

## BAYESIAN ESTIMATION OF DYNAMIC ERROR COMPONENT MODEL WITH MOVING AVERAGE DISTURBANCES

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**ABSTRACT:** This paper explores the estimation of Dynamic Panel Data Error Component Model with Autocorrelated disturbance of Moving Average of order one MA(1) using Bayesian approach; the Bayesian approach produced a very efficient estimator that was not plagued by moment conditions as in other popular Generalized method of moments (GMM) estimators. The Markov Chain Monte Carlo (MCMC) with Gibbs sampler algorithm was used to carry out the analysis as shown in the empirical illustrations. The estimation was done at varying degrees of Autocorrelation coefficients that is from mild to severe. The result showed that the Bayesian Estimator performed excellently as the Numerical Standard Error (NSE) decreased with increasing sample sizes.

**KEYWORDS:** Dynamic Panel Data, Error Component, Autocorrelated Disturbance, Moving Average, Markov Chain Monte Carlo (MCMC), Gibbs Sampler.

### 1. INTRODUCTION

The process of observing repeated choices or outcomes from the same Economic units over several time periods is called Panel Data. Many economic relationships are dynamic in nature and one of the advantages of panel data is that they allow researcher to better understand the dynamics of adjustment ([6]). Due to the several benefits of panel data, increasing fraction of empirical microeconomic and macroeconomic research is done on the basis of panel data in recent years. The importance of panel data has by now been well established in such diverse areas of empirical research as household-level demand and labor supply decisions, workers' wage processes, firm-level productivity, or cross-country determinants of economic growth.

Nickell ([13]) explored the idea that estimating the dynamic panel data Model by OLS will produce biased and inconsistent estimates because a right hand regressor is correlated with the error term while Anderson and Hsiao ([3]) suggested the instrumental variable (IV) estimation this involves first differencing the model to get rid of the individual specific effects and then using the resulting model as instruments, Arellano and Bond

([2]) proposed an optimal Generalized method of Moment (GMM) estimator arguing that additional instruments can be obtained in a Dynamic Error component model by utilizing the orthogonality conditions existing between the lagged values of the dependent variable and the disturbances. The Arellano-Bond estimator exhibits substantial downward bias when the coefficient on the lagged dependent variable is close to unity, as then the dependent variable follows a random walk and lagged levels correlate poorly with lagged differences, thus creating a weak instrument problem. Also, Ahn and Schmidt ([5]), Blundell and Bond ([7]), Hahn ([9]) solves this problem by imposing additional nonlinear moment restrictions on the dependent variable process and exploiting the resulting moment conditions; however, these restrictions may not hold in practice. Hahn, Hausman and Kuersteiner ([10]) like Griliches and Hausman ([8]) took long differences of the data to enhance the correlation between levels and differences; however, this procedure does not make use of all the available data. Hence the estimation of dynamic panel data models is still an open challenge.

All these estimators hinges on the strict condition of no serial correlation in the disturbance term which is not practicable in Dynamic panel Data Model. But, besides the general problem to obtain consistent estimates in the context of dynamic panel data models, a large time series dimension often comes along with specific time series issues like serial correlation or non-stationarity. In order to obtain consistent estimates, serial correlation especially is a severe problem in dynamic panel data models. Serial correlation remains a severe problem that leads to inconsistent estimates, this paper has been able to derive Bayesian Estimator which is consistent in the presence of Autocorrelation.

Bayesian probability is one of the major theoretical and practical frameworks for reasoning and decision making under uncertainty. Bayesian analysis is done by generating a posterior Distribution which is proportional to the product of the likelihood and

Prior Distributions. The major difficulty that arises most often than not is that the joint posterior distribution does not take the form of any well-known density that is the conditionals of the posterior cannot be solved analytically but only through a posterior simulation method. In this work we employed the use of Gibbs sampler, one of the Markov Chain Monte Carlo algorithms.

