ANALYTICAL SOLUTION OF ONE-DIMENSIONAL BURGERS' EQUATION WITH THE HELP OF MAHGOUB TRANSFORM METHOD

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ABSTRACT: The objectives of this article are to show the effectiveness of the mixture of Mahgoub Transform and Homotopy perturbation method in delivering the exact solution of one-dimensional Burger's equation and furthermore to give another technique for solving Burger's equation. A few issues of one-dimensional Burger's equations were considered and used to show the pertinence and unwavering quality of the proposed technique which is combination of Mahgoub transform and Homotopy perturbation method. The analytical solutions gotten with the accessible Laplace decomposition method and Aboodh Homotopy perturbation technique were examined made and the solutions were seen as in great concurrence with one another. The outcomes acquired demonstrated that this method is effective and profoundly encouraging for such sort of challenges. In view of the illustrative models considered, we suggest that this method is increasingly compelling, helpful, efficient and of high precision with lesser computational work.

KEYWORDS: Mahgoub transform, Homotopy, Homotopy Perturbation Method, Burgers equation, nonlinear, partial differential equation.

1. INTRODUCTION

The nearness of Burger's equation was first recognized by Bateman, see ([Bat15]) who gave it steady courses of action. It was as such reviewed by Burgers ([Bur48]). Much the same as a numerical model for unevenness which showed in fields, for instance, acoustics which can be found in ([Sug91]), steady stochastic procedure in ([W+00]), heat conduction in ([BC69]), dispersive water in ([Whi11]), and disturbance in ([BK07]).

Burgers equation is essentially the interlacing of non-linear wave motion with linear scattering. It is also a model which examines the continued with effect of non-linear move in climate conditions and scattering. The responsibilities of nonlinearity thought are so magnificent in the field of sciences, humanistic systems, pharmaceutical sciences, clinical sciences, and so forth. As an issue of such, it mixes the eagerness of various authorities towards gaining a couple of logical and numerical strategies for dealing with non-linear problems. Various systems, have been strategized by a couple of experts in dealing with a couple of issues including both linear and non-linear partial differential equations; some of which are Backlund transformation procedure in ([RS82]), Hirota's bilinear methodology in ([Hir71]), Variational Iteration Method in ([He00b]), Adomian Decomposition Method in ([Ado88]), Approximate Analytic Method in ([He98]), Homotopy Analysis Method in ([Lia92, Lia99]), Homotopy Analysis strategy in ([Lia04]), Series arrangement method in ([Lia06]), Homotopy Perturbation Method (HPM) in ([He00a, He03, He04, He05]).

A couple of systems have been used to clarify Burger's equation in the continuous events, some of which are; Aboodh Homotopy Pertubation Method in ([Olu19]), explicit and exact finite difference strategies in ([KBÖ99]), (G'/G, 1/G) – extension strategy which was used to develop distinct wave answer for the non-linear Burger's and Banjamin-Bona-Mahony equations in ([HAS18]), Laplace Decomposition Method in ([ZA13]). However, it was seen that a run of the mill technique used in the past systems was linearization of the nonlinear terms which can be so tricky/problematic when points of view, for instance, irreversibility are being considered. Thusly, in this article, the mix of Mahgoub transform and Homotopy perturbation method was used to handle Burger's condition. This course of action was possible without linearizing the nonlinear points of view and therefore Mahgoub transform is seen as continuously fitting appeared differently in relation to the past methods. Mahgoub Transform is another irreplaceable transform gotten from the old-style Fourier integral and was exhibited by Mohand Mahgoub in ([Mah16]). It was utilized to give solutions for partial differential equations in ([MA17]) and too was utilized to understand linear ordinary differential equations with variable co-efficient in ([A+18b]), linear integral Volterra equations in ([ASC18a]). It was additionally applied in treating population growth and decay issues in ([A+18a]), used to give answer for linear Volterra integro-differential equations of second kind in ([ASC18c]), was likewise utilized in illuminating linear Volterra integral equations of first kind in ([ASC18b]) and was also considered in ([KS18]).

