IMPROVED MODIFIED RATIO ESTIMATOR FOR ESTIMATING POPULATION MEAN IN DOUBLE SAMPLING USING INFORMATION ON AUXILIARY ATTRIBUTE

F. E. Amoyedo, A. A. Adewara

Department of Statistics, University of Ilorin, Ilorin, Nigeria

Corresponding Author: Amoyedo F. E., amoyedofemi@gmail.com

ABSTRACT: This paper proposes a modified family of ratio estimator of population mean using information on auxiliary attribute and the estimation of population mean in double sampling in ratio form. The proposed modified ratio estimator is a family of estimator which results to different estimators at different value of alpha. For the proposed estimator, when $\alpha = 0, 0.5$ and 1, the estimators of Naik and Gupta (1996), Nirmala Sawan (2010) and Sample mean were recovered respectively. When the auxiliary attribute is a variable, the estimator result to that of Subhash *et al.* (2016). When $\alpha = 0$ and the auxiliary attribute is a variable, it results to conventional double sampling. The expression for the Bias, and Mean Square Error of the proposed modified estimator were obtained up to the first order of approximation. An efficiency comparison of both the theoretical and empirical was carried out with some related existing estimators in double sampling. It has been established that the proposed modified ratio estimator is more efficient when compared with the existing ones at optimum value of alpha.

KEYWORDS: ratio estimator, bias, mean square error and double sampling.

1. INTRODUCTION

It is well established in sample surveys that auxiliary information is often used to improve the precision of estimators of population parameters. Neyman ([Nay38]) was the first to propose double sampling. The use of auxiliary information at the estimation stage appears to have started with the work of Cochran ([Coc40]) and Cochran ([Coc77]) presented the basic result of double sampling, including the simplest regression estimators for this type of sampling design. Double sampling is a sampling method or technique which makes use of auxiliary data where the auxiliary information is obtained through sampling. More precisely, we take a sample of units' (n′) strictly to obtain auxiliary information, and then take a second sample (n) where the variable(s) of interest are observed. It will often be the case that this second sample is a *subsample* of the preliminary sample used to acquire auxiliary information. Two common situations where double sampling is employed to use auxiliary information to improve the estimate of some response variables are:-.

i. If the variable of interest is expensive to measure or unaffordable, but a related variable is much cheaper, we might first sample many of the sampling units and measure the cheaper (auxiliary) variable and only measure the response variable on a subsample (or smaller sample).

ii. A common problem in many surveys is that of potential bias from non-response. Double sampling can be used with stratification principles to adjust for non-response in surveys by taking a second sample of the non-respondents.

The use of auxiliary information can increase the precision of an estimator when study variable y is highly correlated with auxiliary variable x. There exist situations when information is available in the form of attribute or Qualitative (φ), which is highly correlated with y. For example

a) Sex and height of the persons,
b) Amount of milk produced and a particular breed of the cow,
c) Amount of yield of wheat crop and a particular variety of wheat etc. (Jhajj *et al* ([JSG06])).
d) Age of Mother and Type of Delivery
e) Land areas and number of households (Jhajj *et al* ([JSG06]))

Consider a sample of size n drawn by simple random sampling without replacement (SRSWOR) from a population of size N. Let y_i and ϕ_i denote the observations on the variables y and ϕ respectively for i^{th} unit (i=1,2,…N). Suppose there is a complete dichotomy in the population with respect to the presence or absence of an attribute say ϕ, and it is assumed that attribute or quality ϕ takes only the two values 0 and 1 accordingly as

$$\phi_i = \begin{cases} 1, \text{if unit of the population possesses attribute } \phi \\ 0, \text{otherwise} \end{cases}$$

Let $A = \sum_i^N \phi_i$ and $a = \sum_i^n \phi_i$ denote the total number of units in the population and sample respectively possessing attribute ϕ. Let $P = \frac{A}{N}$ and $p = \frac{a}{n}$ denote the proportion of units in the population and sample respectively possessing attribute ϕ (Singh *et al*, ([SK08]))

Auxiliary information:- in sample survey design, information about the sampling unit which is supplementary to the characteristics under investigation in the survey is called auxiliary information. This information can be qualitative called auxiliary attributes.

1.1. Definition of Terms Used

$n^1 = the\ first\ sample\ size$

$n = the\ second\ sample\ size\ or\ the\ sub - sample\ size$

N = Size of the Population

Y = Variable of Interest or $Study\ Variable$

$P^1 = \frac{\sum a^1}{n^1}$ =The first phase sample proportion of the auxiliary attribute

$p = \frac{\sum a}{n}$ = the second phase sample proportion of the auxiliary attribute

Φ= auxiliary attribute

$\bar{y} = \frac{\sum_{i=1}^n y_i}{n}$ = Second phase Sample mean or Sub-Sample mean of study Variable

a = $\sum_i^n \phi_i$ denote the total number of units in the second sample that possessed the attribute

$a^1 = \sum_i^n \phi_i$ denote the total number of units in the first sample that possessed the attribute

2. METHODOLOGY

Double sampling for ratio estimator:-

The use of ratio estimators, using auxiliary information, for estimating the mean of a finite population is well known. The efficiency of ratio estimator is high depending on whether the auxiliary character is highly positively correlated with the main character of interest. This work proposed a modify family of ratio estimator in double sampling using auxiliary attribute.