The outline of the paper is as follows: section 2 presents the methodology and the derivation of the Bayesian Estimator. Section 3 contains the Simulation Study while Section 4 has the Discussions of the Findings and Finally the Conclusion of the work is in Section 5.

## 2. METHODOLOGY

### The Model:

The model is based on a balanced design with T observations for individual unit. Thus, the observation in the panel can be represented in the

$$(y_{it}, x_{it}), i = 1, 2, \dots, N; t = 1, 2, \dots, T;$$

where the index  $i$  denotes the  $i^{th}$  individual unit, and the index  $t$  denotes the  $t^{th}$  time period.

### The dynamic Panel Error Component Model:

The dynamic error component model considered is:

$$y_{it} = \delta y_{i,t-1} + \beta_i x_{it} + \epsilon_{it} \quad \epsilon \sim (0, h^{-1}I) \quad (1)$$

$y_{it}$  is the dependent variable for the  $i^{th}$  individual at the  $t^{th}$  time period,  $\delta$  is the coefficient of the lagged dependent variable,  $x_{it}$  is the exogenous unit specific regressor,  $\epsilon_{it}$  is the idiosyncratic error term. Equation (1) can be written more compactly as:

$$y_{it} = X_{it} \theta_{it} + \epsilon_{it} \quad (2)$$

$$\text{where } X_{it} = (x_{it}, y_{i,t-1}), \theta_{it} = (\delta, \beta_i)^I$$

We assume the error term  $\epsilon_{it}$  follows a one-way error component model  $\epsilon_{it} = u_i + v_{it}$  where  $u_i$  denotes unobserved time-invariant homogeneity,  $v_{it}$ , the idiosyncratic error component. Some of the typical assumptions are:

- Stationarity:  $|\delta| < 1$ ,
- Disturbance term has mean zero:  $E(\epsilon_{it} | y_{i,0}, \dots, y_{i,t-1}, u_i) = 0$ ,
- No serial correlation:  $\epsilon_{it} \sim iid(0, \sigma_i^2)$ ,
- Homoscedasticity:  $u_i \sim iid(0, \sigma_u^2)$ .

Violating the assumption of independence of the error term, we introduce serial correlation of moving average of order one MA(1) into the equation. The

moving average process of order 1 is denoted MA(1) and defined by:

$$X_t = v_t + \theta v_{t-1} \quad (3)$$

However, this gives;

$$v_t = X_t - \theta v_{t-1} \quad (4)$$

Also,

$$v_{t-1} = X_{t-1} - \theta v_{t-2} \quad (5)$$

Equation (3) becomes:

$$v_t = X_t - \theta(X_{t-1} - \theta v_{t-2}) \quad (6)$$

$$v_t = X_t - \theta X_{t-1} + \theta^2 v_{t-2} \quad (7)$$

Successively substituting in expressions for  $X_{t-s}$  in this manner, we have eqn (7) written as:

$$v_t = X_t - \theta X_{t-1} - \theta^2 X_{t-2} - \dots - \theta^{s-1} X_{t-(s-1)} + \theta^s v_{t-s} \quad (8)$$

This is only valid for  $|\theta| < 1$  a so-called, **invertible process**. No two invertible processes have the same autocorrelation function. Note that  $v_t$  is a sequence of independent (or uncorrelated) random variables with mean 0 and variance  $\sigma^2$ . These results can be used to write the covariance matrix of  $\Omega$  as

$$\Omega = \frac{1}{1+\theta^2} \begin{bmatrix} 1 & -\theta & -\theta^2 & \dots & \theta^{s-1} \\ -\theta & 1 & -\theta & \dots & \theta^{s-2} \\ -\theta^2 & -\theta & 1 & \dots & -\theta^2 \\ \theta^{s-1} & -\theta^2 & -\theta & \dots & 1 \end{bmatrix}$$