2. MAHGOUB HOMOTOPY PERTURBATION METHOD

The essential thought of this technique can be shown by considering the general nonlinear homogenous partial differential equation with initial conditions of the structure ([ABN14]),

$$DU(x,t) + RU(x,t) + NU(x,t) = g(x,t), \qquad (1)$$

and

$$U(x,0) = h(x). \qquad (2)$$

Where D is the second order linear differential operator $D = \frac{\partial^2}{\partial t^2}$, R is the linear differential operator of less order than D, N represents the general nonlinear differential operator and $g(x,t)$ is the source term.
Taking the Mahgoub Transform of both sides of equation (1), we have

$$M[DU(x,t) + RU(x,t) + NU(x,t)] = M[g(x,t)], \qquad (3)$$

$$vH(x,v) - vU(x,0) = M[g(x,t)] - RU(x,t) - NU(x,t). \qquad (4)$$

Substituting the initial condition in equation (2) in equation (4) yields,

$$vH(x,v) = vh(x) + M[g(x,t)] - RU(x,t) - NU(x,t),$$

and

$$H(x,v) = h(x) + \frac{1}{v}[M[g(x,t)] - RU(x,t) - NU(x,t)]. \qquad (5)$$

Taking the inverse mahgoub transform of equation (5), we get,

$$M^{-1}[H(x,v)] = M^{-1}[h(x)] + M^{-1}\left[\frac{1}{v}[M[g(x,t)] - RU(x,t) - NU(x,t)]\right], \qquad (6)$$

Where

$$U(x,t) = h(x) + M^{-1}\left[\frac{1}{v}[M[g(x,t)] - RU(x,t) - NU(x,t)]\right]. \qquad (7)$$

Now applying the Homotopy perturbation method in equation (7), we let

$$U(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t). \qquad (8)$$

Putting equation (8) into equation (7), we have;

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = h(x) + M^{-1}\left\{\frac{M}{v}\left[\sum_{n=0}^{\infty} p^n U_n(x,t)_{xx}\right] - \frac{M}{v}[U_n U_n(x,t)_x]\right\}. \qquad (9)$$

Computing the co-efficient of the corresponding powers of p in equation (9), we have

$$p^0: U_0(x,t) = h(x),$$

$$p^1: U_1(x,t) = M^{-1}\left\{\frac{M}{v}[U_0(x,t)_{xx}] - \frac{M}{v}[U_0(x,t)U_0(x,t)_x]\right\},$$

$$p^2: U_2(x,t) = M^{-1}\left\{\frac{M}{v}[U_1(x,t)_{xx}] - \frac{M}{v}[U_1(x,t)U_0(x,t)_x + U_0(x,t)U_1(x,t)_x]\right\},$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{M}{v}[U_2(x,t)_{xx}] - \frac{M}{v}[U_2(x,t)U_0(x,t)_x + U_1(x,t)U_1(x,t)_x + U_0(x,t)U_2(x,t)_x]\right\}$$

and

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}[U_3(x,t)_{xx}] - \frac{M}{v}[U_3(x,t)U_0(x,t)_x + U_2(x,t)U_1(x,t)_x + U_1(x,t)U_2(x,t)_x + U_0(x,t)U_3(x,t)_x]\right\}.$$

$$\vdots$$

Then, the final solution can be written in closed form as

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + \cdots, \qquad (10)$$

3. APPLICATION

To diagram the sufficiency of the proposed system named Mahgoub Homotopy perturbation method in unwinding Burger's equation, we will watch three particular models.

3.1a Example 1

Consider the one-dimensional Burgers' equation as in ([ABN14]),

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x}. \qquad (11)$$

Subject to the initial condition

$$u(x,0) = 1 - \frac{2}{x}. \qquad (12)$$

3.1b Solution

Taking the Mahgoub transform of both sides of equation (11), we have

$$M\left[\frac{\partial u}{\partial t}\right] = M\left[\frac{\partial^2 u}{\partial x^2}\right] - M\left[u\frac{\partial u}{\partial x}\right],$$

and

$$vH(x,v) - vU(x,0) = M\left[\frac{\partial^2 u}{\partial x^2}\right] - M\left[u\frac{\partial u}{\partial x}\right]. \qquad (13)$$

Substituting the initial conditions of equation (12) into (13), we have

$$vH(x,v) = v\left(1 - \frac{2}{x}\right) + M\left[\frac{\partial^2 u}{\partial x^2}\right] - M\left[u\frac{\partial u}{\partial x}\right],$$

and

$$H(x,v) = \left(1 - \frac{2}{x}\right) + \frac{1}{v}\left\{M\left[\frac{\partial^2 U}{\partial x^2}\right] - M\left[u\frac{\partial u}{\partial x}\right]\right\}. \qquad (14)$$

