SIMULATION OF THE EFFECT OF DISEASE TRANSMISSION COEFFICIENT ON THE SUSCEPTIBLE-EXPOSED-INFECTED-RECOVERED (SEIR) EPIDEMIC MODEL USING VARIATIONAL ITERATION METHOD

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ABSTRACT: In the research work, (VIM) is used to solve the system of linear differential equations to investigate the effect of transmission coefficient on the model with a permanent immunity.
KEYWORDS: Permanent immunity, Incidence rate, Transmission Coefficient.

1. INTRODUCTION

The existing model as proposed by ([OKP17, KO16a]) is modified by making δ and d equal zero to have a permanent immunity on the model i.e. anybody cured is no longer moving to the susceptible class again, hence SEIR epidemic model. Individual in the model class can be categorized into subclasses of susceptible s(t), exposed E(t), Infected I(t), recovered R(t) as shown in the transmission diagram below in Figure 1. The Analysis of a SEIRS epidemic model with a saturated incidence rate, considering the initial state of the disease was studied by ([KO16b]). Different model comparing SIR, SEIRS epidemic model was studied by ([KN12, AWG13, PKO16]).
In ([KO16c]), the numerical simulation was carried out using variational iteration method which showed the effect of the tested parameter and its significant effect on the model.

2. THE BASIC MATHEMATICAL MODEL

In this paper, SEIRS model by Olayiwola *et al* ([OKP17]) was adopted and modified making δ (rate of loosing immunity) and d (disease induced death)=0.

Existing model (SEIRS) ([OKP17])

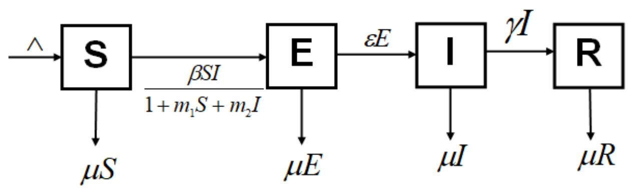
$$\left.\begin{aligned} \frac{dS}{dt} &= N - \frac{\beta SI}{1+m_1S+m_2I} - \mu S + \delta R \\ \frac{dE}{dt} &= \frac{\beta SI}{1+m_1S+m_2I} - (\mu+\varepsilon)E \\ \frac{dI}{dt} &= \varepsilon E - (\mu+\gamma+d)I \\ \frac{dR}{dt} &= \gamma I - (\mu+\delta)R \end{aligned}\right\} \quad (1)$$

When $\delta = 0$ and $d = 0$ gives the proposed model in (2).

Modified Model (SEIR)

$$\left.\begin{aligned} \frac{dS}{dt} &= \Lambda - \frac{\beta SI}{1+m_1S+m_2I} - \mu S \\ \frac{dE}{dt} &= \frac{\beta SI}{1+m_1S+m_2I} - (\mu+\varepsilon)E \\ \frac{dI}{dt} &= \varepsilon E - (\mu+\gamma)I \\ \frac{dR}{dt} &= \gamma I - \mu R \end{aligned}\right\} \quad (2)$$

Here the parameters are the same as used in [1]. This is shown in the transmission diagram below:



3. MODEL SIMULATION (SEIR) EPIDEMIC MODEL

The effect of disease transmission coefficient in the susceptible individual in SEIR epidemic model will be studied using Variational Iteration Method,
Let consider the differential equation with L and N as linear and non-linear operators respectively with $g(\tau)$ being homogeneous term. With VIM, let,

$$L(u) + N(u) = g(\tau) \quad (3)$$

Equation (1) can be constructed as:

$$U_{n+1}(s) = U_n(s) + \int_0^s \lambda[LU_n(\tau) + N\bar{u}_n(\tau) - g(\tau)]d\tau \quad (4)$$

Using (4) in (1), to identify the general Lagrange multiplier λ by variational calculus where $\bar{u}_n(\tau)$ is known as the restricted variation i.e. $\delta\bar{u}_n(\tau) = 0$.

$$S_{n+1}(t) = S_n(t) + \int_0^1 \lambda_1(\tau)\left[\frac{d\bar{S}_n(\tau)}{dt} - N + \frac{\beta\bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)} - \mu\bar{S}_n(\tau)\right]d\tau$$

$$E_{n+1}(t) = E_n(t) + \int_0^1 \lambda_2(\tau)\left[\frac{d\bar{E}_n(\tau)}{dt} - \frac{\beta\bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)} + (\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$
$$I_{n+1}(t) = I_n(t) + \int_0^1 \lambda_3(\tau)\left[\frac{d\bar{I}_n(\tau)}{dt} - \xi\bar{E}_n(\tau) + (\mu+\gamma)\bar{I}_n(\tau)\right]d\tau$$
$$R_{n+1}(t) = R_n(t) + \int_0^1 \lambda_4(\tau)\left[\frac{d\bar{R}_n(\tau)}{dt} - \gamma\bar{I}_n(\tau) + (\mu)\bar{R}_n(\tau)\right]d\tau \quad (5)$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are general Lagrange Multiplier, $\bar{S}_n, \bar{E}_n, \bar{I}_n$ and $\bar{R}_n$ denote restricted variation i.e.