The Proposed modified family of ratio estimator is defined as:-

$$\bar{Y}_{adR} = \alpha\bar{y} + (1-\alpha)\,\bar{y}(\frac{p^1}{p})$$

Existing Related Ratio Estimators in Double Sampling

Conventional estimator of population mean for Simple Random Sampling without Replacement.

$$\bar{Y}_a = \bar{y} = Sample\ mean \quad (1)$$

$$MSE(\bar{Y}_a) = f_n \bar{Y}^2 C_y^2 \quad (2)$$

Double sampling ratio estimator using auxiliary variable.

Ratio Estimator:-

$$\bar{Y}_{DSR} = \bar{y}\frac{\bar{x}^1}{\bar{x}} \quad (3)$$

$$B(\bar{Y}_{DSR}) = \bar{Y} f_n (C_x^2 - C_{yx}) \quad (4)$$

$$MSE(\bar{Y}_{DSR}) = \bar{Y}^2 (f_n c_y^2 + f_{nn^1}(c_x^2 - 2\rho_{yx} c_y c_x) \quad (5)$$

Naik and Gupta ([NG96]), suggested a double sampling ratio estimator using auxiliary attribute as:-

$$\bar{Y}_{NGR} = \bar{y}\frac{p^1}{p} \quad (6)$$

$$B(\bar{Y}_{NGR}) = \bar{Y} f_n (C_p^2 - C_{py}) \quad (7)$$

$$MSE(\bar{Y}_{NGR}) = \bar{Y}^2 (f_n c_y^2 + f_{nn^1}(c_p^2 - 2\rho_{yp} c_y c_p)) \quad (8)$$

Nirmala Sawan ([NS10]), suggested ratio double sampling exponential estimator using auxiliary attribute. The estimators were just an improvement on Bahl and Tuteju([BT91]) suggested exponential estimators using auxiliary attribute

$$\bar{Y}_{NSR} = \bar{y} exp^{(\frac{p^1 - p}{p^1 + p})} \quad (9)$$

$$B(\bar{Y}_{NSR}) = \bar{Y}\left(\frac{1}{n} - \frac{1}{n^1}\right)[\frac{1}{2}\rho_{py} C_y C_p - C_p^2] \quad (10)$$

$$MSE(\bar{Y}_{NSR}) = \bar{Y}^2 (f_n c_y^2 + f_{nn^1}(\frac{1}{4} c_p^2 - 2\rho_{yp} c_y c_p)) \quad (11)$$

Solanki, Singh ([SS13]), suggested improved estimation of population mean using population proportion of an auxiliary character

$$\bar{Y}_{RH} = \bar{y} exp^{(\frac{\alpha(p^1 - p)}{p^1 + p})} \quad (12)$$

$$B(t_{RH}) = (\frac{K_p C_p^2}{2})(1 - K_p) \quad (13)$$

$$MSE(\bar{t}_{RH}) = \theta S_y^2 (1 - \rho_{pb}^2) + \theta^1 \rho_{pb}^2 S_y^2 \quad (14)$$

Where $K_p = \frac{S_{y\phi} p}{S_{\phi}^2 \bar{y}}$

Its limitation is that the range of α is an open range between -1.25 and 2.25.

Subhash *et al* ([S+16]) suggested ratio estimator for estimating population mean in double sampling. The suggested estimator was motivated by Kalita, Singh ([KS13]) exponential dual to ratio type estimator for finite population mean in double sampling:-

$$\bar{Y}_{YADR} = \propto \bar{y} + (1 - \propto) t_{Re}^{d} \quad (15)$$

$$B(\bar{Y}_{YADR}) = \bar{Y}[\frac{1}{8} g^2 \gamma^* C_x^2 + \frac{1}{8} g^2 \gamma C_x^2 - \frac{1}{2} g \gamma^{**} C C_x^2](1 - \frac{A}{gB}) \quad (16)$$

$$MSE(\bar{Y}_{YADR}) = \bar{Y}^2[\gamma C_y^2 + g\gamma^{**}C_x^2(\tfrac{1}{4}g - c) - \gamma^{**}\tfrac{A^2}{4B})] \quad (17)$$

Where $C = \rho yx\frac{Cy}{Cx}$, $\gamma^{**}=\left(\frac{1}{n}-\frac{1}{n^1}\right)$, $\gamma=\left(\frac{1}{n}-\frac{1}{N}\right)$, $\gamma^{*}=\left(\frac{1}{n^1}-\frac{1}{N}\right)$, $D = C_x^2(g+2c)$, $B = C_x^2$, $A = C_x^2(g-2c)$.

In this study the optimum value of the characteristics scalar α was obtained as $\alpha = \frac{A}{gB}$ and the results were compared with Choudhury and Singh ([CS12]), Singh and Vishwakarma ([SV07]). Hence the proposed estimator was preferred over the existing estimators' earlier mentioned.

Bias and MSE of the Proposed modified family of ratio estimator

The proposed modified ratio estimator $\bar{Y}_{adR}$ is defined as:-

$$\bar{Y}_{adR} = \alpha\bar{y} + (1-\alpha)\bar{y}(\tfrac{p^1}{p}) \quad (18)$$

In order to obtain the Bias and mean square error (MSE) of the modified estimator $\bar{Y}_{adR}$ we defined the error terms $\bar{y} = \bar{Y}(1+\Delta\bar{y})$, $p^1 = P(1+\Delta p^1)$, $p = P(1+\Delta p)$ such that