Taking the inverse Mahgoub transform of equation (14), we obtain

$$M^{-1}[H(x,v)] = M^{-1}\left[\left(1 - \frac{2}{x}\right)\right]$$
$$+M^{-1}\left[\frac{1}{v}\left\{M\left[\frac{\partial^2 U}{\partial x^2}\right] - M\left[u\frac{\partial u}{\partial x}\right]\right\}\right], \qquad (15)$$

and

$$U(x,t) = \left(1 - \frac{2}{x}\right) + M^{-1}\left[\frac{1}{v}\left\{M\left[\frac{\partial^2 U}{\partial x^2}\right] - M\left[u\frac{\partial u}{\partial x}\right]\right\}\right]. \qquad (16)$$

Now applying Homotopy perturbation method in equation (16), we let

$$U(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t), \qquad (17)$$

Substitute equation (17) into equation (16) to obtain

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = \left(1 - \frac{2}{x}\right) + M^{-1}\left\{\frac{M}{v}\left[\sum_{n=0}^{\infty} p^n U_n(x,t)_{xx}\right] - \frac{M}{v}\left[U_n U_n(x,t)_x\right]\right\}. \qquad (18)$$

Computing the co-efficient of the corresponding powers of p in equation (8), we have

$$p^0: U_0(x,t) = 1 - \frac{2}{x},$$

$$p^1: U_1(x,t) = M^{-1}\left\{\frac{M}{v}\left[U_0(x,t)_{xx}\right] - \frac{M}{v}\left[U_0(x,t)U_0(x,t)_x\right]\right\}$$

$$p^1: U_1(x,t) = M^{-1}\left\{\frac{M}{v}\left[-\frac{4}{x^3}\right] - \frac{M}{v}\left[\left(1 - \frac{2}{x}\right)\left(\frac{2}{x^3}\right)\right]\right\},$$

$$p^1: U_1(x,t) = M^{-1}\left\{\left[-\frac{4}{vx^3}\right] - \frac{1}{v}.\frac{2}{x^3}\left(1 - \frac{2}{x}\right)\right\}$$
$$= -\frac{4t}{x^3} - \frac{2t}{x^2}\left(1 - \frac{2}{x}\right)$$
$$= -\frac{4t}{x^3} - \frac{2t}{x^2} + \frac{4t}{x^3} = -\frac{2t}{x^2},$$

$$p^2: U_2(x,t) = M^{-1}\left\{\frac{M}{v}\left[U_1(x,t)_{xx}\right] - \frac{M}{v}\left[U_1(x,t)U_0(x,t)_x + U_0(x,t)U_1(x,t)_x\right]\right\},$$

$$p^2: U_2(x,t) = M^{-1}\left\{\frac{M}{v}\left[-\frac{12t}{x^4}\right] - \frac{M}{v}\left[-\frac{2t}{x^2}.\frac{2}{x^2} + \left(1 - \frac{2}{x}\right)\frac{4t}{x^3}\right]\right\}$$

$$p^2: U_2(x,t) = M^{-1}\left\{\frac{M}{v}\left[\frac{-12t}{vx^4}M[t]\right] + \frac{4}{vx^4}M[t] - \frac{4}{vx^3}\left(1 - \frac{2}{x}\right)M[t]\right\}$$

$$p^2: U_2(x,t) = M^{-1}\left\{\left[-\frac{12}{vx^4}.\frac{1}{v} + \frac{4}{vx^4}.\frac{1}{v} - \frac{4}{vx^3}.\frac{1}{v} + \frac{8}{vx^4}.\frac{1}{v}\right]\right\}$$

$$p^2: U_2(x,t) = M^{-1}\left\{\left[\frac{-12}{v^2x^4} + \frac{4}{v^2x^4} - \frac{4}{v^2x^3} + \frac{8}{v^2x^4}\right]\right\}$$

$$p^2: U_2(x,t) = M^{-1}\left\{\frac{-4}{v^2x^3}\right\} = \frac{-4}{x^3}M^{-1}\left\{\frac{1}{v^2}\right\}$$