### The Model with $\Omega$ :

Given a model,

$$y^* = X^* \theta + \epsilon^* \quad (9)$$

where  $y^* = Py, X^* = PX$  and  $\epsilon^* = P\epsilon$ . It can be verified that  $\epsilon$  is  $N(0_N, h^{-1}|_N)$ . Hence, the transformed model given in eqn (9) is identical to the normal linear regression model. Since  $\Omega$  is a positive definite matrix, it follows that there exist an  $N \times N$  matrix  $P$  such that  $P \Omega P^I = |_N$

### The Likelihood Function:

Using the properties of the Multivariate Normal distribution, the likelihood function can be written as;

$$P(y|\theta, h, \Omega) = p(y|\theta, h, \Omega) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} |\Omega|^{-\frac{1}{2}} \left\{ \exp \left[ -\frac{h}{2} (y - X\theta)' \Omega^{-1} (y - X\theta) \right] \right\} \quad (10)$$

or in terms of the transformed data,

$$(y^*|\theta, h, \Omega) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{ \exp \left[ -\frac{h}{2} (y^* - X^*\theta)' (y^* - X^*\theta) \right] \right\} \quad (11)$$

Using GLS, we derive:

$$p(y|\theta, h, \Omega) = \frac{1}{(2\pi)^{\frac{N}{2}}} \left\{ h^{\frac{1}{2}} \exp \left[ -\frac{h}{2} (\theta - \hat{\theta}(\Omega))' X' \Omega^{-1} X (\theta - \hat{\Omega}(\Omega)) \right] \times h^{\frac{v}{2}} \exp \left[ \frac{hv}{2s(\Omega)^{-2}} \right] \right\} \quad (12)$$

Eqn (12) gives the likelihood of the model, where  $h = \frac{1}{\sigma^2}$  is the error precision and  $v = N - k$ .

### The Prior:

A Bayesian analysis of the Dynamic panel data model set up above requires a specification of the prior distributions over the parameters. The non informative prior used in this study is an independent Normal- Gamma prior for  $\theta$  and  $h$ , and we use the general notation  $P(\Omega)$ , to indicate the prior for  $\Omega$ . In other words, the prior used for this study is:

$$p(\theta, h, \Omega) = P(\theta)P(h)P(\Omega)$$

Where,

$$P(\theta) = \frac{1}{(2\pi)^{\frac{1}{2}}} |\underline{V}|^{-\frac{1}{2}} \left\{ \exp \left[ -\frac{1}{2} (\theta - \underline{\theta})' \underline{V}^{-1} (\theta - \underline{\theta}) \right] \right\} \quad (13)$$

$$P(h) = \left\{ h^{\frac{v}{2}} \exp \left[ \frac{hv}{2(\underline{s})^{-2}} \right] \right\} \quad (14)$$

$$P(\Omega) = \frac{1}{(2\pi)^{\frac{1}{2}}} |\underline{V}|^{-\frac{1}{2}} \left\{ \exp \left[ -\frac{1}{2} (\rho - \underline{\rho})' \underline{V}^{-1} (\rho - \underline{\rho}) \right] \right\} \quad (15)$$

it is deduced that:  $E[\beta^*|y^*] = \beta^*$  is the prior mean of  $\theta$  and  $Var(\beta^*|h^*) = V^*$  is the prior covariance matrix of  $\theta$  with which the mean of  $h^*$  is  $s^{-2}$  and  $v^*$  degree of freedom.

### The Posterior:

The Posterior is proportional to the product of the prior and the likelihood. Mathematically, it is usually denoted by  $p(\theta, h, \Omega|y)$ .

$$p(\theta, h, \Omega|y) \propto p(y|\theta, h, \Omega) \times p(\Omega) \times p(\theta) \times p(h)$$

Multiplying equations (12), (13), (14) and (15), we have;

$$p(\theta, h, \Omega|y) \propto p(\Omega) \times \left\{ \exp \left[ -\frac{1}{2} h (y^* - X^*\theta)' (y^* - X^*\theta) \right] \times (\theta - \underline{\theta})' \underline{V}^{-1} (\theta - \underline{\theta}) \right\} \times h^{\frac{N+v-2}{2}} \exp \left[ -\frac{hv}{2s^{-2}} \right] \quad (16)$$

The joint posterior density for  $\Omega$ ,  $\theta$  and  $h$  does not take the form of any well-known density i.e., the conditionals of the posterior cannot be solved analytically but only through a posterior simulation method.