$$\delta\bar{S}_n = \delta\bar{E}_n = \delta\bar{I}_n = \delta\bar{R}_n = 0$$

The stationary values that corresponds to (5) are:

$$\delta S_{n+1}(t) = \delta S_n(t) + \delta\int_0^1 \lambda_1(\tau)\left[\frac{d\bar{S}_n(\tau)}{dt} - N + \frac{\beta\bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)} - \mu\bar{S}_n(\tau)\right]d\tau$$
$$\delta E_{n+1}(t) = \delta E_n(t) + \delta\int_0^1 \lambda_2(\tau)\left[\frac{d\bar{E}_n(\tau)}{dt} - \frac{\beta\bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)} + (\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$
$$\delta I_{n+1}(t) = \delta I_n(t) + \delta\int_0^1 \lambda_3(\tau)\left[\frac{d\bar{I}_n(\tau)}{dt} - \xi\bar{E}_n(\tau) + (\mu+\gamma)\bar{I}_n(\tau)\right]d\tau$$
$$\delta R_{n+1}(t) = \delta R_n(t) + \delta\int_0^1 \lambda_4(\tau)\left[\frac{d\bar{R}_n(\tau)}{dt} - \gamma\bar{I}_n(\tau) + (\mu)\bar{R}_n(\tau)\right]d\tau \quad (6)$$

Equation (6) gives $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -1$. With $\lambda_i, i = 1,\dots,4$ we obtained the following iterative scheme:,

$$S_{n+1}(t) = S_n(t) - \int_0^1 \left[\frac{d\bar{S}_n(\tau)}{dt} - N + \frac{\beta\bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)} - \mu\bar{S}_n(\tau)\right]d\tau$$
$$E_{n+1}(t) = E_n(t) - \int_0^1 \left[\frac{d\bar{E}_n(\tau)}{dt} - \frac{\beta\bar{S}_n(\tau)\bar{I}_n(\tau)}{1+m_1\bar{S}_n(\tau)} + (\mu+\xi)\bar{E}_n(\tau)\right]d\tau$$
$$I_{n+1}(t) = I_n(t) - \int_0^1 \left[\frac{d\bar{I}_n(\tau)}{dt} - \xi\bar{E}_n(\tau) + (\mu+\gamma)\bar{I}_n(\tau)\right]d\tau$$
$$R_{n+1}(t) = R_n(t) - \int_0^1 \left[\frac{d\bar{R}_n(\tau)}{dt} - \gamma\bar{I}_n(\tau) + (\mu)\bar{R}_n(\tau)\right]d\tau \quad (7)$$

For numerical results, we used the following parameters:

$$S_0(t) = 15;\ I_0(t) = 10;\ E_0(t) = 13;\ R_0(t) = 11;\ \varepsilon = 0.25;\ \mu = 0.3;\ N = 49;\ \gamma = 0.1;\ m_1 = 0.1;\ m_2 = 0.2 \quad (8)$$

When $n = 4$, the following results can be obtained by maple 18.

$$\begin{aligned} S_1(t) &= 15 + 44.35t - 150\beta t \\ E_1(t) &= 13 - 8.85t - 150\beta t \\ I_1(t) &= 10 - 0.75t \\ R_1(t) &= 11 - 2.85t \end{aligned} \quad (9)$$

$$\begin{aligned} S_2(t) &= 15 + 44.35t - 150\beta t + 11.0875\beta t^3 - 37.5\beta^2 t^2 - 186.125\beta t^2 + 750\beta^2 t^2 - 8.93375t^2 \\ E_2(t) &= 13 - 8.85t - 150\beta t + 11.0875\beta t^3 - 37.5\beta^2 t^3 - 167.375\beta t^2 + 0.22375t^2 \\ I_2(t) &= 10 - 0.75t + 11.0875\beta t^3 - 0.95625t^2 - 18.75\beta^2 t^2 \\ R_2(t) &= 11 - 2.85t + 0.46125t^2 \end{aligned} \quad (10)$$

