

## RECONCILIATION OF ECONOMIC DATA WITH IRREGULAR BENCHMARKS: A NEW APPROACH

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**ABSTRACT:** Benchmarking problem arises when the sum of sub-annual totals of a particular economic variable from a source are not equal to the annual totals from another source. As a result, there is a need for reconciliation of the two datasets from different sources for such data to be reliably used for research purposes. Earlier research showed that Proportional First Difference (PFD) was found to be the best method for solving benchmarking problem in indicator series when compared with other methods. In this paper, we proffered an alternative solution to the benchmarking problem. This proposed benchmarking method is based on the generalized least squares regression model which is grounded on a statistical model that allows for the presence of bias and autocorrelated errors in the indicator, and the presence of non-binding benchmarks. Using the simulated quarterly series by Dagum and Cholette ([DC06]), the results show that the method exactly preserves the quarter-to-quarter movement, a residual adjustment is made, and the final benchmarked series converges to the bias-corrected series at the end. None of these desirable features was exhibited by the existing methods of solving benchmarking problem.

**KEYWORDS:** Bias-corrected, Autocorrelated errors, Benchmarking, Growth rate, indicator.

### INTRODUCTION

Benchmarking is an adjustment of the level of a sub-annual series to the level of an annual series ([L+07]). Both time series measure the same variable, but at different time intervals and different levels of accuracy. The annual series is usually more accurate than the sub-annual series and, for this reason, it is considered a benchmark. Because the two series are coming from different sources, discrepancies are usually observed between the annual benchmarks and the annual sums of the sub-annual series. Benchmarking, however, can be defined more broadly as the process of optimally combining two sources of measurements, in order to achieve improved estimates of the series under investigation ([CD94]). This adjustment process is called benchmarking and the more accurate values are called benchmarks. Typically, benchmarks are either yearly totals or values observed at a particular time-point each year ([AAI17]). The sub-annual

series is modified so that the annual sums of the sub-annual series are equal to the corresponding benchmarks ([L+07]). In time series works of literatures, this problem, involving either one or many time series, is generally known as temporal disaggregation ([AAI15]). This is done considering two main challenges. The first one is to preserve the movement in the sub-annual series as much as possible which Denton tried to do but does not allow non-binding benchmarks ([Den71]). The second one is to account for the timeliness of annual benchmarks, in the sense that the benchmarks for the observations at the end of the series may not be available yet. The presented method is a special case of the general regression-based benchmarking model proposed by Dagum and Cholette ([DC06]). Causey ([CT81]) developed a numerical algorithm that was later revised by Trager ([CT81]), showing that there was an iterative solution to the problem of minimizing the changes to the month-to-month changes subject to a set of constraints (the benchmarks). The procedure minimizes the changes iteratively using steepest descent. Though the first solutions were not published, their notes are available as appendixes in a research report by Bozik and Otto ([BO88]). This method is referred to generally as the Causey-Trager method ([CT81]). The numerical algorithm was eventually programmed into FORTRAN, and the U.S. Census Bureau still supports and maintains the FORTRAN code. The Census Bureau has used the program for a production of benchmarked numbers since the early 1980s. The program was not designed for benchmarking the seasonally adjusted series to annual totals from the original series since the Census Bureau does not do this type of benchmark ([HH05]). The aim of this paper is to force the sub-annual totals (benchmarks) to obey the annual totals (benchmarking) using the derived model.

### 1. METHOD

The sub-annual and annual series are respectively denoted by:

$$s_t, t = 1, 2, 3, \dots, T \quad (1)$$

$$s_m, m = 1, 2, 3, \dots, M \quad (2)$$

where 1, 2, ..., T refers to a set of contiguous months, quarter, days, etc; and 1, 2, ..., M to set of not necessarily of contiguous periods, e.g. there may be a benchmark every “year”.