It can be verified that the posterior of  $\theta$ , conditional on the other parameter of the model is multivariate Normal; by ignoring the terms that do not involve  $\theta$  in (16):

$$\theta|y, h, \Omega \sim N(\bar{\theta}, \bar{V}) \quad (17)$$

Where,  $\bar{V} = (\underline{V}^{-1} + h X' \Omega^{-1} X)^{-1}$  and  $\bar{\theta} = \bar{V} (\underline{V}^{-1} \underline{\theta} + h X' \Omega^{-1} X \hat{\theta}(\Omega))$ .

Also, ignoring terms that do not involve  $h$  in eqn(16), the posterior of  $h$  conditional on the other parameter in the model is Gamma:

$$h|y, \theta, \Omega \sim G(\bar{s}^{-2}, \bar{v}) \quad (18)$$

where,  $\bar{v} = N + \underline{v}$  and  $\bar{s}^2 = \frac{(y-X\theta)' \Omega^{-1} (y-X\theta) + \underline{v} \underline{s}^2}{\bar{v}}$ .

Similarly, by treating Eqn (16) as a function of  $(\Omega)$  and ignoring the terms that do not involve  $(\Omega)$ , we have;

$$p(\Omega|y, \theta, h) \propto f_N(\rho|\bar{\rho}, \bar{V}_\rho) 1 \quad (\rho \in \phi) \quad (19)$$

Where,  $\bar{V}_\rho = (\underline{V}_\rho^{-1} + h E' E)^{-1}$  and  $\bar{\rho} = \bar{V}_\rho (\underline{V}_\rho^{-1} \underline{\rho} + h E' E)$ , note that  $E$  is a  $(T - p) \times K$  matrix with  $t^{th}$  row given by  $(\varepsilon_{t-1}, \dots, \varepsilon_{t-p})$ .

The formulae of equations (17), (18) and (19) look familiar to the conjugate of normal-gamma priors but does not relate directly to the posterior of interest. These conditional posteriors in the equations above do not directly tell us everything about the posterior of interest. We will be using a Markov Chain Monte Carlo Algorithm called the Gibbs sampler to perform posterior simulations.

The Gibbs sampler is a powerful tool for posterior simulation used in many Econometric Models. The Gibbs sampler makes use of the conditional posteriors to produce random draws which can be averaged to obtain the estimate of the posterior  $E[g(\theta)/y]$ .

In Bayesian inference the Gibbs sampler is commonly used because of the desirable results that iterative sampling from the conditional distributions will lead to a sequence of random variables converging to the joint distribution.

### 3. SIMULATION STUDY

#### 3.1. Design of the Monte Carlo Experiment:

Data would be generated using a Monte Carlo simulation procedure from the model:

$$y_{it} = \delta y_{i,t-1} + \beta_i x_{it} + \varepsilon_{it} \quad \varepsilon \sim (0, h^{-1}I)$$

The explanatory variables were drawn from uniform (0, 0.25), the assumed value for the lag parameter ( $\delta$ ) is from normal distribution with mean 0 and variance 0.25  $N(0,0.25)$ . The error term followed an autoregressive process with parameter  $\theta$  at 0.3, 0.5 and 0.8 with all possible combinations of cross-sections (N=10, 50, 100) and time periods (T=5, 10, 20). Each combination was replicated 1000 with the first 100 samples ignored, which is commonly known as burn-in periods. The reason for discarding the first several periods is that it may take a while to reach the stationary distribution of the Markov chain, which is the desired joint distribution.