$$E(\Delta p^1) = 0,\ E(\Delta p) = 0,$$
$$E(\Delta\bar{y}) = 0,\ E(\Delta_{\bar{y}}^2) = (\tfrac{1}{n}-\tfrac{1}{N})\tfrac{S_y^2}{\bar{Y}^2},$$
$$E\left(\Delta_{p^1}^2\right) = (\tfrac{1}{n^1}-\tfrac{1}{N})\tfrac{S_P^2}{p^2},$$
$$E(\Delta_P^2) = (\tfrac{1}{n}-\tfrac{1}{N})\tfrac{S_P^2}{P^2},$$
$$E(\Delta\bar{y}\Delta p^1) = (\tfrac{1}{n^1}-\tfrac{1}{N})\tfrac{S_{yp}}{\bar{Y}P},$$
$$E(\Delta\bar{y}\Delta p) = (\tfrac{1}{n}-\tfrac{1}{N})\tfrac{S_{yp}}{\bar{Y}P},$$
$$E(\Delta p\Delta p^1) = (\tfrac{1}{n^1}-\tfrac{1}{N})\tfrac{S_P^2}{p^2} \quad (19)$$

To the first degree of approximation, the modified ratio estimator $\bar{Y}_{adR}$ can be expressed in term of error term $\Delta\bar{y}$, Δp^1 and Δp as

$$\bar{Y}_{adR} = \alpha\bar{Y}(1+\Delta\bar{y}) + (1-\alpha)\bar{Y}(1+\Delta\bar{y})\frac{P(1+\Delta p^1)}{P(1+\Delta p)} \quad (20)$$

This can be written homogeneously as

$$\bar{Y}_{adR} = \alpha\bar{Y}(1+\Delta\bar{y}) + (1-\alpha)\bar{Y}(1+\Delta\bar{y})(1+\Delta p^1)(1+\Delta p)^{-1} \quad (21)$$

Using power series, the expansion of $(1+\Delta p)^{-1}$ up to the first order approximation $0(n^{-1})$ is given by $(1-\Delta p + \Delta_P^2)$.

Therefore equation (2.3) now result to

$$\bar{Y}_{adR} = \bar{Y}[\alpha(1+\Delta\bar{y}) + (1-\alpha)\bar{Y}(1+\Delta\bar{y})(1+\Delta p^1)(1-\Delta p + \Delta_P^2)] \quad (22)$$

Further expansion of equation (22)

$$\bar{Y}_{adR} = \bar{Y}[\alpha(1+\Delta\bar{y})\ (1-\alpha)(1-\Delta p + \Delta_P^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta p^1\Delta_P^2 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta_P^2\Delta\bar{y} + \Delta\bar{y}\Delta p^1 + \Delta\bar{y}\Delta p\Delta p^1 + \Delta\bar{y}\Delta p^1\Delta_P^2)] \quad (23)$$

Limiting equation (23) to the power of two or order two (2)

$$\bar{Y}_{adR} = \bar{Y}[\alpha(1+\Delta\bar{y}) + (1-\alpha)\left(1-\Delta p + \Delta_P^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1\right) \quad (24)$$

Bias of the Proposed modified family of ratio estimator

$$\text{Bias}(\bar{Y}_{adR}) = E(\bar{Y}_{adR} - \bar{Y})$$

$$(\bar{Y}_{adR} - \bar{Y}) = \{\bar{Y}\left[\alpha(1+\Delta\bar{y}) + (1-\alpha)\left(1-\Delta p + \Delta_P^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1\right)\right] - \bar{Y}\} \quad (25)$$

$$(\bar{Y}_{adR} - \bar{Y}) = \bar{Y}\{\left[\alpha(1+\Delta\bar{y}) + (1-\alpha)\left(1-\Delta p + \Delta_P^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1\right)\right] - 1\}$$

Take the expectation of both sides of equation (25):

$$E(\bar{Y}_{adR} - \bar{Y}) = \bar{Y}\{\left[\alpha(E(1) + E(\Delta\bar{y})) + (1-\alpha)(E(1) - E(\Delta p) + E(\Delta_P^2) + E(\Delta p^1) - E(\Delta p\Delta p^1) + E(\Delta\bar{y}) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1)\right] - E(1) \quad (26)$$

Substitute for equation (19) in equation (26):

$$E(\bar{Y}_{adR} - \bar{Y}) = \bar{Y}\{\left[\alpha(0) + (1-\alpha)\left(0 + E(\Delta_P^2) + 0 - E(\Delta p\Delta p^1) + 0 - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1)\right] - 0\}$$

$$E(\bar{Y}_{adR} - \bar{Y}) = \bar{Y}\left[(1-\alpha)\left(E(\Delta_P^2) - E(\Delta p\Delta p^1) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1)\right)\right] \quad (27)$$

Further substitution in equation (27) gives:

$E(\Delta_P^2) = (\frac{1}{n}-\frac{1}{N})\frac{S_P^2}{P^2}$, $E(\Delta\bar{y}\Delta p^1) = (\frac{1}{n^1}-\frac{1}{N})\frac{S_{yp}}{\bar{Y}P}$, $E(\Delta\bar{y}\Delta p) = (\frac{1}{n}-\frac{1}{N})\frac{S_{yp}}{\bar{Y}P}$ and $E(\Delta p\Delta p^1) = (\frac{1}{n^1}-\frac{1}{N})\frac{S_P^2}{p^2}$

$$E(\bar{Y}_{adR} - \bar{Y}) = \bar{Y}\left[(1-\alpha)\left((\tfrac{1}{n}-\tfrac{1}{N})\tfrac{S_P^2}{P^2} - (\tfrac{1}{n^1}-\tfrac{1}{N})\tfrac{S_P^2}{p^2} - (\tfrac{1}{n}-\tfrac{1}{N})\tfrac{S_{yp}}{\bar{Y}P} + (\tfrac{1}{n^1}-\tfrac{1}{N})\tfrac{S_{yp}}{\bar{Y}P}\right)\right] \quad (28)$$