The regression-based model consists of the following two equations:

$$s_t = \sum_{h=1}^h r_{th} B_h + \theta_t + e_t = \sum_{k=1}^h r_{tk} B_k + \theta_t + \sigma_t^h e_t^*, t = 1, \dots, T \quad (3)$$

Equation (3) means that the true values are corrupted with error and bias, where:

$$E(e_t) = 0, E(e_t e_{t-1}) = \sigma_t^\lambda \sigma_{t-1}^\lambda \omega_t$$

$$a_m = \sum_{t=t_{1m}}^{t_{Lm}} j_{mt} \theta_t + \varepsilon_m, m = 1, \dots, M \quad (4)$$

Where

$$E(\varepsilon_m) = 0, E(\varepsilon_m^2) = \sigma_{\varepsilon_m}^2, E(\varepsilon_m e_t) = 0$$

The errors  $\varepsilon_m$  and  $e_t$  are mutually independent because the annual and quarterly data come from two separate sources, that is, from a survey carried out and from record.

#### Matrix representation of the model

Eqns. (3) and (4) can be written thus:

$$S = R\beta + \theta + e, E(e) = 0, E(ee') = V_e \quad (5)$$

$$a = J\theta + \varepsilon, E(\varepsilon) = 0, E(\varepsilon\varepsilon') = V_\varepsilon, E(e\varepsilon') = 0 \quad (6)$$

Where

$S = [S_1, \dots, S_T]'$ ,  $\theta = [\theta_1, \dots, \theta_T]'$ ,  $a = [a_1, \dots, a_M]'$   
Matrix  $R$  of dimension T by H contains the H regressors (the X's or the sums of the related series) of equation (3) and  $\beta = [\beta_1, \dots, \beta_H]$  the regression parameters. The covariance matrix has this:

$$V_e = \Sigma^\lambda \Omega \Sigma^\lambda \quad (7)$$

Where:

$\Sigma$  is a diagonal matrix of standard deviations,  $\sigma_1, \dots, \sigma_T$  and  $\Omega$  contains the autocorrelations usually following an ARMA error model.

For an AR(1) error model with parameter  $|\phi| < 1$ , the elements of  $\Omega$  are given by  $\omega_{ij} = \phi^{|i-j|}$ .

#### Temporal sum operator

Matrix  $J$  is a temporal sum operator of dimension M by T, containing the coverage fractions,

$$J = \begin{bmatrix} j_{11} & \cdots & j_{1T} \\ \vdots & \ddots & \vdots \\ j_{M1} & \cdots & j_{MT} \end{bmatrix}$$

Where

$$j_{mt} = 0 \text{ for } t < t_{1m} \text{ or } t > t_{Lm}, \text{ and } t_{1m} \leq t_{1m} \leq t_{Lm}.$$

Each coverage fraction  $j_{mt}$  is placed in row m and column t of  $J$ .

$J$  is a design matrix with ones and zeros such that, for any variable, say,  $s$ ,  $J_s$  yields the annual sums of  $s$ .

Equations (5) and (6) can be re-written as

$$y = X\alpha + U \quad (8)$$

$$E(U) = 0, E(UU) = V_u = \text{block}(V_e, V_\varepsilon)$$

Where

$$y = \begin{bmatrix} s \\ a \end{bmatrix}, X = \begin{bmatrix} R & I_T \\ 0 & J \end{bmatrix}, \alpha = \begin{bmatrix} \beta \\ \theta \end{bmatrix}, V_u = \begin{bmatrix} V_e & 0 \\ 0 & V_\varepsilon \end{bmatrix}$$

Where

$y, X, \alpha, I_T, V_u$  are indicators, regressors, parameters, identity matrix of dimension T by T, and covariance matrix for sub-annual and annual series respectively.