**Table 1: Posterior Means and Numerical Standard Errors of the Parameters for N=10,50,100;  $\delta \sim N(0,0.25)$ ,  $\beta_i \sim \beta(2,3)$ ,  $\theta = 0.3$**

T=5			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.1414352 (0.00150161)	0.0164455 (0.00165342)	0.9887677 (0.00072411)
$\beta_1$ {Mean NSE	1.9008805 (0.00907502)	2.1145423 (0.00325315)	2.0005433 (0.00297368)
$\beta_2$ {Mean NSE	3.2216544 (0.00733885)	2.5002557 (0.00603246)	2.8036580 (0.00119378)
T=10			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.1421281 (0.00155105)	0.01728419 (0.00185051)	1.1774463 (0.00058747)
$\beta_1$ {Mean NSE	1.9056525 (0.00920651)	2.13597958 (0.00419000)	2.0062053 (0.00273351)
$\beta_2$ {Mean NSE	3.2264262 (0.00753039)	2.51066222 (0.00616843)	3.28050425 (0.00248312)
T=20			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.1424591 (0.00155813)	0.01728419 (0.00185052)	1.1774463 (0.00058747)
$\beta_1$ {Mean NSE	1.9065653 (0.00925996)	2.13597958 (0.00419102)	2.2710425 (0.00248312)
$\beta_2$ {Mean NSE	3.2258338 (0.00758104)	2.51066222 (0.00616856)	2.9062053 (0.00273351)

**Table 2: Posterior Means and Numerical Standard Errors of the Parameters for N=10,50,100;  $\delta \sim N(0,0.25)$ ,  $\beta_i \sim \beta(2,3)$ ,  $\theta = 0.5$**

T=5			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.1084332 (0.00144075)	0.05132242 (0.00170913)	1.7008789 (0.00030463)
$\beta_1$ {Mean NSE	1.7483256 (0.00900413)	1.92766567 (0.00387029)	2.0100030 (0.00281419)
$\beta_2$ {Mean NSE	3.1473208 (0.00724547)	2.61565346 (0.00568972)	3.2082237 (0.00301693)
T=10			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.1085731 (0.00148964)	0.05248499 (0.00170913)	1.04935322 (0.00055743)
$\beta_1$ {Mean NSE	1.7504805 (0.00913690)	1.9323948 (0.00387029)	2.449556 (0.00252711)
$\beta_2$ {Mean NSE	3.22452009 (0.00745140)	2.62725872 (0.00568972)	3.223312 (0.00228948)
T=20			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.1088941 (0.00149614)	0.05248499 (0.00170913)	1.0493530 (0.00055746)
$\beta_1$ {Mean NSE	1.7505009 (0.00918996)	1.93239484 (0.00387029)	2.4495564 (0.00252711)
$\beta_2$ {Mean NSE	3.4447830 (0.00750047)	2.62725872 (0.00568972)	3.2233124 (0.00228948)

**Table 3: Posterior Means and Numerical Standard Errors of the Parameters for N=10,50,100;  $\delta \sim N(0,0.25)$ ,  $\beta_i \sim \beta(2,3)$ ,  $\theta = 0.8$**

T=5			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.0553916 (0.00134927)	0.05176466 (0.00149943)	1.8973758 (0.00045838)
$\beta_1$ {Mean NSE	1.65509501 (0.00856370)	1.83548265 (0.00149943)	2.0200398 (0.00222406)
$\beta_2$ {Mean NSE	3.08106894 (0.00696807)	2.77061621 (0.00433577)	3.2238074 (0.00200097)
T=10			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.05608186 (0.00137302)	0.05176466 (0.00150115)	0.7085615 (0.000510505)
$\beta_1$ {Mean NSE	1.65483295 (0.00873006)	1.83548265 (0.00337097)	2.0460532 (0.00223192)
$\beta_2$ {Mean NSE	3.08211352 (0.00708501)	2.77061621 (0.00499573)	3.0335998 (0.00200438)
T=20			
MA(1)			
	N=10	N=50	N=100
$\delta$ {Mean NSE	0.05638131 (0.00137936)	0.05176466 (0.00150120)	0.7085615 (0.00051051)
$\beta_1$ {Mean NSE	1.65285566 (0.00878069)	1.83548265 (0.00337099)	2.0460532 (0.00223192)
$\beta_2$ {Mean NSE	3.68185480 (0.00713080)	2.77061621 (0.00499577)	3.0335998 (0.00200558)