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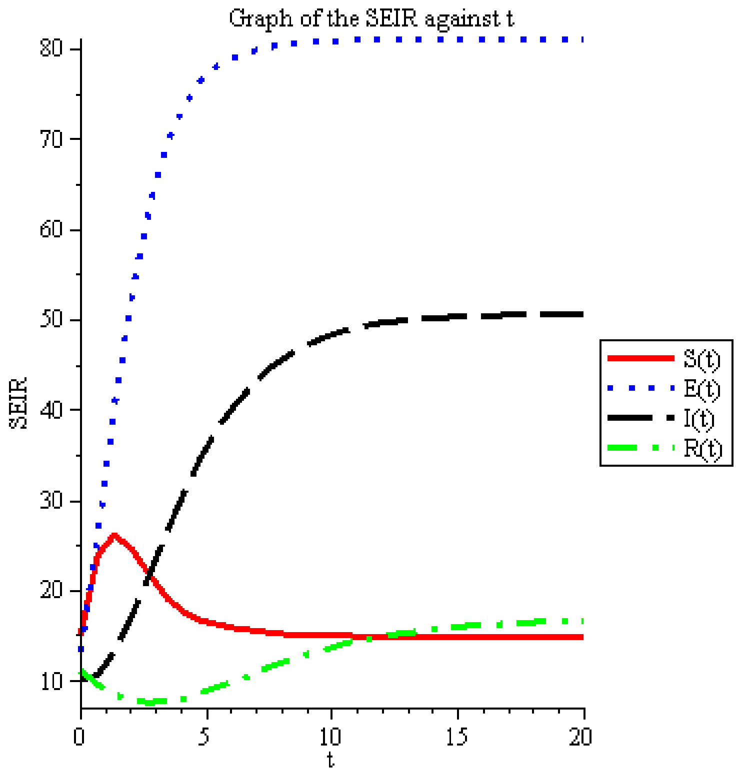