The models (5) and (6) can be written as

$$\begin{bmatrix} s \\ a \end{bmatrix} = \begin{bmatrix} R & I_T \\ 0 & J \end{bmatrix} \times \begin{bmatrix} \beta \\ \theta \end{bmatrix} + \begin{bmatrix} e \\ \varepsilon \end{bmatrix} \quad (9)$$

Model (8) has the form of a standard regression model, with the generalized least squares solution and can be written as:

$$\hat{\alpha} = (X'V_u^{-1}X)^{-1}X'V_u^{-1}y = \begin{bmatrix} \hat{\beta} \\ \hat{\theta} \end{bmatrix} \quad (10)$$

$V_u$  is the known or true covariance matrix of the disturbances  $u$ , the covariance matrix of the estimates  $\hat{\alpha}$  is

$$\text{var}(\hat{\alpha}) = (X'V_u^{-1}X)^{-1} = \begin{bmatrix} \text{var}(\hat{\beta}) & \text{cov}(\hat{\beta}\hat{\theta}) \\ \text{cov}(\hat{\theta}\hat{\beta}) & \text{var}(\hat{\theta}) \end{bmatrix} \quad (11)$$

The  $H$  first elements of the vector  $\hat{\alpha} = [\hat{\beta}' \ \hat{\theta}']$  contain the estimates of  $\hat{\beta}_1, \dots, \hat{\beta}_H$ ; the last  $T$  elements, contains the estimates of  $\hat{\theta}_1, \dots, \hat{\theta}_T$ ; similarly the first  $H$  rows and columns of  $\text{var}(\hat{\alpha})$  contains the covariance matrix of vector  $\hat{\beta}$ ; and the last  $T$  rows and columns of  $\text{var}(\hat{\alpha})$ , contains the covariance matrix of a matrix of the vector  $\hat{\theta}$ .

The solution (10) is obtained only if  $V_u$  is invertible, which requires that both  $V_e$  and  $V_\varepsilon$  be invertible. The implication of the latter is that all the diagonal elements of the  $V_e$  must be greater than zero i.e. all the benchmarks must be non-binding.

Given equation (9), the estimator in (10) can be written as

$$\begin{bmatrix} \hat{\beta} \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} R & I_T \\ 0 & J \end{bmatrix}^{-1} \begin{bmatrix} s \\ a \end{bmatrix} - \begin{bmatrix} e \\ \varepsilon \end{bmatrix}$$

Making substitution the partitions of  $X$ ,  $V_U$  and  $\hat{\alpha}$  in equation (10) the transformations produces

$$\begin{bmatrix} \hat{\beta} \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} R'V_e^{-1}R & R'V_e^{-1} \\ V_e^{-1}R & (V_e^{-1} + J'V_\varepsilon^{-1}J) \end{bmatrix}^{-1} \begin{bmatrix} R'V_e^{-1}s & 0 \\ V_e^{-1}s & J'V_\varepsilon^{-1}a \end{bmatrix} \quad (12a)$$

Therefore

$$\begin{bmatrix} \hat{\beta} \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} R'V_e^{-1}R & R'V_e^{-1} \\ V_e^{-1}R & (V_e^{-1} + J'V_\varepsilon^{-1}J) \end{bmatrix}^{-1} \begin{bmatrix} R'V_e^{-1}s \\ V_e^{-1}s + J'V_\varepsilon^{-1}a \end{bmatrix} \quad (12b)$$

Having the following dimensions:

$$R = T \times 1, V_e \text{ and } V_\varepsilon = T \times T, J = M \times T$$

From (11)

$$var(\hat{\alpha}) = \begin{bmatrix} R'V_e^{-1}R & R'V_e^{-1} \\ V_e^{-1}R & (V_e^{-1} + J'V_\varepsilon^{-1}J) \end{bmatrix}^{-1} \quad (13)$$

$$= \begin{bmatrix} E_{11} & E_{12} \\ E'_{12} & E_{22} \end{bmatrix}^{-1} = \begin{bmatrix} E^{11} & E^{12} \\ E'^{12} & E^{22} \end{bmatrix} \quad (14)$$

Combining (11) and (12)

$$\hat{\beta} = var[\hat{\beta}]R'V_e^{-1}s + cov[\hat{\beta}, \hat{\theta}](V_e^{-1}s + J'V_\varepsilon^{-1}a) \quad (15)$$

$$\hat{\theta} = cov[\hat{\theta}, \hat{\beta}]R'V_e^{-1}s + var[\hat{\theta}](V_e^{-1}s + J'V_\varepsilon^{-1}a) \quad (16)$$

