MODELLING OF BOREHOLE COMPUTER EXPERIMENTS THROUGH A MODIFIED BOREHOLE MODEL

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ABSTRACT: This study aimed at performing a borehole computer experiment via a modified borehole model. An Orthogonal Array Latin Hypercube Design, OA (49, 8) LHD with parameters' specification N=49 and k=8 was used to develop a borehole computer experiment using the existing borehole model and a modified borehole model with the same assumed ranges of values for the 8 input variables of a borehole model. The borehole model was modified based on the data generated from the existing model and no assumption of a borehole model was relaxed. The modified model has four newly introduced parameters and a nonlinear regression fit in MATLAB (The MathWorks, Inc. 2016) was used to estimate the values of the four parameters. The four parameters β_1, β_2, β_3 and β_4 have the estimated values of 0.4538, -0.9986, -0.1176 and 9.2245, respectively. The modified borehole model performs well in developing a borehole computer experiment with the results obtained which are close to those of the existing model as shown in Figure 1.This study concludes that a borehole experiment can be implemented through a modified borehole model which gives a fair representation of the existing borehole model without relaxing the assumptions of the model. This approach on the modification of an existing borehole model is novel in the world of research in this field.

KEYWORDS: Computer experiment, Existing borehole model, Modified borehole model, orthogonal array Latin hypercube design, Parameters.

1. INTRODUCTION

Experiments are currently performed virtually in every field of human endeavour as an efficient tool for studying and improving various processes. Fang *et al.* ([FLS06]) classified experiments into physical and computer experiments. While a physical experiment is conventionally performed in a laboratory, a factory or an agricultural field where different responses are obtained under the same experimental setting due to the existence of random errors, a computer experiment serves as a practical alternative to mimic a physical experiment when it seems infeasible to perform. To give a few examples of such situations, it is difficult to employ physical experimentation to predict climate and weather, the performance of integrated circuits, the behaviour of controlled nuclear fusion devices, the properties of thermal energy storage devices and the stresses in prosthetic devices. Computer experiments can also be conducted to serve as a prototype before a physical experiment is implemented. Computer experiments are known for long, not only in science and technology but also in statistics and modern industry. Chemical kinetics was modelled as a computer experiment by Miller and Frenklach ([MF83]) and Sacks *et al.* ([SSW89]). Several researchers used a computer simulation in the design of analog integrated circuit behaviour where *x* represents various circuit parameters and *y* is the measurement of circuit performance such as output voltage (([S+89], [LZH00]). The environmental experiment which shows applications of design and modelling techniques for computer experiments to industrial experiments has also been discussed by Fang and Wang ([FW94]) and Li ([Li02]). Computer experiments are also used in the design of engine block and head joint sealing assembly containing multiple components ([C+02]). Qian *et al.* ([Q+06]) gave an example which deals with designing a heat exchanger for a representative electronic cooling application.

2. MODELLING OF COMPUTER EXPERIMENTS

Computer model refers to a model of a physical system. This model mimics a physical experiment. In this study, a borehole model (BM) and a modified borehole model (MBM) were used as computer models for simulating a real life borehole experiment. The OA (49, 8) LHD constructed by Osuolale et al. ([OYA15]) was adopted in the implementation of borehole computer experiments. Many authors including Worley ([Wor87]), Morris et al. ([MMY93]), Ho and Xu ([HX00]), An and Owen ([AO01]), Gramacy and Haaland ([GH14]) and Osuolale *et al.* (OYA17) among others have investigated a borehole experiment as a good example for computer experiments. The borehole model discussed in this study is deterministic and thus responses from the model lacks random error; that is, repeated experimental runs for identical input parameters give the same responses from the model. The borehole model is a simple example of flow rate of water through a borehole from an upper aquifer to a lower aquifer separated by an impermeable rock layer. It is assumed that the two aquifers are separated by an impermeable rock layer and the borehole is drilled from the ground surface.