**Note:** The values indicate: the posterior means of the parameters with the posterior numerical standard error in parenthesis.

#### 4. DISCUSSIONS AND FINDINGS

Table 1 above shows the posterior means and the numerical standard error (NSE) of the parameters when the degree of coefficient of autocorrelation is mild that is  $\theta=0.3$ . We have results for different values of N and T. It can be observed that when T is small (T=5); the Bayesian estimator produced values that are very close to the true values assumed, we can see that the values were closest to the true value at N=100 (i.e. when N is very large compared with T). As the sample size increases, the NSE decreases for the parameters except for the lag parameter that fluctuated when N=50. This implies that the Bayesian estimator is very efficient and consistent. Also, when we have T=10, for varying sample sizes of N=10, 50 & 100.  $\beta_1$  produced the closest value at N=100 while  $\beta_2$  got its closest value at N=10. The Numerical Standard Error (NSE) also steadily reduced as the sample size increased. For T=20 (large T), it shows that the Bayesian Estimator produced values that are very close to the true values, a careful look at the NSE to reveals that the values consistently reduced as the sample size increased.

Table 2 shows the posterior means and Numerical Standard Error (NSE) of the parameters when the degree of coefficient of autocorrelation is moderate that is  $\theta=0.5$ . We have result for varying values of N and T. It can be observed that when T is small (T=5); the Bayesian estimator produced values that are very close to the true values assumed, we can see that the values were closest to the true value at N=100 (i.e. when N is very large compared with T). As the sample size increases, the NSE decreases for the parameters except for the lag parameter that fluctuated when N=50. This implies that the Bayesian estimator is very efficient and consistent. Also, when we have T=10 & 20 for varying sample sizes of N=10, 50 & 100.  $\beta_1$  and  $\beta_2$  produced the closest value at N=100 and the Numerical Standard Error (NSE) decreased steadily as sample size increased.

Table 3 above is a scenario of when the degree of coefficient of autocorrelation is severe that is  $\theta=0.8$ . The results show the posterior means and Numerical Standard Error (NSE) of the parameters for varying values of N and T. The Bayesian Estimator produced estimates that are close to the true values in all the different combinations of N and T. The closest values produced as observed is when the sample size is very large (N=100). This implies that as the sample size (N) increases, the values of the

parameters gets closer to the true values. For the Numerical Standard Error (NSE), across the varying values of the time dimension (T= 5, 10 & 20), as the sample size increased, the NSE steadily reduced.

#### CONCLUSIONS

This paper has proposed a Bayesian approach to analyze Dynamic Error Component model with serial correlation in the disturbance term. We consider the Moving Average process of order one MA(1) and we utilize the Gibbs Sampling method to implement the Bayesian inferences.

In conclusion, the Bayesian approach has been able to produce consistent and efficient estimator for the Dynamic Error Component Model in the presence of serial correlation as can be seen in the results, the posterior estimates produced have values that are very close to the true values and as the sample size increased we observed a consistent decrease in the NSE.

Significantly, we found that for higher degree of the first order Moving Average process we have the lowest values for the Numerical Standard Error (NSE). This implies that the when the degree of autocorrelation is severe;  $\theta= 0.8$ , the Bayesian estimator is most efficient. The Bayesian approach has been able to take care of one of the major limitations that had plagued the Dynamic Panel Model Estimators for a long time.

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