The solution (10) requires that  $V_\varepsilon$  be positive definite, which excludes the possibility of binding benchmarks. The estimator (10) can be expressed as:

$$\hat{\beta} = -(R'J'V_d^{-1}JR)^{-1}R'J'V_d^{-1}[a - Js] \quad (17)$$

$$var[\hat{\beta}] = (R'J'V_d^{-1}JR)^{-1} \quad (18)$$

Where  $s$  = indicator series and  $V_d = J'V_e^{-1}J + V_\varepsilon$ . The derivation assumes that both  $V_e$  and  $V_\varepsilon$  are invertible. When  $V_\varepsilon$  is not invertible, it can be replaced by a matrix  $V_\delta$  equal to  $V_\varepsilon + \delta I_M$  with  $\delta$  close to zero. Parameter  $\delta$  is then set to 0 after the derivation.

The lower-right partition of (13) corresponding to partition  $E_{22}$  in (14) may be inverted using the matrix identities ([Jaz70]) and expansion of the algebra

$$(D + BCB')^{-1} \equiv D^{-1} - D^{-1}B(B'D^{-1}B + C^{-1})^{-1}B'D^{-1} \quad (19)$$

Proof:

From R.H.S.

$$= D^{-1} - D^{-1}BB'D^{-1}(B'D^{-1}B + C^{-1})^{-1} = D^{-1}(1 - D^{-1}BB'(\frac{1}{B'D^{-1}B+C^{-1}})) \quad (20)$$

$$= D^{-1} \left( 1 - \frac{D^{-1}BB'}{B'D^{-1}B+C^{-1}} \right) = D^{-1} \left( \frac{B'D^{-1}B+C^{-1}-D^{-1}BB'}{B'D^{-1}B+C^{-1}} \right) \quad (21)$$

$$= \frac{D^{-1}C^{-1}}{B'D^{-1}B+C^{-1}} = \frac{(DC)^{-1}}{B'D^{-1}B+C^{-1}} \quad (22)$$

$$= \frac{1/DC}{B'D^{-1}B+C^{-1}} = \left( \frac{1}{DC} \right) \left( \frac{1}{B'D^{-1}B+C^{-1}} \right) \quad (23)$$

$$= \frac{1}{DCB'D^{-1}B + DCC^{-1}} = \frac{1}{IBCB' + ID} \quad (24)$$

$$(since CC^{-1}=I and DD^{-1}=I)$$

$$= \frac{1}{I(BCB'+D)} = \frac{1}{BCB'+D} \quad (24)$$

$$= (BCB' + D)^{-1} \text{ or } = (D + BCB')^{-1} \quad (25)$$

Therefore

$$(V_e^{-1} + J'V_\varepsilon^{-1}J)^{-1} \equiv V_e - V_eJ'(J'V_\varepsilon^{-1}J + V_e)^{-1}JV_e \equiv V_e - V_eJ'V_d^{-1}JV_e = E_{22}^{-1} \quad (26)$$

Given  $E_{22}^{-1}$  from (26), the matrix inversion in (14) is performed by parts using the following formulae:

$$\begin{bmatrix} E_{11} & E_{12} \\ E'_{12} & E_{22} \end{bmatrix}^{-1} = \begin{bmatrix} E_{22} & -E_{12} \\ -E'_{12} & E_{11} \end{bmatrix} \quad (27)$$

$$E^{11} = \frac{E_{22}}{E_{11}E_{22} - E'_{12}E_{12}} \quad (28)$$

$$= \frac{E_{22}}{E_{22}(E_{11} - E'_{12}E_{12}E_{22}^{-1})} \quad (29)$$

$$= \frac{1}{E_{11} - E'_{12}E_{12}E_{22}^{-1}} = (E_{11} - E'_{12}E_{12}E_{22}^{-1})^{-1} \quad (30)$$

Therefore

$$E^{11} = (E_{11} - E'_{12}E_{12}E_{22}^{-1})^{-1} \quad (31)$$

$$E^{12} = \frac{-E_{12}}{E_{11}E_{22} - E'_{12}E_{12}} = \frac{-E_{12}}{E_{22}(E_{11} - E'_{12}E_{12}E_{22}^{-1})} = -E_{12}E_{22}^{-1}(E_{11} - E'_{12}E_{12}E_{22}^{-1})^{-1} \quad (32)$$