3. METHODOLOGY

The borehole model originally involves an 8-dimensional input variable and the output variable y in m^3/yr. This output measures the rate of flow of water from an upper aquifer to a lower one and the model assumes that the flow is steady-state, laminar and isothermal and is mathematically given as:

$$y = \frac{2\pi T_u\ (H_u - H_l\ )}{ln\left(\frac{r}{r_w}\right)\left[1 + \frac{2LT_u}{ln\left(\frac{r}{r_w}\right)r_w^2\ k_w} + \frac{T_u}{T_l}\right]} \tag{1}$$

Where:
y (m^3/yr) = flow rate of water
r_w (m) = radius of borehole
r (m) = radius of influence
T_l (m^2/yr) = transmissivity of lower aquifer
T_u (m^2/yr) = transmissivity of upper aquifer
H_l (m) = potentiometric head of lower aquifer
H_u (m) = potentiometric head of upper aquifer
L (m) = length of borehole and
K_w (m/yr) = hydraulic conductivity of borehole

The OA (49, 8) LHD was used to implement the model in equation 1 with specified assumed range of values for the input variables as provided in Table 1 as well as the experimental data for borehole computer experiments in Table 2, respectively. The modified model is also given in Equation 2.

Table 1: Input and Output variables for borehole model

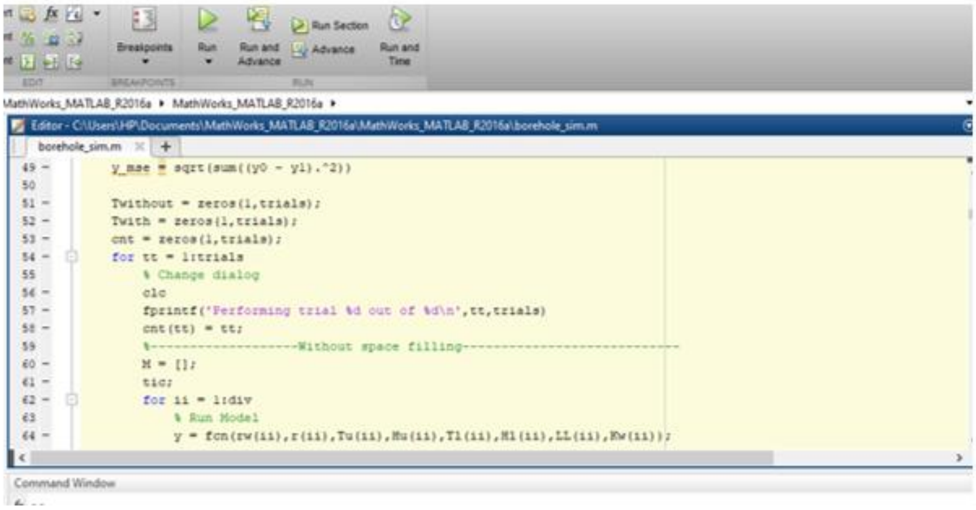
Variable	Variable name	Minimum	Maximum
r_w	Radius of Borehole (m)	0.05	0.15
R	Radius of Influence (m)	100	50000
T_u	Transmissivity of Upper Aquifer (m^2/yr)	63070	115600
H_u	Potentiometric Head of Upper Aquifer (m)	990	1100
T_l	Transmissivity of Lower Aquifer (m^2/yr)	63.1	116
H_l	Potentiometric Head of Lower Aquifer (m)	700	820
L	Length of Borehole (m)	1120	1680
K_w	Hydraulic Conductivity of Borehole (m/yr)	9855	12045
Y	Flow Rate of Water (m^3/yr)	-	-

$$y = \frac{\beta_1 T_u\ (H_u - H_l\ )}{\beta_2 ln\left(\frac{r}{r_w}\right)\left[1 + \frac{\beta_3 LT_u}{\beta_4 r_w^2\ k_w} + \frac{T_u}{T_l}\right]} \tag{2}$$

The borehole model was modified based on the experimental data generated from the existing model without relaxing any of its assumptions. A nonlinear regression fit in MATLAB was used to estimate the values of the four parameters β_1, β_2, β_3 and β_4 newly introduced to the existing model using the function given in equation 3.

mdl = NonLinearModel.fit (ds,modelfcn,beta0) (3)

