IMPROVED ESTIMATORS OF FINITE POPULATION VARIANCE USING UNKNOWN WEIGHT OF AUXILIARY VARIABLE

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ABSTRACT: In this paper, we proposed a class of ratio estimators for finite population variance. The proposed estimators were obtained by transforming [S+18] estimators. The properties (Bias and Mean Square Errors (MSEs)) of the proposed estimators have been obtained and the conditions for their efficiencies over some existing variance estimators have been established. The efficiencies of proposed estimators based on optimal value of the constant, exhibit significant improvement over the estimators considered in the study. The numerical illustration was also conducted to corroborate the theoretical results. The results of the empirical study show that the proposed estimators are more efficient over existing estimators.

***KEYWORDS*:** Efficiency, Auxiliary variable, Median, Mean Square Error, Ratio Estimator.

1. INTRODUCTION

The use of auxiliary information, being constant with unit (e.g. population variance, population mean, population standard deviation, etc.) or unit free constant (e.g. Coefficients of variation, Kurtosis, Skewness etc.), can enhance the efficiency at the estimation stage. [AAS16, GS08, SK12, TS12, YK13] utilized this concept to improve the efficiency of ratio and product type estimators for estimating the population variance as well as population mean of study variable. To effectively estimate the population parameter of the variable of interest, there is need for the population values of the auxiliary variables. When auxiliary information is available researchers are able to utilize it in methods of estimation to obtain the most efficient estimator [Coc77]. In many situations, information on the auxiliary is required either at the designing stage or estimation stage or both stages, to increase precision of the estimators. Ratio, Product and regression estimators are often used when advance knowledge of population variance of the auxiliary variable is readily available.

Estimation of the population variance of the study variable Y has received a considerable attention from experts engaged in survey statistics. For example, in agriculture the variation in production of crop is required for further planning or in manufacturing industries and pharmaceutical laboratories, the variation life of their products is a necessity for their quality control. Although, in literature, a great variety of techniques have been used mentioning the use of auxiliary information by means of ratio, product and regression methods for estimating population variance and other parameters [KO13].

In this paper, improved class of ratio estimators for estimating finite population variance has been proposed with objective to produce efficient estimators and properties have been established.

Let $\Omega = (1, 2, 3, \ldots, N)$ be a population of size N and Y, X be two real valued functions having values$(Y_i, X_i) \in \mathbb{R}^+ > 0$ on the i^{th} unit of $U(1 \leq i \leq N)$. Let S_y^2 and S_x^2 be the finite population variance of Y and X respectively and s_y^2 and s_x^2 be respective sample variances based on the random sample of size n drawn without replacement. The following are the other notations used throughout this paper.

N : Population size
n : Sample size
Y : Study variable
X : Auxiliary variable
$\bar{y}, \bar{x}$: Sample means of study and auxiliary variables
$\bar{Y}, \bar{X}$: Population means of study and auxiliary variables
ρ : Coefficient of correlation
C_y, C_x : Coefficient of variations of study and auxiliary variables
$\beta_{2(y)}$: Coefficient of kurtosis of study variable
$\beta_{2(x)}$ Coefficient of kurtosis of auxiliary variable
M_d : Median of the auxiliary variable

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i\,, \qquad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i,$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i\,, \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i\,, \qquad \gamma = \frac{1-f}{n},$$

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2\,, s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2,$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2\,, S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2$$

1.1 Existing Estimator in Literature

The sample variance estimator of the finite population variance is defined as

$$t = s_y^2 \tag{1}$$

which is an unbiased estimator of finite population variance $\left(S_y^2\right)$ and its variance is

$$Var(t) = \gamma S_y^4\left(\beta_{2(y)} - 1\right) \tag{2}$$

[Isa83] proposed a ratio type variance estimator for the finite population variance $\left(S_y^2\right)$ when the finite population variance $\left(S_x^2\right)$ of auxiliary variable X is known. The bias and its mean squared error are given below:

$$\hat{S}_R^2 = s_y^2\frac{S_x^2}{s_x^2} \tag{3}$$

$$Bias\ (\hat{S}_R^2) = \gamma S_y^2\left[\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{4}$$

$$MSE\ (\hat{S}_R^2) = \gamma S_y^4\begin{bmatrix}\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right)\\ -2\left(\lambda_{22} - 1\right)\end{bmatrix} \tag{5}$$