Collecting the like terms,

$$E(\bar{Y}_{adR}-\bar{Y}) = \bar{Y}\left[(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_P^2}{P^2} - \left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_{yp}}{\bar{Y}P}\right)\right] \quad (29)$$

Further simplification result to

$$E(\bar{Y}_{adR}-\bar{Y}) = \bar{Y}\left[(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_P^2}{P^2} - \left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_{yp}}{\bar{Y}P}\right)\right] \quad (30)$$

$$E(\bar{Y}_{adR}-\bar{Y}) = \bar{Y}\left[(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)\left(\frac{S_P^2}{P^2} - \frac{S_{yp}}{\bar{Y}P}\right)\right)\right] \quad (31)$$

$$E(\bar{Y}_{adR}-\bar{Y}) = \bar{Y}\left[(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)\left(C_p^2 - C_{yp}\right)\right)\right] \quad (32)$$

The Bias denoted by $Bias(\bar{Y}_{adR})$ of the modified ratio estimator $(\bar{Y}_{adR})$ is

$$Bias(\bar{Y}_{adR}) = \bar{Y}\left[(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)\left(C_p^2 - C_{yp}\right)\right)\right] \quad (33)$$

MSE of the Proposed modified family of ratio estimator $(\bar{Y}_{adR})$

To obtain mean square error (MSE) of the modified ratio estimator $\bar{Y}_{adR}$, repeat the steps of equations (20), (21), (22), (23), (24) and (25).

$$(\bar{Y}_{adR}-\bar{Y}) = \bar{Y}\{[\alpha(1+\Delta\bar{y}) + (1-\alpha)(1-\Delta p + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1)] - 1\} \quad (34)$$

$$MSE(\bar{Y}_{adR}) = E(\bar{Y}_{adR}-\bar{Y})^2$$

Squaring both sides of equation (34)

$$\left\{(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[\alpha^2(1+\Delta\bar{y})^2 + 2\alpha(1-\alpha)(1+\Delta\bar{y})(1-\Delta p + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1) - 2\alpha - 2(1-\alpha)(1+\Delta\bar{y})(1-\Delta p + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1) + (1-\alpha)^2(1-\Delta p + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1)^2 + 1]\right\} \quad (35)$$

Expanding equation (35) and limit the equation to order two

$$\{(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[\alpha^2(1+2\Delta\bar{y} + \Delta_{\bar{y}}^2) + 2\alpha(1-\alpha)(1-\Delta p + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1 + \Delta_{\bar{y}}^2) - 2\alpha(1+\Delta\bar{y}) - 2(1-\alpha)(1-\Delta p + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1) + (1-\alpha)^2(1-\Delta p + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1 - \Delta p + \Delta_p^2 - \Delta p\Delta p^1 - \Delta p\Delta\bar{y} + \Delta_p^2 + \Delta p^1 - \Delta p\Delta p^1 + \Delta_{p^1}^2 + \Delta p^1\Delta\bar{y} - \Delta p\Delta p^1 + \Delta\bar{y} - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1 + \Delta_{\bar{y}}^2 - \Delta\bar{y}\Delta p + \Delta\bar{y}\Delta p^1)]\} \quad (36)$$

Taking the expectation of equation (36) above

$$E(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[E(1) + 2E(\Delta\bar{y}) + E(\Delta_{\bar{y}}^2) + 2\alpha(1-\alpha)E(1) - E(\Delta p) + E(\Delta_p^2) + E(\Delta p^1) - E(\Delta p\Delta p^1) - 2E(\Delta\bar{y}\Delta p) + 2E(\Delta\bar{y}\Delta p^1) + 2E(\Delta\bar{y}) + E(\Delta_{\bar{y}}^2) - 2\alpha(E(1) + E(\Delta\bar{y}) - 2(1-\alpha)(E(1) - E(\Delta p) + E(\Delta_p^2) + E(\Delta p^1) - E(\Delta p\Delta p^1) + E(\Delta\bar{y}) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1) + (1-\alpha)^2(E(1) - E(\Delta p) + E(\Delta_p^2) + E(\Delta p^1) - E(\Delta p\Delta p^1) + E(\Delta\bar{y}) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1) - E(\Delta p) + E(\Delta_p^2) - E(\Delta p\Delta p^1) - E(\Delta p\Delta\bar{y}) + E(\Delta_p^2) + E(\Delta p^1) - E(\Delta p\Delta p^1) + E(\Delta_{p^1}^2) + E(\Delta p^1\Delta\bar{y}) - E(\Delta p\Delta p^1) + E(\Delta\bar{y}) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1) + E(\Delta_{\bar{y}}^2) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1)] \quad (37)$$

substituting for equation (19) in equation (37)

$$E(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[0 + 0 + E(\Delta_{\bar{y}}^2) + 2\alpha(1-\alpha)(0 - 0 + E(\Delta_p^2) + 0 - E(\Delta p\Delta p^1) - 2E(\Delta\bar{y}\Delta p) + 2E(\Delta\bar{y}\Delta p^1) + 0 + E(\Delta_{\bar{y}}^2) - 2(1-\alpha)(0 - 0 + E(\Delta_p^2) + 0 - E(\Delta p\Delta p^1) + 0 - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1)) + (1-\alpha)^2(0 - 0 + E(\Delta_p^2) + 0 - E(\Delta p\Delta p^1) + 0 - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1) - 0 + E(\Delta_p^2) - E(\Delta p\Delta p^1) - E(\Delta p\Delta\bar{y}) + E(\Delta_p^2) + 0 - E(\Delta p\Delta p^1) + E(\Delta_{p^1}^2) + E(\Delta p^1\Delta\bar{y}) - E(\Delta p\Delta p^1) + 0 - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1) + E(\Delta_{\bar{y}}^2) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1)] \quad (2.19)$$