The parameters β_1, β_2, β_3 and β_4 gave the estimated values of 0.4538, -0.9986, -0.1176 and 9.2245, respectively. All the parameters in the existing model remained the same as those in the modified model. In the experimental results given in Table 2, Y_e is the experimental output from existing model while Y_m is the output of a modified borehole model.



```
    y_mse = sqrt(sum((y0 - y1).^2))

    Twithout = zeros(1,trials);
    Twith = zeros(1,trials);
    cnt = zeros(1,trials);
    for tt = 1:trials
        % Change dialog
        clc
        fprintf('Performing trial %d out of %d\n',tt,trials)
        cnt(tt) = tt;
        %------------------Without space filling------------------------
        M = [];
        tic;
        for ii = 1:div
            % Run Model
            y = fcn(rw(ii),r(ii),Tu(ii),Hu(ii),Tl(ii),Hl(ii),LL(ii),Kw(ii));
```

Figure 1: MATLAB Code for the development of the experiment using the existing model and modified borehole model

Table 2: Experimental data and outputs for borehole computer experiments using the existing and modified models

Run	r_w	r	T_u	H_u	T_l	H_l	L	K_w	Y_e	Y_m
1	0.08	3218.25	64821.00	992.50	64.01	701.67	1126.51	10551.32	56.96	66.53
2	0.08	3219.62	64821.00	992.50	64.01	701.67	1126.51	10667.60	58.65	68.75
3	0.08	3220.99	64821.00	992.50	64.01	701.67	1126.51	10783.88	60.38	71.02
4	0.08	3222.36	64821.00	992.50	64.01	701.67	1126.51	10900.16	62.14	73.35
5	0.08	3223.73	64821.00	992.50	64.01	701.67	1126.51	11016.45	63.93	75.74
6	0.09	3225.11	64821.00	992.50	64.01	701.67	1126.51	11132.73	65.75	78.19
7	0.09	3226.48	64821.00	992.50	64.01	701.67	1126.51	11249.01	67.61	80.70
8	0.09	3219.82	64821.00	992.50	64.01	701.67	1126.51	10567.93	64.65	76.89
9	0.09	3221.19	64821.00	992.50	64.01	701.67	1126.51	10684.21	66.50	79.37
10	0.09	3222.56	64821.00	992.50	64.01	701.67	1126.51	10800.49	68.38	81.92
11	0.09	3223.93	64821.00	992.50	64.01	701.67	1126.51	10916.78	70.29	84.53
12	0.09	3225.30	64821.00	992.50	64.01	701.67	1126.51	11033.06	72.24	87.21
13	0.09	3226.67	64821.00	992.50	64.01	701.67	1126.51	11149.34	74.22	89.96
14	0.09	3218.44	64821.00	992.50	64.01	701.67	1126.51	11265.62	76.24	92.80
15	0.09	3221.38	64821.00	992.50	64.01	701.67	1126.51	10584.54	72.84	88.25
16	0.09	3222.75	64821.00	992.50	64.01	701.67	1126.51	10700.82	74.84	91.03
17	0.09	3224.13	64821.00	992.50	64.01	701.67	1126.51	10817.11	76.88	93.88
18	0.09	3225.50	64821.00	992.50	64.01	701.67	1126.51	10933.39	78.95	96.80
19	0.10	3226.87	64821.00	992.50	64.01	701.67	1126.51	11049.67	81.06	99.80
20	0.10	3218.64	64821.00	992.50	64.01	701.67	1126.51	11165.95	83.20	102.90
21	0.10	3220.01	64821.00	992.50	64.01	701.67	1126.51	11282.24	85.38	106.05
22	0.10	3222.95	64821.00	992.50	64.01	701.67	1126.51	10601.15	81.52	100.68
23	0.10	3224.32	64821.00	992.50	64.01	701.67	1126.51	10717.44	83.68	103.78
24	0.10	3225.69	64821.00	992.50	64.01	701.67	1126.51	10833.72	85.88	106.97
25	0.10	3227.07	64821.00	992.50	64.01	701.67	1126.51	10950.00	88.12	110.23
26	0.10	3218.84	64821.00	992.50	64.01	701.67	1126.51	11066.28	90.39	113.61
27	0.10	3220.21	64821.00	992.50	64.01	701.67	1126.51	11182.56	92.70	117.04
28	0.10	3221.58	64821.00	992.50	64.01	701.67	1126.51	11298.85	95.05	120.55
29	0.10	3224.52	64821.00	992.50	64.01	701.67	1126.51	10617.76	90.69	114.26
30	0.10	3225.89	64821.00	992.50	64.01	701.67	1126.51	10734.05	93.02	117.73
31	0.10	3227.26	64821.00	992.50	64.01	701.67	1126.51	10850.33	95.38	121.28
32	0.11	3219.03	64821.00	992.50	64.01	701.67	1126.51	10966.61	97.79	124.95
33	0.11	3220.40	64821.00	992.50	64.01	701.67	1126.51	11082.89	100.24	128.68
34	0.11	3221.77	64821.00	992.50	64.01	701.67	1126.51	11199.18	102.72	132.50
35	0.11	3223.15	64821.00	992.50	64.01	701.67	1126.51	11315.46	105.24	136.42
36	0.11	3226.09	64821.00	992.50	64.01	701.67	1126.51	10634.38	100.35	129.09
37	0.11	3227.46	64821.00	992.50	64.01	701.67	1126.51	10750.66	102.85	132.95
38	0.11	3219.23	64821.00	992.50	64.01	701.67	1126.51	10866.94	105.39	136.94
39	0.11	3220.60	64821.00	992.50	64.01	701.67	1126.51	10983.22	107.97	140.99
40	0.11	3221.97	64821.00	992.50	64.01	701.67	1126.51	11099.51	110.59	145.14
41	0.11	3223.34	64821.00	992.50	64.01	701.67	1126.51	11215.79	113.25	149.40
42	0.11	3224.71	64821.00	992.50	64.01	701.67	1126.51	11332.07	115.95	153.77
43	0.11	3227.65	64821.00	992.50	64.01	701.67	1126.51	10650.99	110.51	145.27
44	0.11	3219.42	64821.00	992.50	64.01	701.67	1126.51	10767.27	113.18	149.60
45	0.12	3220.80	64821.00	992.50	64.01	701.67	1126.51	10883.55	115.90	154.00
46	0.12	3222.17	64821.00	992.50	64.01	701.67	1126.51	10999.84	118.66	158.50
47	0.12	3223.54	64821.00	992.50	64.01	701.67	1126.51	11116.12	121.46	163.13
48	0.12	3224.91	64821.00	992.50	64.01	701.67	1126.51	11232.40	124.30	167.86
49	0.12	3226.28	64821.00	992.50	64.01	701.67	1126.51	11348.68	127.18	172.72