Figure 1: Graph of SEIR against t when $\beta = 1$

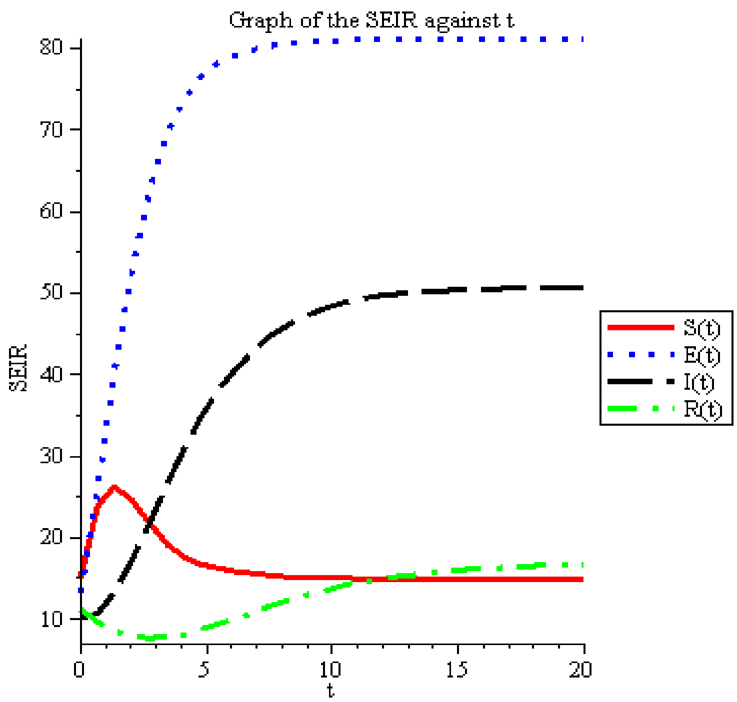

Figure 2: Graph of SEIR against t when $\beta = 0.75$

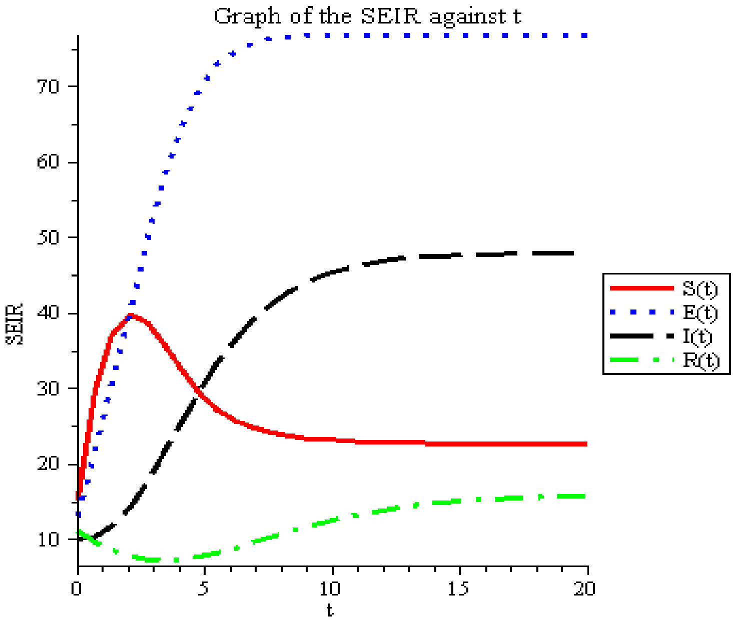

Figure 3: Graph of SEIR against t when $\beta = 0.5$

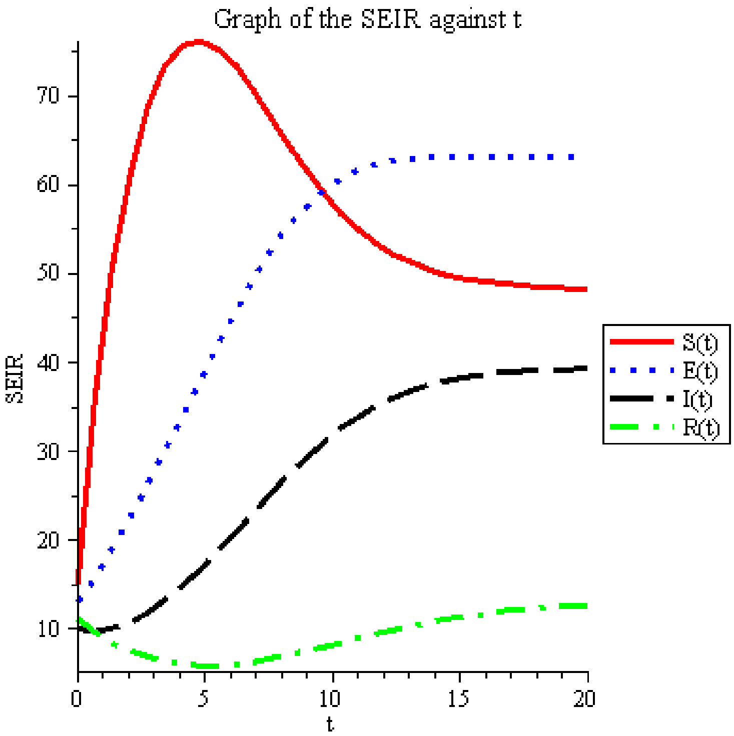

Figure 4: Graph of SEIR against t when $\beta = 0.25$

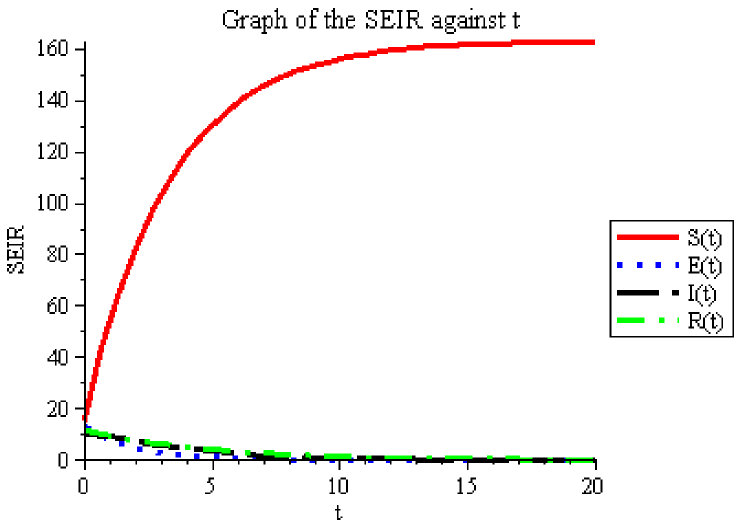

Figure 5: Graph of SEIR against t when $\beta = 0.0005$

4. DISCUSSION OF RESULTS

Figure 1 reveals the simulation result when $\beta = 1$, the exposed class increases drastically with infected class, while susceptible and recovered classes were at minimum.
Figure 2 reveals the simulation results when $\beta = 0.75$. It was observed that exposed class is also on increment also with infected class.
Figures 2-5 reveals a drastically changes in susceptible class when $\beta = 0.5$, $\beta = 0.25$ and $\beta = 0.005$. That is, the lower the β, the better the stability of the disease free equilibrium.
It was therefore observed that, in the presence of a permanent immunity β plays a vital role in disease eradication.

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