[KC06] proposed a class of ratio type estimators for finite population variance by imposing Coefficient of variation and Coefficient of kurtosis on the work of [Isa83] as:

$$\hat{S}_{kc_1}^2 = s_y^2\left(\frac{S_x^2+C_x}{s_x^2+C_x}\right) \tag{6}$$

$$\hat{S}_{kc_2}^2 = s_y^2\left(\frac{S_x^2+\beta_{x(2)}}{s_x^2+\beta_{x(2)}}\right) \tag{7}$$

$$\hat{S}_{kc_3}^2 = s_y^2\left(\frac{S_x^2\beta_{x(2)}+C_x}{s_x^2\beta_{x(2)}+C_x}\right) \tag{8}$$

$$\hat{S}_{kc_4}^2 = s_y^2\left(\frac{S_x^2C_x+\beta_{x(2)}}{s_x^2C_x+\beta_{x(2)}}\right) \tag{9}$$

$$Bias\ (\hat{S}_{kc_i}^2) = \gamma A_i S_y^2\left[A_i\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{10}$$

where i= 1,2,3,4

$$MSE\ (\hat{S}_{kc_i}^2) = \gamma S_y^4\begin{bmatrix}\left(\beta_{2(y)} - 1\right) + A_i^2\left(\beta_{2(x)} - 1\right)\\ -2A_i\left(\lambda_{22} - 1\right)\end{bmatrix} \tag{11}$$

where i= 1,2,3,4

Here:

$$A_1 = \frac{S_x^S}{s_x^2+C_x},\quad A_2 = \frac{S_x^2}{S_x^2+\beta_{2(x)}},\quad A_3 = \frac{S_x^2\beta_{2(x)}}{S_x^2\beta_{2(x)}+C_x},$$

$$A_4 = \frac{S_x^2C_x}{S_x^2C_x+\beta_{2(x)}}$$

[SK12] proposed ratio type estimator for finite population variance using quartile median of auxiliary variable as:

$$\hat{S}_{sk}^2 = s_y^2\left(\frac{S_x^2+Md}{s_x^2+Md}\right) \tag{12}$$

$$Bias\ (\hat{S}_{sk}^2) = \gamma A_{sk}S_y^2\left[A_{sk}\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{13}$$

$$MSE\ (\hat{S}_{sk}^2) = \gamma S_y^4\begin{bmatrix}\left(\beta_{2(y)} - 1\right) + A_{sk}^2\left(\beta_{2(x)} - 1\right)\\ -2A_{sk}\left(\lambda_{22} - 1\right)\end{bmatrix} \tag{14}$$

where

$$A_{sk} = \frac{S_x^2}{S_x^2 + Md}$$

[S+18] proposed a class of ratio type variance estimators utilizing different parameters of auxiliary variable as:

$$\hat{S}_{NK1}^2 = s_y^2\left(\frac{S_x^2+C_xS_x}{s_x^2+C_xS_x}\right) \tag{15}$$

$$\hat{S}_{NK2}^2 = s_y^2\left(\frac{S_x^2+C_x\bar{X}}{s_x^2+C_x\bar{X}}\right) \tag{16}$$

$$\hat{S}_{NK3}^2 = s_y^2\left(\frac{S_x^2+C_xMd}{s_x^2+C_xMd}\right) \tag{17}$$

$$Bias\ (\hat{S}_{NKi}^2) = \gamma A_{NKi}S_y^2\begin{bmatrix}A_{NKi}\left(\beta_{2(x)} - 1\right)\\ -\left(\lambda_{22} - 1\right)\end{bmatrix} \tag{18}$$

i=1, 2, 3

$$MSE\ (\hat{S}_{NKi}^2) = \gamma S_y^4\begin{bmatrix}\left(\beta_{2(y)} - 1\right) + A_{NKi}^2\left(\beta_{2(x)} - 1\right)\\ -2A_{NKi}\left(\lambda_{22} - 1\right)\end{bmatrix} \tag{19}$$

i=1,2,3

$$A_{NK1} = \frac{S_x^2}{S_x^2 + C_xS_x},\qquad A_{NK2} = \frac{S_x^2}{S_x^2 + C_x\bar{X}},$$