$$p^2: U_2(x,t) = -\frac{4}{x^3}.\frac{t^2}{2}$$

$$p^2: U_2(x,t) = -\frac{2t^2}{x^3},$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{M}{v}[U_2(x,t)_{xx}] - \frac{M}{v}[U_2(x,t)U_0(x,t)_x + U_1(x,t)U_1(x,t)_x + U_0(x,t)U_2(x,t)_x]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{M}{v}\left[\frac{-24t^2}{x^5}\right] - \frac{M}{v}\left[-\frac{2t^2}{x^3}\left(\frac{2}{x^2}\right) + \left(-\frac{2t}{x^2}\right)\left(\frac{4t}{x^3}\right) + \left(1-\frac{2}{x}\right)\frac{6t^2}{x^4}\right]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{M}{v}\left[\frac{-24t}{vx^5}M[t^2]\right] + \frac{4}{vx^5}M[t^2] + \frac{8}{vx^5}M[t^2] - \frac{6}{vx^4}\left(1-\frac{2}{x}\right)M[t]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\left[\frac{-24}{vx^5}.\frac{2}{v^2} + \frac{4}{vx^5}.\frac{2}{v^2} - \frac{8}{vx^5}.\frac{2}{v^2} - \frac{6}{vx^4}\left(1-\frac{2}{x}\right)\frac{2}{v^2}\right]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\left[\frac{-48}{v^3x^5} + \frac{8}{v^3x^5} + \frac{16}{v^3x^5} + \frac{-12}{v^3x^4} + \frac{24}{v^3x^5}\right]\right\} = M^{-1}\left\{\frac{-12}{v^3x^4}\right\}$$

$$p^3: U_3(x,t) = \frac{-12}{x^4}M^{-1}\left\{\frac{1}{v^3}\right\} = -\frac{12}{x^4}\left[\frac{t^2}{6}\right] = \frac{-2t^3}{x^4},$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}[U_3(x,t)_{xx}] - \frac{M}{v}[U_3(x,t)U_0(x,t)_x + U_2(x,t)U_1(x,t)_x + U_1(x,t)U_2(x,t)_x + U_0(x,t)U_3(x,t)_x]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}\left[\frac{-40t^3}{x^6}\right] - \frac{M}{v}\left[\frac{-2t^3}{x^4}\left(\frac{2}{x^2}\right) + \left(-\frac{2t^2}{x^3}\right)\left(\frac{4t}{x^3}\right) + \left(1-\frac{2}{x}\right)\left(\frac{6t^2}{x^4}\right) + \left(1-\frac{2}{x}\right)\frac{8t^3}{x^5}\right]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}\left[\frac{-40}{vx^6}M[t^3]\right] + \frac{4}{vx^6}M[t^3] + \frac{8}{vx^6}M[t^3] + \frac{12}{vx^6}M[t^3] - \frac{8}{vx^5}\left(1-\frac{2}{x}\right)M[t^2]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\left[\frac{-40}{vx^6}.\frac{6}{v^3} + \frac{4}{vx^6}.\frac{6}{v^3} + \frac{8}{vx^6}.\frac{6}{v^3} + \frac{12}{vx^6}.\frac{6}{V^3} - \frac{8}{vx^5}\left(1-\frac{2}{x}\right)\frac{6}{v^3}\right]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\left[\frac{-240}{v^4x^6} + \frac{24}{v^4x^6} + \frac{48}{v^4x^6} + \frac{72}{v^4x^6} - \frac{48}{v^4x^5} + \frac{96}{v^4x^6}\right]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{-48}{v^4x^5}\right\}$$

$$p^4: U_4(x,t) = \frac{-48}{x^5}M^{-1}\left\{\frac{1}{v^4}\right\}$$

$$p^4: U_4(x,t) = -\frac{48}{x^5}\left[\frac{t^4}{24}\right]$$

$$p^4: U_4(x,t) = \frac{-2t^4}{x^5}.$$

Then, the solution $U(x,t)$ is expressed as

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + \cdots,$$

Where

$$U(x,t) = \left(1-\frac{2}{x}\right) + \left(-\frac{2}{x^2}t\right) + \left(-\frac{2}{x^3}t^2\right) + \left(-\frac{2}{x^4}t^3\right) + \left(\frac{-2}{x^5}t^4\right) + \cdots,$$

$$U(x,t) = 1 - \frac{2}{x} - \frac{2}{x^2}t - \frac{2}{x^3}t^2 - \frac{2}{x^4}t^3 - \frac{2}{x^5}t^4 + \cdots.$$

Thus, the solution can be written in the closed form as

$$U(x,t) = 1 - \left(\frac{2}{x-t}\right). \quad (19)$$

Equation (19) gives the exact solution for equation (10) which totally conforms with the result in ([ABN14]).