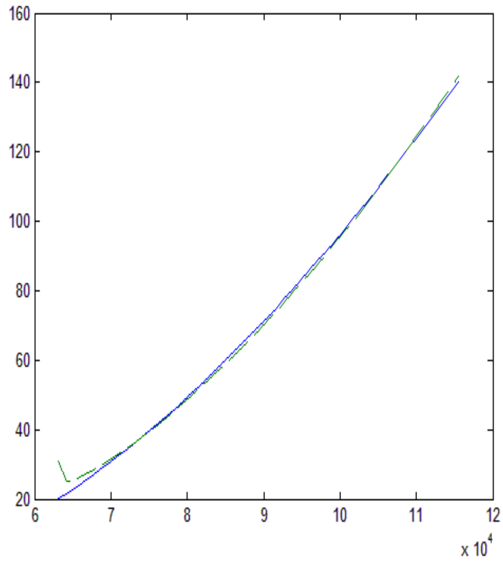


Figure 2: Representation of the output using the existing model and modified borehole model

The dotted line in Figure 1 represents the output of a modified borehole model while the thick line represents the output of an existing borehole model.

4. CONCLUSION

This study presents a technique for modifying a borehole model to develop a borehole computer experiment using orthogonal array-based Latin hypercube design (OALHD) with 49 runs and 8 input variables. Both the standard and a modified model are used with specified ranges of values for the borehole variables. The two models were used to compare their output which is the flow rate of water and see how close or accurate the output from a modified borehole model is relative to the output of the standard model. The modified model has performed well in developing a borehole computer experiment following the results obtained in this study which are close to those of the standard model. This study concludes that a modified borehole model can be used to develop a borehole computer experiment.

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