$$A_{NK3} = \frac{S_x^2}{S_x^2 + C_xMd}$$

2. PROPOSED ESTIMATOR

Motivated by the work of [S+18], we proposed a class of ratio estimators of finite population variance as:

$$\hat{S}_{MJ_1}^2 = k_1s_y^2\left(\frac{S_x^2+C_xS_x}{s_x^2+C_xS_x}\right) \tag{20}$$

$$\hat{S}_{MJ_2}^2 = k_2s_y^2\left(\frac{S_x^2+C_x\bar{X}}{s_x^2+C_x\bar{X}}\right) \tag{21}$$

$$\hat{S}_{MJ_3}^2 = k_3s_y^2\left(\frac{S_x^2+C_xMd}{s_x^2+C_xMd}\right) \tag{22}$$

Where $k_i\ (i = 1, 2, 3)$ are unknown weights to be determined such that the MSEs of the proposed estimators $\hat{S}_{MJ_i}^2$ are minimized.

2.1 Properties of The Proposed Estimators

2.1.1 Bias and MSE of $\hat{S}^2_{MJ_i} = 1, 2, 3.$

In order to determine the bias and MSE of $\hat{S}^2_{MJ_i} = 1, 2, 3$, need to express equations (20), (21), and (22) in general form as:

$$\hat{S}^2_{MJ_i} = k_i s_y^2 \left(\frac{S_x^2 + AB}{s_x^2 + AB}\right) \qquad \text{i= 1, 2, 3} \qquad (23)$$

Table 1: Values of A and B in Proposed Estimators

Estimator	A	B
$\hat{S}^2_{MJ_1}$	C_x	S_x
$\hat{S}^2_{MJ_2}$	C_x	$\bar{X}$
$\hat{S}^2_{MJ_3}$	C_x	Md

Defining $e_0 \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 \frac{s_x^2 - S_x^2}{S_x^2}$ such that $s_y^2 = S_y^2(1 + e_0)$, and $s_x^2 = S_x^2(1 + e_1)$, from the definition of e_0 and e_1, obtaining

$$\left.\begin{array}{l} E(e_0) = E(e_1) = 0,\ E(e_0^2) = \gamma(\beta_{2(y)} - 1) \\ E(e_1^2) = \gamma(\beta_{2(x)} - 1), E(e_0 e_1) = \gamma(\lambda_{22} - 1) \end{array}\right\} \quad (24)$$

Now expressing (23) in error terms as:

$$\hat{S}^2_{MJ_i} = S_y^2(1 + e_0)\, k_i \left(\frac{S_x^2 + AB}{S_x^2(1 + e_1) + AB}\right) \qquad (25)$$

Simplifying (25) up to first order approximation, it reduces to (26) as:

$$\hat{S}^2_{MJ_i} = S_y^2(k_i + k_i e_0 - k_i t_i e_1 - k_i t_i e_0 e_1 + k_i t_i^2 e_1^2) \quad (26)$$

Where

$$t_1 = \frac{S_x^2}{S_x^2 + C_x S_x}, t_2 = \frac{S_x^2}{S_x^2 + C_x \bar{X}}, t_3 = \frac{S_x^2}{S_x^2 + C_x Md}$$

Subtracting S_y^2 from both sides of (26):

$$\hat{S}^2_{MJ_i} - S_y^2 = S_y^2 \begin{pmatrix} k_i + k_i e_0 - k_i t_i e_1 \\ -k_i t_i e_0 e_1 + k_i t_i^2 e_1^2 \end{pmatrix} - S_y^2 \qquad (27)$$

Taking Expectation of both sides of (27)

$$E(\hat{S}^2_{MJ_i} - S_y^2) = S_y^2 E \begin{pmatrix} (k_i - 1) + k_i e_0 - k_i t_i e_1 \\ -k_i t_i e_0 e_1 + k_i t_i^2 e_1^2 \end{pmatrix} \qquad (28)$$

Applying the results of (24), obtaining the $Bias(\hat{S}^2_{MJ_i})$ as:

$$Bias\,(\hat{S}^2_{MJ_i}) = S_y^2 \begin{bmatrix} (k_i - 1) + \gamma k_i t_i^2(\beta_{2(x)} - 1) \\ -\gamma k_i t_i(\lambda_{22} - 1) \end{bmatrix} \qquad (29)$$