$$E(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[E(\Delta_{\bar{y}}^2) + 2\alpha(1-\alpha)(E(\Delta_p^2) - E(\Delta p\Delta p^1) - 2E(\Delta\bar{y}\Delta p) + 2E(\Delta\bar{y}\Delta p^1) + E(\Delta_{\bar{y}}^2)) - 2(1-\alpha)(E(\Delta_p^2) - E(\Delta p\Delta p^1) - E(\Delta\bar{y}\Delta p) + E(\Delta\bar{y}\Delta p^1)) + (1-\alpha)^2(3E(\Delta_p^2) - 4E(\Delta p\Delta p^1) - 4E(\Delta\bar{y}\Delta p) + 4E(\Delta\bar{y}\Delta p^1) + E(\Delta_{p^1}^2) + E(\Delta_{\bar{y}}^2)] \quad (38)$$

Further substituting for equation (19) in equation (38):

$$E(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[\alpha^2\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{\bar{y}}^2}{\bar{Y}^2} + 2\alpha(1-\alpha)(\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_P^2}{P^2} - \left(\frac{1}{n^1}-\frac{1}{N}\right)\frac{S_P^2}{p^2} - 2\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{yp}}{\bar{Y}P} + 2\left(\frac{1}{n^1}-\frac{1}{N}\right)\frac{S_{yp}}{\bar{Y}P} + \left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{\bar{y}}^2}{\bar{Y}^2}) - 2(1-\alpha)((\frac{1}{n}-\frac{1}{N})\frac{S_P^2}{P^2} - \left(\frac{1}{n^1}-\frac{1}{N}\right)\frac{S_P^2}{p^2} - \left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{yp}}{\bar{Y}P} + (\frac{1}{n^1}-\frac{1}{N})\frac{S_{yp}}{\bar{Y}P}) + (1-\alpha)^2(3\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_P^2}{P^2} - 4\left(\frac{1}{n^1}-\frac{1}{N}\right)\frac{S_P^2}{p^2} -$$

$$4\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_{yp}}{\overline{YP}}+4\left(\frac{1}{n^1}-\frac{1}{N}\right)\frac{S_{yp}}{\overline{YP}}+\left(\frac{1}{n^1}-\frac{1}{N}\right)\frac{S_P^2}{p^2}+\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2})] \qquad (39)$$

Collecting the like terms in equation (39)

$$E(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[\alpha^2\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}+2\alpha(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_P^2}{P^2}--2\left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_{yp}}{\overline{YP}}+\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}\right)-2(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_P^2}{P^2}-\left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_{yp}}{\overline{YP}}\right)+(1-\alpha)^2\left(3\left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_P^2}{P^2}-4\left(\frac{1}{n}-\frac{1}{N}-\frac{1}{n^1}+\frac{1}{N}\right)\frac{S_{yp}}{\overline{YP}}+\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}\right)] \qquad (40)$$

Further simplifying equation (40)

$$E(\bar{Y}_{adR}-\bar{Y})^2 = \bar{Y}^2[\alpha^2\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}+2\alpha(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_P^2}{P^2}--2\left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_{yp}}{\overline{YP}}+\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}\right)-2(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_P^2}{P^2}-\left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_{yp}}{\overline{YP}}+\right.$$
$$\left.(1-\alpha)^2\left(3\left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_P^2}{P^2}-4\left(\frac{1}{n}-\frac{1}{n^1}\right)\frac{S_{yp}}{\overline{YP}}+\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}\right)\right] \qquad (41)$$

$$MSE(\bar{Y}_{adR}) = E(\bar{Y}_{adR}-\bar{Y})^2 = \alpha^2\left(\frac{1}{n}-\frac{1}{N}\right)\frac{S_y^2}{\bar{Y}^2}+2\alpha(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)C_p^2--2\left(\frac{1}{n}-\frac{1}{n^1}\right)C_{yp}+\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2\right)-2(1-\alpha)\left(\left(\frac{1}{n}-\frac{1}{n^1}\right)C_p^2-\left(\frac{1}{n}-\frac{1}{n^1}\right)C_{yp}\right)+(1-\alpha)^2\left(3\left(\frac{1}{n}-\frac{1}{n^1}\right)C_p^2-4\left(\frac{1}{n}-\frac{1}{n^1}\right)C_{yp}+\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2\right) \qquad (42)$$

Mean Square Error (MSE) of the modified ratio estimator ($\bar{Y}_{adR}$) denoted by $MSE(\bar{Y}_{adR})$ is derived as :-

$$MSE(\bar{Y}_{adR}) = \bar{Y}^2[\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+(1-\alpha)\left(\frac{1}{n}-\frac{1}{n^1}\right)\left((1-\alpha)C_p^2-2C_{yp}\right) \qquad (43)$$

The most favourable value of the characterizing scalar α is obtained by minimizing Mean square error $MSE(\bar{Y}_{adR})$ using the method of maxima-minima.

