF (11). Then there exists a BISCOD with two initial sequences (1, 4, 5, 9, 3) and (2, 8, 10, 7, 6).
Consider the first initial sequence, (1, 4, 5, 9, 3), then ten initial sequences will be formed as below.

Table 5. Balanced crossover design for prime number of treatment, v= 11

1	2	3	4	5	6	7	8	9	10
4	8	1	5	9	2	6	10	3	7
5	10	4	9	3	8	2	7	1	6
9	7	5	3	1	10	8	6	4	2
3	6	9	1	4	7	10	2	5	8

We get a universally optimal balanced COD of first order with parameters v = 11, N = 110, K = 5 by developing the ten initial sequences above mod 11 cyclically.

3.2 Case 2 Design Construction when x=3

Consider the balanced incomplete block design (BIBD) with v = 2k+1, b = 2(2k+1), r = 2k, $k = \frac{v-1}{2}$, λ = k-1, where v is any prime number in which primitive root, x=3 generates its non-zero elements The two initial sequences are

$$I_1 = (x^0, x^m, x^{2m}, \ldots, x^{2k+m-x-1}) \quad (2a)$$
$$I_2 = (x^1, x^{m+1}, x^{2m+1}, \ldots, x^{2k+m-x}) \quad (2b)$$

(1) From the above two initial sequences suppose any one is chosen
(2) Multiplying with every non-zero element of GF(v) to obtain (v-1) initial sequences
(3) A universally optimal balanced incomplete sequence crossover design of first order with parameters v = 2k+1, N = 2k(2k+1), $k = \frac{v-1}{2}$ is constructed by developing the (v-1) initial sequences.

Example 2: For v = 17, m = 2, x = 3 is a primitive element of GF (17). Then there exist a BISCOD with two initial sequences (1, 9, 13, 15, 16, 8, 4, 2) and (3, 10, 5, 11, 14, 7, 12, 6).
Consider the second initial sequence (3, 10, 5, 11, 14, 7, 12, 6), then sixteen initial sequences will be:

Table 6. Balanced crossover design for prime number of treatment, v=17

3	6	9	12	15	1	4	7	10	13	16	2	5	8	11	14
10	3	13	6	16	9	2	12	5	15	8	1	11	4	14	7
5	10	15	3	8	13	1	6	11	16	4	9	14	2	7	12
11	5	16	10	4	15	9	3	14	8	2	13	7	1	12	6
14	11	8	5	2	16	14	10	7	4	1	15	12	9	6	3
7	14	4	11	1	8	15	5	12	2	9	16	6	13	3	10
12	7	2	14	9	4	16	11	6	1	13	8	3	15	10	5
6	12	1	7	13	2	8	14	3	9	15	4	10	16	5	11

We get a universally optimal balanced COD of first order with parameters v = 17, N = 272, K = 8 by developing the sixteen initial sequences above mod 17 cyclically.

4. DISCUSSION OF RESULT

For the two methods of construction, it was discovered that taken one out of the two initial sequences from balance incomplete block design and multiply with every non-zero element there exist balance incomplete sequence crossover design with v-1 initial sequences. Universally optimal balanced crossover design of first order was obtained through cyclic development of v-1 initial sequences.

5. CONCLUSION

The two methods of construction can be used to construct number of treatments, v which is any prime number in which primitive element, x=2 and x= 3 generates its non-zero elements to obtain universally optimal balanced crossover design of first order residual effect.

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