$i = 1, 2, 3$

Squaring, taking expectation using the results in equation (24), and differentiating partially with respect to k_i and equate to zero, we obtain the $MSE(\hat{S}^2_{MJ_i})_{min}$ of the proposed estimators as:

$$MSE(\hat{S}^2_{MJ_i})_{min} = S_y^4\left(1 - \frac{P_i^2}{Q_i}\right), i = 1, 2, 3 \qquad (30)$$

$$k_i^{opt} = \frac{1 + \gamma[t_i^2(\beta_{2(x)} - 1) - t_i(\lambda_{22} - 1)]}{1 + \gamma[3t_i^2(\beta_{2(x)} - 1) - 4t_i(\lambda_{22} - 1) + (\beta_{2(y)} - 1)]}$$

$$k_i^{opt} = \frac{P_i}{Q_i}$$

3. EFFICIENCY COMPARISONS

In this section efficiency of the proposed estimator is compared with efficiencies of some estimators in the literature.

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than sample variance if,

$$MSE(\hat{S}^2_{MJ_i})_{min} < Var(t) \quad i = 1, 2, 3$$

$$S_y^4\left(1 - \frac{P_i^2}{Q_i}\right) < \gamma S_y^4(\beta_{2(y)} - 1) \qquad (31)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_R$ if,

$$MSE(\hat{S}^2_{MJ_i})_{min} < MSE(\hat{S}^2_R) \quad i = 1, 2, 3$$

$$S_y^4\left(1 - \frac{P_i^2}{Q_i}\right) < \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) \\ -2(\lambda_{22} - 1) \end{bmatrix} \qquad (32)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{kc_i}$ if,

$$MSE(\hat{S}^2_{MJ_i})_{min} < MSE(\hat{S}^2_{kc_i})$$

$$S_y^4\left(1 - \frac{P_i^2}{Q_i}\right) < \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + A_i^2(\beta_{2(x)} - 1) \\ -2A_i(\lambda_{22} - 1) \end{bmatrix} \qquad (33)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{sk}$ if,

$$S_y^4\left(1 - \frac{P_i^2}{Q_i}\right) < \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + A_{sk}^2(\beta_{2(x)} - 1) \\ -2A_{sk}(\lambda_{22} - 1) \end{bmatrix} \qquad (34)$$

The $\hat{S}^2_{MJ_i}$ of estimators of the finite population variance is more efficient than $\hat{S}^2_{NKi}$ if,

$$MSE(\hat{S}^2_{MJ_i})_{min} < MSE\,(\hat{S}^2_{NKi})$$

$$S_y^4\left(1 - \frac{P_i^2}{Q_i}\right) < S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + A_{NKi}^2(\beta_{2(x)} - 1) \\ -2A_{NKi}(\lambda_{22} - 1) \end{bmatrix} \qquad (35)$$

When conditions (31), (32), (33), (34), and (35) are satisfied, we can conclude that the proposed estimators are more efficient than some selected existing estimators.

4. NUMERICAL ILLUSTRATION

In order to investigate the merits of the proposed finite population variance estimators over sample variance and some selected existing estimators under simple random sampling without replacement (SRSWOR), we have considered the following real populations:

Table 2: Populations: Population 1: [SC86], Population 2: [Coc77], Population 3: [Mur67]

Characteristics	Population 1	Population 2	Population 3
N	22	49	80
n	5	20	20
$\bar{Y}$	22.5	116.1633	51.8264
$\bar{X}$	1467.5	98.6765	11.2624
ρ	0.9022	0.6904	0.9413
S_y	32.8	98.8286	18.3569
C_y	1.4578	0.8508	0.3542
S_x	2503.2	102.9709	8.4542
C_x	1.7058	1.0435	0.7507
$\beta_{2(y)}$	13.2	5.9878	2.2667
$\beta_{2(x)}$	5.57	4.9245	2.8664
λ_{22}	7.71	4.6977	2.2209
Md	534.5	64	7.5750