Expanding equation (43)

$$MSE(\bar{Y}_{adR}) = \bar{Y}^2[\left(\frac{1}{n}-\frac{1}{N}\right)C_y^2+(1-\alpha)^2\left(\frac{1}{n}-\frac{1}{n^1}\right)C_p^2-2(1-\alpha)\left(\frac{1}{n}-\frac{1}{n^1}\right)C_{yp}]$$

Where $F_{nn^1}=\left(\frac{1}{n}-\frac{1}{n^1}\right)$, $F_n=\left(\frac{1}{n}-\frac{1}{N}\right)$

$$MSE(\bar{Y}_{adR}) = \bar{Y}^2[F_n C_y^2+(1-\alpha)^2F_{nn^1}C_p^2-2(1-\alpha)F_{nn^1}C_{yp}] \qquad (44)$$

Differentiating $MSE(\bar{Y}_{adR})$ with respect to α

$$\frac{\Delta(MSE(\bar{Y}_{adR}))}{\Delta\alpha} = 2(1-\alpha)(-1)F_{nn^1}C_p^2-2(-1)F_{nn^1}C_{yp} \qquad (45)$$

Limiting $\frac{\Delta(MSE(\bar{Y}_{adR}))}{\Delta\alpha}=0$

$$2(1-\alpha)(-)F_{nn^1}C_p^2-2(-1)F_{nn^1}C_{yp}=0 \qquad (46)$$
$$2F_{nn^1}C_{yp}-2(1-\alpha)F_{nn^1}C_p^2=0 \qquad (47)$$
$$2F_{nn^1}(C_{yp}-(1-\alpha)C_p^2)=0$$
$$C_{yp}-(1-\alpha)C_p^2=0 \qquad (48)$$
$$(1-\alpha)=\frac{C_{yp}}{C_p^2} \qquad (49)$$

The optimum value of the characteristics scalar is

$$\alpha = 1-\frac{C_{yp}}{C_p^2}=\frac{C_p^2-C_{yp}}{C_p^2} \qquad (50)$$

Substituting for optimum alpha α_{opt} in equation (33), where $F_{nn^1}=\left(\frac{1}{n}-\frac{1}{n^1}\right)$, $F_n=\left(\frac{1}{n}-\frac{1}{N}\right)$

$$Bias(\bar{Y}_{adR})=\bar{Y}(1-\alpha)F_{nn^1}(C_p^2-C_{yp}) \qquad (51)$$
$$Bias(\bar{Y}_{adR})=\bar{Y}\frac{C_{yp}}{C_p^2}F_{nn^1}(C_p^2-C_{yp}) \qquad (52)$$

Simplifying equation (52)

$$Bias(\bar{Y}_{adR})=\bar{Y}F_{nn^1}\left(C_{yp}-\frac{{C_{yp}}^2}{C_p^2}\right) \qquad (53)$$

Further simplification of equation (53)

$$Bias(\bar{Y}_{adR})=\bar{Y}F_{nn^1}(\rho_{yp}C_yC_p-\rho_{yp}^2C_y^2) \qquad (54)$$

The minimum $Bias(\bar{Y}_{adR})$ written as $Bias(\bar{Y}_{adR})_{min}$ is obtained as

$$Bias(\bar{Y}_{adR})=\bar{Y}F_{nn^1}\rho_{yp}C_y(C_p-\rho_{yp}C_y) \qquad (55)$$

Substituting for optimum alpha α_{opt} in equation (44), where $F_{nn^1}=\left(\frac{1}{n}-\frac{1}{n^1}\right)$, $F_n=\left(\frac{1}{n}-\frac{1}{N}\right)$

$$MSE(\bar{Y}_{adR}) = \bar{Y}^2[F_nC_y^2+(1-\alpha)^2F_{nn^1}C_p^2-2(1-\alpha)F_{nn^1}C_{yp}] \qquad (56)$$

$$MSE(\bar{Y}_{adR}) = \bar{Y}^2[F_nC_y^2+\left(\frac{C_{yp}}{C_p^2}\right)^2F_{nn^1}C_p^2-2\left(\frac{C_{yp}}{C_p^2}\right)F_{nn^1}C_{yp}] \qquad (57)$$

Simplifying equation (57)

$$MSE(\bar{Y}_{adR}) = \bar{Y}^2[F_n C_y^2 + \frac{{C_{yp}}^2}{C_p^2} F_{nn^1} - 2\left(\frac{{C_{yp}}^2}{C_p^2}\right) F_{nn^1}] \quad (58)$$

Further simplification of equation (58)

$$MSE(\bar{Y}_{adR}) = \bar{Y}^2[F_n C_y^2 - \left(\frac{{C_{yp}}^2}{C_p^2}\right) F_{nn^1}] \quad (59)$$

The minimum $MSE(\bar{Y}_{adR})$ written as $MSE(\bar{Y}_{adR})_{min}$ is obtained as

$$MSE(\bar{Y}_{adR})_{min} = \bar{Y}^2 C_y^2[F_n - \rho_{yp}^2 F_{nn^1}] \quad (60)$$

where $\rho_{yp}^2 = \frac{{C_{yp}}^2}{C_y^2 C_p^2}$, $C_y^2 \rho_{yp}^2 = \frac{{C_{yp}}^2}{C_p^2}$

3. EFFICIENCY AND EMPIRICAL COMPARISON

To determine the efficiency of the proposed ratio estimator with some existing estimators of population mean in double sampling, we have considered the following two populations

Population 1 (Source: [Coc77] page 34)
Y=food cost, X= Family Income, ϕ =Family of size more than 3
$\bar{Y}$=27.40, $\bar{X}$=72.55, P=0.52, C_x=0.146, C_y=0.369, C_p=0.985, ρ_{yx}=0.2521, ρ_{yp}=0.388, ρ_{xp}=-0.153, n=15, n^1=25, N=33