3.2a Example 2

Consider the one-dimensional Burger's equation of the form ([ABN14])

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}-U\frac{\partial u}{\partial x}, \qquad (20)$$

subject to the initial condition

$$U(x,0)=x. \qquad (21)$$

3.2b Solution

Applying Mahgoub Transform to both sides of equation (20), will give

$$M\left[\frac{\partial u}{\partial t}\right]=M\left[\frac{\partial^2 u}{\partial x^2}\right]-M\left[U\frac{\partial u}{\partial x}\right],$$

and

$$vH(x,v)-vU(x,0)=M\left[\frac{\partial^2 u}{\partial x^2}\right]-M\left[U\frac{\partial u}{\partial x}\right]. \qquad (22)$$

Substituting the initial condition in equation (21) into equation (22), we obtain

$$vH(x,v)-v(x)=M\left[\frac{\partial^2 u}{\partial x^2}\right]-M\left[U\frac{\partial u}{\partial x}\right],$$

$$vH(x,v)=v(x)+M\left[\frac{\partial^2 u}{\partial x^2}\right]-M\left[U\frac{\partial u}{\partial x}\right]$$

and

$$H(x,v)=x+\frac{1}{v}\left[M\left[\frac{\partial^2 u}{\partial x^2}\right]-M\left[U\frac{\partial u}{\partial x}\right]\right]. \qquad (23)$$

Taking the Mahgoub inverse of both sides of equation (23), we get

$$M^{-1}\left[vH(x,v)\right]=M^{-1}(x)+M^{-1}\left\{\frac{1}{v}\left[M\left[\frac{\partial^2 u}{\partial x^2}\right]-M\left[U\frac{\partial u}{\partial x}\right]\right]\right\},$$

and

$$U(x,t)=x+M^{-1}\left\{\frac{1}{v}\left[M\left[\frac{\partial^2 u}{\partial x^2}\right]-M\left[U\frac{\partial u}{\partial x}\right]\right]\right\}. \qquad (24)$$

Now, applying Homotopy Perturbation method, let

$$U(x,t)=\textstyle\sum_{n=0}^{\infty}P^nU_n(x,t). \qquad (25)$$

Putting equation (25) into equation (24), we have

$$\textstyle\sum_{n=0}^{\infty}P^nU_n(x,t)=x+M^{-1}\left[\frac{M}{v}\left[\frac{\partial^2 u}{\partial x^2}\right]-\frac{M}{v}\left[U\frac{\partial u}{\partial x}\right]\right]. \qquad (26)$$

Comparing the co-efficient of the corresponding powers of p in equation (26), we have

$$p^0: U_0(x,t)=x,$$

$$p^1: U_1(x,t)=M^{-1}\left\{\frac{M}{v}\left[U_0(x,t)_{xx}\right]-\frac{M}{v}\left[U_0(x,t)U_0(x,t)_x\right]\right\}$$

$$p^1: U_1(x,t)=M^{-1}\left\{\frac{M}{v}[0]-\frac{M}{v}[x.1]\right\}$$

$$p^1: U_1(x,t)=M^{-1}\left\{-\frac{x}{v}.1\right\}$$

$$p^1: U_1(x,t)=xM^{-1}\left\{\frac{-1}{v}\right\}$$

$$p^1: U_1(x,t)=-xt,$$

$$p^2: U_2(x,t)=M^{-1}\left\{\frac{M}{v}\left[U_1(x,t)_{xx}\right]-\frac{M}{v}\left[U_1(x,t)U_0(x,t)_x+U_0(x,t)U_1(x,t)_x\right]\right\}$$

$$p^2: U_2(x,t)=M^{-1}\left\{\frac{M}{v}[0]-\frac{M}{v}\left[(-xt)(1)+(x)(-t)\right]\right\}$$

$$p^2: U_2(x,t)=M^{-1}\left\{\frac{x}{v}M(t)+\frac{x}{v}M(t)\right\}$$

$$p^2: U_2(x,t)=M^{-1}\left\{\left(\frac{x}{v}\right)\left(\frac{1}{v}\right)+\left(\frac{x}{v}\right)\left(\frac{1}{v}\right)\right\}$$