Table 3: Bias of the Reviewed and Proposed Estimators

Estimator	Bias		
	Population 1	Population 2	Population 3
Sample variance	0	0	0
Isaki ([Isa83]) $\hat{S}_R^2$	-.460.45952	110.7588	10.8759
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_1}^2$	1181.2713	629.7246	10.4394
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_2}^2$	1181.2643	629.3153	9.2908
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_3}^2$	1181.2722	629.9759	10.7218
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_4}^2$	1181.2676	628.3687	8.8105
Subramani and Kumarapandiyan ([SK12]) $\hat{S}_{sk}^2$	1180.9477	611.7195	7.1090
Shahzad *et al.* ([S+18]) $\hat{S}_{NK1}^2$	1178.681558	599.514104	7.6334647
Shahzad *et al.* ([S+18]) $\hat{S}_{NK2}^2$	1179.752763	600.764668	6.7509439
Shahzad *et al.* ([S+18]) $\hat{S}_{NK3}^2$	1180.718622	610.932096	7.927605
Proposed Estimator $\hat{S}_{MJ_1}^2$	-643.8688364	-674.392618	-9.219566
Proposed Estimator $\widehat{S}_{MJ_2}^2$	-6436792818	-674.339338	-8.950364
Proposed Estimator $\widehat{S}_{MJ_3}^2$	-643.5084101	-673.920646	-9.311667

Table 4: MSE of the Reviewed and Proposed Estimators

Estimator	Mean Square Error (MSE)		
	Population 1	Population 2	Population 3
Sample variance	2824133.362	23790830.79	7191.85865
Isaki ([Isa83]) $\hat{S}_R^2$	775479.2427	7235316.417	3924.9482
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_1}^2$	775478.5508	7234105.562	3849.90665
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_2}^2$	775473.8884	7228377.799	3658.078981
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_3}^2$	775479.1903	7235114.148	3898.329219
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_4}^2$	775476.1037	7228666.588	3480.481068
Subramani and Kumarapandiyan ([SK12]) $\hat{S}_{sk}^2$	775262.4693	7162332.866	3319.859578
Shahzad *et al.* ([S+18]) $\hat{S}_{NK1}^2$	773749.7250	7114243.297	3398.78445
Shahzad *et al.* ([S+18]) $\hat{S}_{NK2}^2$	774464.7103	7119149.271	3269.313622
Shahzad *et al.* ([S+18]) $\hat{S}_{NK3}^2$	775109.5267	7159216.595	3443.444496
Proposed Estimator $\hat{S}_{MJ_1}^2$	692699.851	6586854.887	3106.770333
Proposed Estimator $\widehat{S}_{MJ_2}^2$	692495.9187	6586334.681	3016.055758
Proposed Estimator $\widehat{S}_{MJ_3}^2$	692312.0891	6582245.200	3137.806329

Table 5: PRE of Estimators with respect to s_y^2

Estimator	Percentage Relative Efficiency (PRE)		
	Population 1	Population 2	Population 3
Sample variance	100	100	100
Isaki ([Isa83]) $\hat{S}_R^2$	364.1791	328.8153	183.2345
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_1}^2$	364.1794	328.8704	186.8061
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_2}^2$	364.1816	329.131	196.6021
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_3}^2$	364.1791	328.8245	184.4857
Kadilar and Cingi ([KC06]) $\hat{S}_{kc_4}^2$	364.1806	329.1178	206.634
Subramani and Kumarapandiyan ([SK12]) $\hat{S}_{sk}^2$	364.2809	332.1659	216.6314
Shahzad *et al*. ([S+18]) $\hat{S}_{NK1}^2$	364.9931	334.4113	211.6009
Shahzad *et al*. ([S+18]) $\hat{S}_{NK2}^2$	364.6562	334.1808	219.9807
Shahzad *et al*. ([S+18]) $\hat{S}_{NK3}^2$	364.3528	332.3105	208.8565
Proposed Estimator $\hat{S}_{MJ_1}^2$	407.6994	361.1865	231.4899
Proposed Estimator $\widehat{S}_{MJ_2}^2$	407.8195	361.2150	238.4524
Proposed Estimator $\widehat{S}_{MJ_3}^2$	407.9278	361.4394	229.2002

Tables 2, 3, 4 and 5 show the population, bias, MSE, and PRE respectively of some selected existing estimators and the proposed estimators, The results of tables 4, and 5 show the proposed estimators has minimum MSEs at almost all and having the highest Percentage Relative Efficiency (PRE) respectively.

5. CONCLUSION

From the results of Tables 3, and 4, we conclude that the proposed variance estimators are more efficient than the existing estimators in the sense of having least mean square error and highest PRE compare to other existing estimators considered in this research. We therefore recommend for use in estimating finite population variance.

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