Population 2 (Source: University of Ilorin Teaching Hospital, Ilorin Nigeria 2018)
Y= Length of Labour, X= Mother's Age, ϕ =Type of labour
$\bar{Y}$=44.94, $\bar{X}$=30.34, P=0.62, C_x=0.1843, C_y=0.2122, C_p=0.6452, ρ_{yx}=0.05759, ρ_{yp}=0.21396, ρ_{xp}=0.4041, n=25, n^1=30, N=50

POPULATION 1
From Table 1, it is clear that all the conditions are met. From Table 2:

$MSE(\bar{Y}_{adR})$< $MSE(\bar{Y}_{SS})$< MSE ($\bar{Y}_{Sub}$)< MSE ($\bar{Y}_{DSR}$)< $MSE(\bar{Y}_{srs})$ <$MSE(\bar{Y}_{NS})$< $MSE(\bar{Y}_{NG})$
PRE($\bar{Y}_{adR}$)> PRE($\bar{Y}_{SS}$) > PRE($\bar{Y}_{Sub}$) > PRE($\bar{Y}_{DSR}$) > $PRE(\bar{Y}_{srs})$ > $PRE(\bar{Y}_{NS})$ > $PRE(\bar{Y}_{NG})$
$Bias(\bar{Y}_{SS})$ < Bias($\bar{Y}_{Sub}$) < $Bias(\bar{Y}_{adR})$ < $Bias(\bar{Y}_{adRP})$ < $Bias(\bar{Y}_{DSR})$ <$Bias(\bar{Y}_{NS})$ < $Bias(\bar{Y}_{NG})$

POPULATION 2
From Table 3, it is clear that all the conditions are met. From Table 4:

$MSE(\bar{Y}_{adR})$ < MSE ($\bar{Y}_{Sub}$)<$MSE(\bar{Y}_{SS})$ < MSE ($\bar{Y}_{DSR}$)< $MSE(\bar{Y}_{srs})$ <$MSE(\bar{Y}_{NS})$ < $MSE(\bar{Y}_{NG})$
PRE($\bar{Y}_{adR}$) > PRE ($\bar{Y}_{Sub}$) > PRE($\bar{Y}_{SS}$) > PRE ($\bar{Y}_{DSR}$) > $PRE(\bar{Y}_{srs})$ > $PRE(\bar{Y}_{NS})$ > $PRE(\bar{Y}_{NG})$
$Bias(\bar{Y}_{SS})$ <Bias ($\bar{Y}_{Sub}$) < Bias ($\bar{Y}_{DSR}$) < $Bias(\bar{Y}_{adR})$ < $Bias(\bar{Y}_{NG})$<$Bias(\bar{Y}_{NS})$

Table 1: Theoretical and Empirical Conditions for Efficiency of $\bar{Y}_{adR}$ on $\bar{Y}_{srs}$, $\bar{Y}_{DSR}$, $\bar{Y}_{NG}$, $\bar{Y}_{NS}$, $\bar{Y}_{SS}$, and $\bar{Y}_{Sub}$ using population 1, n=15, n^1=25, N=33

S/No	Author	Estimator	Theoretical or Mathematical Condition	Empirical Condition
1	General Estimator	$\bar{Y}_{srs}$	$\rho_{yp}^2 \geq 0$	0.1505 > 0
2	Double sampling	$\bar{Y}_{DSR}$	$\rho_{yp}^2 \geq \frac{2C_{yx} - C_x^2}{C_y^2}$	0.1505 > 0.0429
3	Naik, Gupta ([NG96])	$\bar{Y}_{NG}$	$\rho_{yp} \leq \frac{C_p}{C_y}$	0.388 < 2.6694
4	Sawan ([Saw10])	$\bar{Y}_{NS}$	$\rho_{yp} \leq \frac{C_p}{2C_y}$	0.388 < 1.3347
5	Solanki, Singh([SS13])	$\bar{Y}_{SS}$	$\rho_{yp}^2(F_n - F_{nn^1}) \geq 0$	0.00146 >0
6	Subhash et al ([SSA16])	$\bar{Y}_{Sub}$	$\rho_{yp}^2 \geq \frac{A^2 - gC_x^2 B(g-4c)}{4BC_y^2}$	0.1505 > 0.0635

Table 2: Bias, Mean Square Error and Percentage Relative Efficiency of $\bar{Y}_{adR}, \bar{Y}_{srs}, \bar{Y}_{DSR}, \bar{Y}_{NG}, \bar{Y}_{NS}, \bar{Y}_{SS}$, and $\bar{Y}_{Sub}$ for Population 1 where ρ > 0

S/No	Author	Estimator	Bias	Mean Square Error (MSE)	Percentage Relative Efficiency (PRE)
1	General Estimator	$\bar{Y}_{srs}$		3.7179	100
2	Double sampling	$\bar{Y}_{DSR}$	0.1221	3.5997	103
3	Naik, Gupta ([NG96])	$\bar{Y}_{NG}$	0.6066	17.5319	21
4	Sawan ([NS10])	$\bar{Y}_{NS}$	-0.6581	5.7530	65
5	Solanki, Singh([SS13])	$\bar{Y}_{SS}$	0.003464	3.5677	104
6	Subhash et al ([S+16])	$\bar{Y}_{Sub}$	0.01005	3.5444	105
7	Modified Ratio Estimator	$\bar{Y}_{adR}$	0.0881	3.3069	112