Therefore

$$E^{12} = -E_{12}E_{22}^{-1}E^{11} \quad (33)$$

$$E^{22} = E_{22}^{-1} - E^{12'}E_{12}E_{22}^{-1} = E_{22}^{-1} + E_{22}^{-1}E'_{12}E^{11}E_{12}E_{22}^{-1} \quad (34)$$

$$= E_{22}^{-1} + E_{22}^{-1}E^{12'}(E_{11} - E_{12}E_{22}^{-1}E'_{12})^{-1}E_{12}E_{22}^{-1} \quad (35)$$

Substituting (31) in the upper left partition of (11) yields:

$$var[\hat{\beta}] = E^{11} = [(R'V_e^{-1}R) - (R'V_e^{-1})(V_e - V_eJ'V_d^{-1}JV_e)(V_e^{-1}R)]^{-1} = (R'J'V_d^{-1}JR)^{-1} \quad (36)$$

this proves (18).

Substituting (33) in the upper right partition of (11) yields:

$$cov[\hat{\beta}, \hat{\theta}] = E^{12} = -var[\hat{\beta}](R'V_e^{-1})(V_e - V_eJ'V_d^{-1}JV_e) = -var[\hat{\beta}]R'(I - J'V_d^{-1}JV_e) \quad (37)$$

$$cov[\hat{\theta}, \hat{\beta}] = E^{12'} = -(I - V_eJ'V_d^{-1}J)R var[\hat{\beta}] \quad (38)$$

Substituting (35) into the lower right partition of (11) yields:

$$\begin{aligned} \text{var}[\hat{\theta}] &= E^{22} = (V_e - V_e J' V_d^{-1} J V_e) \\ &\quad - \text{cov}[\hat{\theta}, \hat{\beta}] (R V_e^{-1}) (V_e \\ &\quad - V_e J' V_d^{-1} J V_e) \\ &= (V_e - V_e J' V_d^{-1} J V_e) + (I - V_e J' V_d^{-1} J) R \text{var}[\hat{\beta}] R' (I - \\ &\quad J' V_d^{-1} J V_e) \end{aligned} \quad (39)$$

Substituting (36), (37), (38) into (15) and expanding leads to cancellations and simplifications

$$\begin{aligned} \hat{\beta} &= \text{var}[\hat{\beta}] R' V_e^{-1} s \\ &\quad - \text{var}[\hat{\beta}] R' (I - J' V_d^{-1} J V_e) (V_e^{-1} s \\ &\quad + V_e^{-1} a) \\ &= \text{var}[\hat{\beta}] R' V_d^{-1} J s + \text{var}[\hat{\beta}] R' J' V_d^{-1} (J V_e J' s - \\ &\quad V_d) V_e^{-1} a \end{aligned} \quad (40)$$

Replacing the inner occurrences of  $V_d$  in the last equation by  $J V_e J' + V_e$  gives:

$$\hat{\beta} = \text{var}[\hat{\beta}] R' J' V_e^{-1} J s + \text{var}[\hat{\beta}] R' J' V_d^{-1} (J V_e J' - J V_e J' - V_e) V_e^{-1} a \quad (41)$$

$$\hat{\beta} = -(R' J' V_d^{-1} J R)^{-1} R' J' V_d^{-1} (a - J s) \quad (42)$$

Substituting (36), (37), (38) into (16) and expanding produces:

$$\hat{\theta} = -(I - B J) R \text{var}[\hat{\beta}] R' V_e^{-1} s + ((V_e - B J V_e) + (I - B J) R \text{var}[\hat{\beta}] R' (I - J' B')) (V_e^{-1} s + J' V_e a) \quad (43)$$

Where  $B = V_e J' V_d^{-1}$ .