$$p^2: U_2(x,t)=M^{-1}\left\{\frac{x}{v^2}+\frac{x}{v^2}\right\}$$

$$p^2: U_2(x,t)=M^{-1}\left\{\frac{2x}{v^2}\right\}$$

$$p^2: U_2(x,t)=2x\left[M^{-1}\left\{\frac{1}{v^2}\right\}\right]$$

$$p^2: U_2(x,t)=(2x)\frac{t^2}{2}$$

$$p^2: U_2(x,t)=xt^2,$$

$$p^3: U_3(x,t)=M^{-1}\left\{\frac{M}{v}\left[U_2(x,t)_{xx}\right]-\frac{M}{v}\left[U_2(x,t)U_0(x,t)_x+U_1(x,t)U_1(x,t)_x+U_0(x,t)U_2(x,t)_x\right]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{M}{v}[0] - \frac{M}{v}[(xt^2.1) + (-xt)(-t) + (x)(t^2)]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{-x}{v}M[t^2] - \frac{x}{v}M[t^2] - \frac{x}{v}M[t^2]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{-x}{v}M[t^2] - \frac{x}{v}M[t^2] - \frac{x}{v}M[t^2]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\left(\frac{-x}{v^2}\right)\left(\frac{2}{v^2}\right) - \left(\frac{x}{v}\right)\left(\frac{2}{v^2}\right) - \left(\frac{x}{v}\right)\left(\frac{2}{v^2}\right)\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{-2x}{v^3} - \frac{2x}{v^3} - \frac{2x}{v^3}\right\}$$

$$p^3: U_3(x,t) = -2x\, M^{-1}\left\{\frac{1}{v^3}\right\} - 2x\, M^{-1}\left\{\frac{1}{v^3}\right\} - 2x\, M^{-1}\left\{\frac{1}{v^2}\right\}$$

$$p^3: U_3(x,t) = -\frac{6xt^3}{6}$$

$$p^3: U_3(x,t) = -xt^3,$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}[U_3(x,t)_{xx}] - \frac{M}{v}[U_3(x,t)U_0(x,t)_x + U_2(x,t)U_1(x,t)_x + U_1(x,t)U_2(x,t)_x + U_0(x,t)U_3(x,t)_x]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}[0] - \frac{M}{v}[(-xt^3)(1) + (xt^2)(-t) + (-xt)(t^2) + (x)(-t^3)]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{x}{v}M[t^3] - \frac{x}{v}M[t^3] - \frac{x}{v}M[t^3] + \frac{x}{v}M[t^3]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{x}{v}.\frac{6}{v^3} + \frac{x}{v}.\frac{6}{v^3} + \frac{x}{v}.\frac{6}{v^3} + \frac{x}{v}.\frac{6}{v^3}\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{x}{v}.\frac{6}{v^3} + \frac{x}{v}.\frac{6}{v^3} + \frac{x}{v}.\frac{6}{v^3} + \frac{x}{v}.\frac{6}{v^3}\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{6x + 6x + 6x + 6x}{v^2}\right\}$$

$$p^4: U_4(x,t) = 24xM^{-1}\left\{\frac{1}{v^4}\right\} = \frac{24x.t^4}{4!} = \frac{24xt}{24}$$

$$p^4: U_4(x,t) = xt^4.$$

Then, the solution $U(x,t)$ is expressed as

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + \cdots,$$

$$U(x,t) = x + (-xt) + xt^2 + (-xt^3) + xt^4 + \cdots,$$

$$U(x,t) = x[1 - t + t^2 - t^3 - t^4 + \cdots].$$

Thus, in the close form, the solution is given as

$$U(x,t) = \frac{x}{1+t}. \tag{27}$$

Equation (27) gives the exact solution of equation (20) which can be verified from [ABN14].

3.3a Example 3

Consider the one-dimensional Burger's equation of the form ([BG09])

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}, \tag{28}$$

subject to the initial condition

$$U(x,0) = 2x. \tag{29}$$

3.3b Solution

Taking the Mahgoub Transform of both sides of equation (28), we have

$$M\left[\frac{\partial u}{\partial t}\right] + M\left[U\frac{\partial u}{\partial x}\right] = M\left[\frac{\partial^2 u}{\partial x^2}\right],$$

and

$$vH(x,v) - vU(x,0) + M\left[U\frac{\partial u}{\partial x}\right] = M\left[\frac{\partial^2 u}{\partial x^2}\right],$$

$$vH(x,v) - vU(x,0) = M\left[\frac{\partial^2 u}{\partial x^2}\right] - M\left[U\frac{\partial u}{\partial x}\right].$$