Table 3: Theoretical and Empirical Conditions for Efficiency of $\bar{Y}_{adR}$ on $\bar{Y}_{srs}, \bar{Y}_{DSR}, \bar{Y}_{NG}, \bar{Y}_{NS}, \bar{Y}_{SS}$, and $\bar{Y}_{Sub}$ using population 2, n=30, n^1=45, N=60

S/No	Author	Estimator	Theoretical or Mathematical Condition	Empirical Condition
1	General Estimator	$\bar{Y}_{srs}$	$\rho_{yp}^2 \geq 0$	0.9932 > 0
2	Double sampling	$\bar{Y}_{DSR}$	$\rho_{yp}^2 \geq \frac{2C_{yx} - C_x^2}{C_y^2}$	0.9932 > 0.7328
3	Naik, Gupta([NG96])	$\bar{Y}_{NG}$	$\rho_{yp} \leq \frac{C_p}{C_y}$	0.9966 < 39.6719
4	Nirmala Sawan ([Saw10])	$\bar{Y}_{NS}$	$\rho_{yp} \leq \frac{C_p}{2C_y}$	0.9966 < 19.8360
5	Solanki, Singh([SS13])	$\bar{Y}_{SS}$	$\rho_{yp}^2(F_n - F_{nn^1}) \geq 0$	0.005518 >0
6	Subhash et al ([S+16])	$\bar{Y}_{Sub}$	$\rho_{yp}^2 \geq \frac{A^2 - gC_x^2 B(g-4c)}{4BC_y^2}$	0.9932 > 0.8411

Table 4: Bias, Mean Square Error and Percentage Relative Efficiency of $\bar{Y}_{adR}, \bar{Y}_{srs}, \bar{Y}_{DSR}, \bar{Y}_{NG}, \bar{Y}_{NS}, \bar{Y}_{SS}$, and $\bar{Y}_{Sub}$ for Population 2 where ρ > 0

S/No	Author	Estimator	Bias	Mean Square Error (MSE)	Percentage Relative Efficiency (PRE)
1	General Estimator	$\bar{Y}_{srs}$		265.4960	100
2	Double sampling	$\bar{Y}_{DSR}$	1.3525	139.3662	191
3	Naik, Gupta (1996)	$\bar{Y}_{NG}$	2988.0396	264,455.3987	0.1
4	Nirmala Sawan (2010)	$\bar{Y}_{NS}$	-3026.5379	62,826.7679	0.4
5	Solanki, Singh(2013)	$\bar{Y}_{SS}$	0.007144	177.5505	150
6	Subhash et al (2016)	$\bar{Y}_{Sub}$	-0.8073	116.8164	227
7	Modified Ratio Estimator	$\bar{Y}_{adR}$	75.0624	89.6849	296

4. CONCLUSION

The present study proposed a modified family of ratio estimator of population mean using information on auxiliary attribute, we have improved estimation of population mean in double sampling in ratio form. The expressions for the Bias and Mean Square Error of the proposed modified estimator have been obtained up to the first order of approximation. An efficiency comparison in both theoretical and empirical has been carried out with some related existing estimators in double sampling. It has been shown that the proposed modified family of ratio estimator is more efficient than some related existing estimators.

REFERENCES

[BT91] **Bahl S., Tuteja R. K.** - *Ratio and product type exponential estimator*, Inform. and Optimize. Science, 12(1), 159-163. 1991.

[Coc40] **Cochran W. G.** - *The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce*, J. Agric. Sci., 30, 262–275, 1940.

[Coc77] **Cochran W. G.** - *Sampling techniques*, 3rd Edition. John Wiley, New York. 1977.

[CS12] **Choudhury S., Singh B. K.** - *A Class of Product-Cum-Dual to Product Estimators of the Population Mean In Survey Sampling using Auxiliary Information*, Asian J Math Stat, 12(3). 9. 2012.

[JSG06] **Jhajj H. S., Sharma M. K., Grover L. K.** - *A family of estimators of population means using information on auxiliary attributes*, Pakistan Journal of Statistics, 22(1), 43-50. 2006.

[Nay38] **Nayman J.** - *Contribution to the theory of sampling human populations*. Jour. Amer. Stat. Assoc. 33, 101-116, 1938.

[NG96] **Naik V. D., Gupta P. C.** - *A note on estimation of mean with known population proportion of an auxiliary character*. Jour. Ind. Soc. Agr. Stat., 48(2), 151-158. 1996.

[Saw10] **Sawan N.** - *Suggested Ratio-product type exponential estimator for estimating finite population mean using information on auxiliary attribute*. Pak. Sta. Oper. Res. 5.47-53, 2010.

[SK08] **Singh H. P., Kumur S.** - *Estimation of mean in presence of non-response using two phase sampling scheme*, Statistical Paper Do1 10.1007/S00362-008040-5, 2008.

[SS13] **Solanki R. S., Singh H. P.** - *Improved estimation of population mean using population proportion of an auxiliary character*, Chilean Journal of Statistics, 1-16, 2013.

[SV06] **Singh H. P., Vishwakarma G. K.** - *Combined ratio-product estimator of finite population mean in stratified sampling*, Metodologia de Encuestas, 7(1), 32-40. 2006.

[SV07] **Singh H. P., Vishwakarma G.** - *Modified exponential ratio and product estimators of finite population mean in double sampling*, Aust. J. of Statist., 36(3), 217-225. 2007.

[S+16] **Subhash K. Y., Sat G., Mishra S. S., Shukla A. K.** - *Modified Ratio and Product Estimators for Estimating Population Mean in Two-Phase Sampling*. American Journal of Operational Research. 6(3): 61-68, 2016.