After lengthy algebraic transformations and collecting the terms in  $J' V_e a$  and  $s$  we obtain:

$$\begin{aligned} \hat{\theta} &= s - V_e J' V_d^{-1} J s + V_e J' V_d^{-1} J R \text{var}[\hat{\beta}] R' J' V_d^{-1} J s - \\ &\quad R \text{var}[\hat{\beta}] R' J' V_d^{-1} J s - V_e J' [V_d^{-1} J V_e J' - I] V_e^{-1} a - \\ &\quad [R \text{var}[\hat{\beta}] R' J'] [V_d^{-1} J V_e J' - I] V_e^{-1} a + \\ &\quad [V_e J' V_d^{-1} J R \text{var}[\hat{\beta}] R' J'] [V_d^{-1} J V_e J' - I] V_e^{-1} a \end{aligned} \quad (44)$$

The above models can be implemented using MATLAB mathematical software for modeling; the aim of this paper is to apply the model described above in solving benchmarking problem of economic data.

## 2. DATA AND ANALYSIS

In order to apply the derived model, the simulated quarterly series by Cholette and Dagum (2006) is used in this paper. It is the indicator series in table 1:

**Table 1: Indicator series, bias-adjusted, benchmarked and the growth rates**

Date	Indicator (s)	Bias-adj (S*)	Benchmarked (alt. method)	Annual Totals	The growth rate in indicator series	The growth rate in benchmarked series
1998-1	85.00	136.75	134.81			
1998-2	95.00	146.75	144.10		0.11	0.06
1998-3	125.00	176.75	173.12		0.24	0.17
1998-4	95.00	146.75	141.77	494.00	-0.32	-0.22
1999-1	85.00	136.75	129.93		-0.12	-0.09
1999-2	95.00	146.75	137.36		0.11	0.05
1999-3	125.00	176.75	163.80		0.24	0.16
1999-4	95.00	146.75	128.90	560.00	-0.32	-0.27
2000-1	85.00	136.75	112.16		-0.12	-0.15
2000-2	95.00	146.75	120.63		0.11	0.07
2000-3	125.00	176.75	154.14		0.24	0.22
2000-4	95.00	146.75	133.07	520.00	-0.32	-0.16
2001-1	85.00	136.75	138.30		-0.12	0.04
2001-2	95.00	146.75	156.70		0.11	0.12
2001-3	125.00	176.75	189.11		0.24	0.17
2001-4	95.00	146.75	155.78	640.00	-0.32	-0.21
2002-1	85.00	136.75	136.38		-0.12	-0.14
2002-2	95.00	146.75	142.21		0.11	0.04
2002-3	125.00	176.75	172.88		0.24	0.18
2002-4	95.00	146.75	148.43	600.00	-0.32	-0.16
2003-1	85.00	136.75	149.44		-0.12	0.01
2003-2	95.00	146.75	165.79		0.11	0.10
2003-3	125.00	176.75	198.12		0.24	0.16
2003-4	95.00	146.75	166.66	680.00	-0.32	-0.19
2004-1	85.00	136.75	151.28		-0.12	-0.10
2004-2	95.00	146.75	157.35		0.11	0.04
2004-3	125.00	176.75	184.48		0.24	0.15
2004-4	95.00	146.75	152.39	661.00	-0.32	-0.21