Applying the initial condition

$U(x,0) = 2x,$

we have

$$vH(x,v) - v(2x) = M\left[\frac{\partial^2 u}{\partial x^2}\right] - M\left[U\frac{\partial u}{\partial x}\right],$$

$$vH(x,v) - 2xv = M[U_{xx}] - M[U\,U_x],$$

$$vH(x,v) = 2xv + M[U_{xx}] - M[U\,U_x],$$

and

$$H(x,v) = 2x + \tfrac{1}{v}\{M[U_{xx}] - M[U\,U_x]\}. \qquad (30)$$

Taking the inverse Mahgoub Transform of both sides of equation (30), we have

$$M^{-1}[H(x,v)] = M^{-1}(2x) + M^{-1}\left\{\frac{1}{v}[M[U_{xx}] - M[UU_x]]\right\},$$

$$U(x,t) = 2xM^{-1}[1] + M^{-1}\left\{\frac{1}{V}[M[U_{xx}] - M[UU_x]]\right\},$$

$$U(x,t) = 2x + M^{-1}\left\{\tfrac{1}{v}[M[U_{xx}] - M[UU_x]]\right\}. \quad (31)$$

Now applying Homotopy perturbation method, we let

$$U(x,t) = \textstyle\sum_{n=0}^{\infty} p^n U_n(x,t). \qquad (32)$$

Putting equation (32) into equation (31), we have

$$\textstyle\sum_{n=0}^{\infty} P^n U_n(x,t) = 2x + M^{-1}\left[\frac{M}{v}[\sum_{n=0}^{\infty} P^n U_n(x,t)_{xx}] - \frac{M}{v}[U_n U_n(x,t)_x]\right]. \qquad (33)$$

Comparing the co-efficient of the corresponding powers of p in equation (33), we have

$$p^0: U_0(x,t) = 2x,$$

$$p^1: U_1(x,t) = M^{-1}\left\{\frac{M}{v}[U_0(x,t)_{xx}] - \frac{M}{v}[U_0(x,t)U_0(x,t)_x]\right\}$$

$$p^1: U_1(x,t) = M^{-1}\left\{\frac{M}{v}[0] - \frac{M}{v}[2x.2]\right\}$$

$$p^1: U_1(x,t) = M^{-1}\left\{\frac{-4x}{v}.M[1]\right\} = M^{-1}\left[\frac{-4x}{v}\right]$$

$$p^1: U_1(x,t) = -4xM^{-1}\left\{\frac{1}{v}\right\}$$

$$p^1: U_1(x,t) = -4xt,$$

$$p^2:\ U_2(x,t) = M^{-1}\left\{\frac{M}{v}[U_1(x,t)_{xx}] - \frac{M}{v}[U_1(x,t)U_0(x,t)_x + U_0(x,t)U_1(x,t)_x]\right\}$$

$$p^2:\ U_2(x,t) = M^{-1}\left\{\frac{M}{v}[0] - \frac{M}{v}[(-4xt)(2) + (2x)(-4t)]\right\}$$

$$p^2:\ U_2(x,t) = M^{-1}\left\{\frac{8x}{v}M(t) + \frac{8x}{v}M(t)\right\}$$

$$p^2:\ U_2(x,t) = M^{-1}\left\{\frac{8x}{v}.\frac{1}{v} + \frac{8x}{v}.\frac{1}{v}\right\}$$

$$p^2:\ U_2(x,t) = M^{-1}\left\{\frac{8x}{v^2} + \frac{8x}{v^2}\right\} = M^{-1}\left\{\frac{16x}{v^2}\right\}$$

$$p^2:\ U_2(x,t) = 16x\left[M^{-1}\left\{\frac{1}{v^2}\right\}\right] = 16x.\frac{t^2}{2!} = \frac{16xt^2}{2}$$

$$p^2:\ U_2(x,t) = 8xt^2,$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{M}{v}[U_2(x,t)_{xx}] - \frac{M}{v}[U_2(x,t)U_0(x,t)_x + U_1(x,t)U_1(x,t)_x + U_0(x,t)U_2(x,t)_x]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{M}{v}[0] - \frac{M}{v}[(8xt^2)(2) + (-4xt)(-4t) + (2x)(8t^2)]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{-16x}{v}M[t^2] - \frac{16x}{v}M[t^2] - \frac{16x}{v}M[t^2]\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{-16x}{v^2}.\frac{2}{v^2} - \frac{16x}{v}.\frac{2}{v^2} - \frac{16x}{v}.\frac{2}{v^2}\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{-16x}{v^2}\cdot\frac{2}{v^2} - \frac{16x}{v}\cdot\frac{2}{v^2} - \frac{16x}{v}\cdot\frac{2}{v^2}\right\}$$