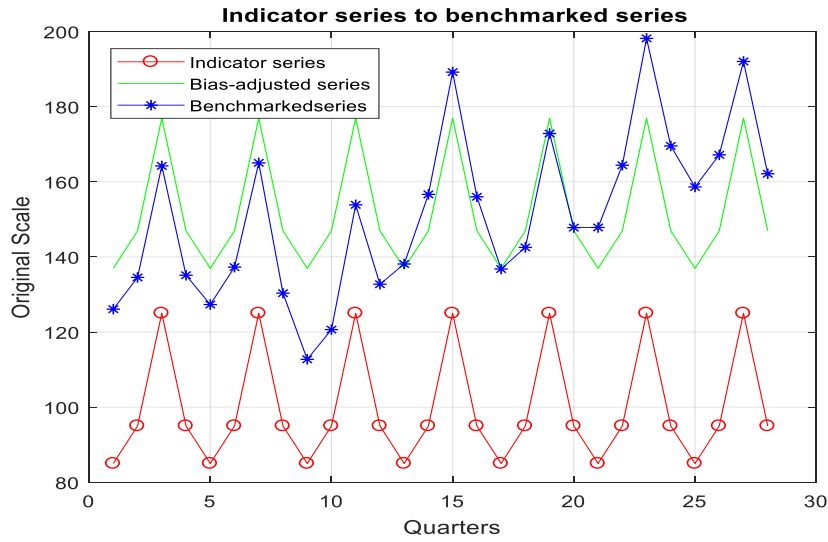


Figure 1: Indicator series to benchmarked series using the alternative method

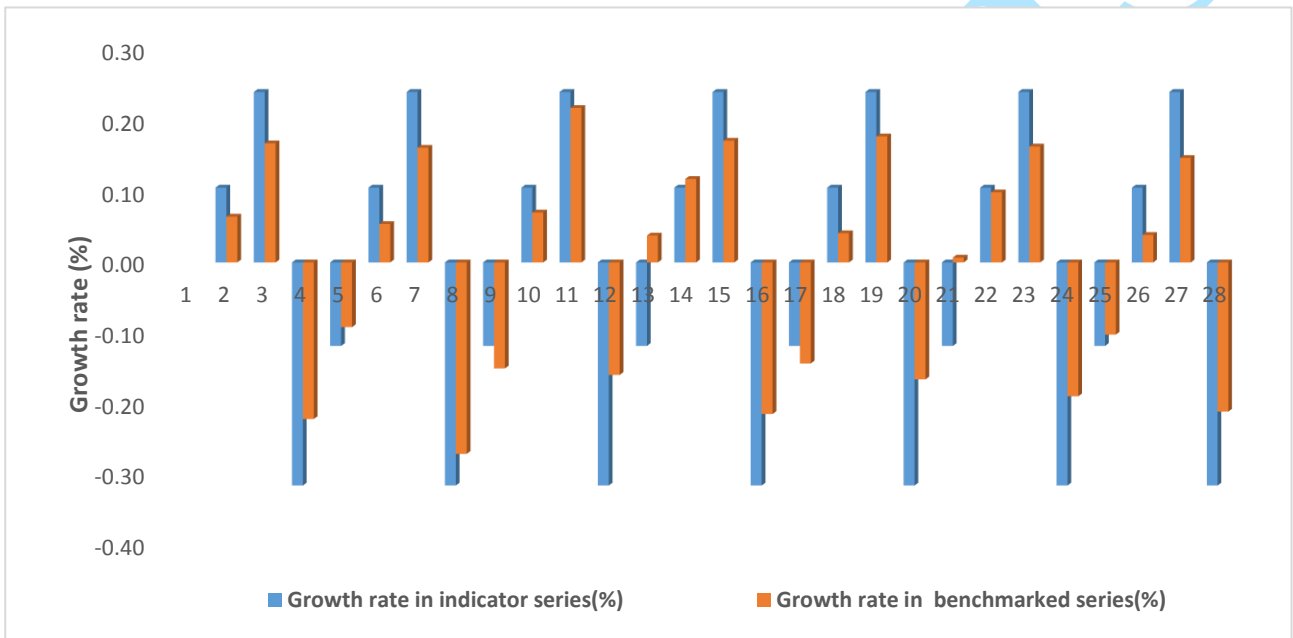


Figure 2: Growth rates graph for indicators and benchmarked series (alt method)