$$p^3: U_3(x,t) = M^{-1}\left\{\frac{-6x.16x}{v^3}\right\} = -96x\, M^{-1}\left\{\frac{1}{v^3}\right\}$$

$$p^3: U_3(x,t) = -96x.\frac{t^3}{3!}$$

$$p^3: U_3(x,t) = 16xt^3,$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}[U_3(x,t)_{xx}] - \frac{M}{v}[U_3(x,t)U_0(x,t)_x + U_2(x,t)U_1(x,t)_x + U_1(x,t)U_2(x,t)_x + U_0(x,t)U_3(x,t)_x]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{M}{v}[0] - \frac{M}{v}[(-16xt^3)(2) + (8xt^2)(-4t) + (-4xt)(8t^2) + (2x)(-16t^3)]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{32x}{v}M[t^3] + \frac{32x}{v}M[t^3] + \frac{32x}{v}M[t^3] + \frac{32x}{v}M[t^3]\right\}$$

$$p^4: U_4(x,t) = M^{-1}\left\{\frac{32x}{v}.4\right\}M[t^3] = \frac{128x}{v}M^{-1}\left\{\frac{3!}{v^3}\right\}$$

$$p^4: U_4(x,t) = 768xM^{-1}\left\{\frac{1}{v^4}\right\} = \frac{768x.t^4}{4!} = \frac{768x}{24}t^4$$

$$p^4: U_4(x,t) = 32xt^4,$$

$$p^5: U_5(x,t) = M^{-1}\left\{\frac{M}{V}[U_4(x,t)_{xx}] - \frac{M}{V}[U_4(x,t)U_0(x,t)_x + U_3(x,t)U_1(x,t)_x + U_2(x,t)U_2(x,t)_x + U_1(x,t)U_3(x,t)_x + U_0(x,t)U_4(x,t)_x]\right\}$$

$$p^5: U_5(x,t) = M^{-1}\left\{\frac{M}{v}[0] - \frac{M}{v}[(32xt^4)(2) + (-16xt^3)(-4xt) + (8xt)(8t^2) + (-4xt)(-16t^3) + (2x)(32t^4)]\right\}$$

$$p^5: U_5(x,t) = M^{-1}\left\{\frac{-64x}{v}M[t^4] - \frac{64x}{v}M[t^4] - \frac{64x}{v}M[t^4] - \frac{64x}{v}M[t^4] - \frac{64x}{v}M[t^4]\right\}$$

$$p^5: U_5(x,t) = M^{-1}\left\{\frac{-320x}{v}\right\}M[t^4]$$

$$p^5: U_5(x,t) = M^{-1}\left\{\frac{-320x}{v}.\frac{24}{v^4}\right\} = (-7680x)M^{-1}\left\{\frac{1}{v^5}\right\}$$

$$p^5: U_5(x,t) = -7680.\frac{t^4}{120}$$

$$p^5: U_5(x,t) = -64xt^5.$$

Then, the solution $U(x,t)$ is expressed as

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + U_5(x,t) \ldots,$$

$$U(x,t) = 2x - 4xt + 8xt^2 - 16xt^3 + 32xt^4 - 64xt^5 + \cdots,$$

$$U(x,t) = 2x[1 - 2xt + 4xt^2 - 8xt^3 + 16xt^4 - 32xt^5 + \cdots].$$

Thus, the solution can be written in closed form as:

$$U(x,t) = \frac{2x}{1+2t}. \qquad (34)$$

Meanwhile, equation (34) shows perfectly the result obtained in [BG09].

4. CONCLUSIONS

In this article, the essential focus was to show the genuine nature of the mix of Mahgoub transform and Homotopy perturbation method in disentangling Burger's equations and to show another procedure for getting solutions for Burgers' equation. The tendency of this procedure over other existing methods is finding the solutions of Burger's equations without enormous computational work and without linearizing the nonlinear terms. From

the illustrative models considered up until this point, the results obtained appeared to us that the blend of Mahgoub transform and Homotopy perturbation method is suitable, genuine, and strong enough to offer symptomatic responses for non-linear issues and can likewise be used gainfully to secure the exact solutions in an immediately restricted gathering, with effectively figured terms.
It is simple but compact. Hence, we recommend the significance of this technique to one another Burger's equation problems which may have not surface in this investigating article.

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