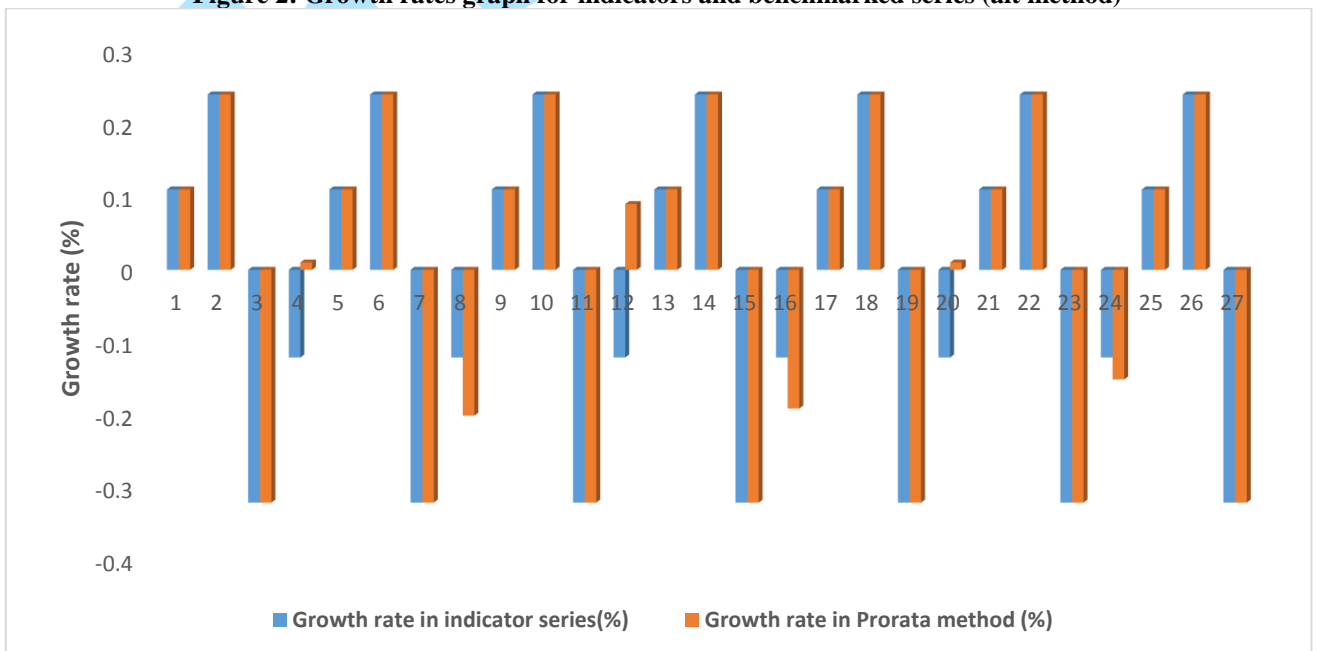


Figure 3: Growth rates graph for indicators and benchmarked series (Prorata)



**Table 2: Standard deviation and coefficient of variation of the benchmarked series methods**

Method	Std.	CV
PLD	34.79	22.90
AFD	20.82	13.70
PFD	27.38	18.02
ASD	21.12	13.90
PSD	27.53	18.11
PRORATA	28.36	19.11
ALT.M	<b>20.71</b>	<b>13.63</b>

Where PLD, AFD, PFD, ASD, PSD, PRO RATA, and ALT.M are the proportional level difference, additive first difference, proportional first difference, additive second difference, proportional second difference, pro rata method and the alternative method respectively.

### 3. DISCUSSION OF RESULTS

It is discovered in table 1 that the benchmarked series later converged to the bias-adjusted series (as shown in fig. 1). The growth rate (the first order difference using the sub-annual series) contains both negative and positive values, this is in order and expected since economic data behave in such manner many times. Another reason behind this is because the benchmarks are non-binding, therefore the variance cannot be equal to zero. The quarterly in the indicator series has been made to obey the annual totals at the application of the alternative method of solving benchmarking problem.

The growth rates of the indicator series to the growth rates of the benchmarked series are presented in fig. 2 and 3. The growth rates are not similar in some quarters, this may be as a result of the difference in the simulated indicator series, but quarters, 2, 9, 14, 17 and 22 similar growth rates which shows that the benchmarking method ensures good movement preservation from the indicator series to the benchmarked series ().

Comparing the alternative method with the existing ones using standard deviation and coefficient of variation, table 2 shows that the ALT.M preserves the quarter-to-quarter movement better than the existing methods.

### 4. CONCLUSION AND RECOMMENDATION

Since the alternative method performs benchmarking better than the other methods it is therefore recommended for use whenever data reconciliation is necessary.

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