# Multiplier-Accelerator Cycle Model: Shocks and Money

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REZUMAT. Modelele economice cu multiplicator-accelerator care se ocupă de teoria ciclurilor în afaceri au apărut ca urmare a nevoii de teoretizare a ciclurilor economice dinamice. Aceste modele reunesc descrierea empirică a ciclurilor economice dinamice, cu descrierea teoretică a echilibrului, privită din punct de vedere static. Principiul modelelor economice cu multiplicator este următorul: dacă investiţiile cresc, vom avea o creştere a ieşirilor (prin ieşiri putem înţelege atât produsul finit cât şi venitul firmei). Creşterea ieşirilor rezultă din relaţia "multiplicator" care arată echilibrul între ieşiri şi intrări (investiţii).

The principle of the multiplier is that if investment increases, there will be an increase in output as a result of a "multiplier" relationship between equilibrium output and the autonomous components of spending, in this case:

$$\Delta Y = \Delta I/(1-c)$$

where c is the marginal propensity to consume, Y is output and I is investment. The principle of the accelerator was that investment decisions on the part of firms are at least in part dependent upon expectations of future increases in demand, which may, in turn, be extrapolated from any current or past increases in aggregate demand or output, e.g.

$$I_t = \beta(Y_t - Y_{t-1})$$

Thus, the multiplier principle implies that investment increases output whereas the acceleration principle implies that increases in output will themselves induce increases in investment.

John Hicks is credited for trying to reveal proper macroeconomic cycles in a linear multiplier accelerator model by essentially aiming for unstable oscillations and adding floors and ceilings to constrain them. However, Hicks proposed a slightly different model. Instead of having the accelerator related to past consumption changes, he had it related to past output changes, in short he argued that:

$$I_t = I_0 + \beta(Y_{t-1} - Y_{t-2})$$

and, in goods market equilibrium:

$$Y_t = C_t + I_t = c_0 + I_0 + cY_{t-1} + \beta(Y_{t-1} - Y_{t-2})$$

There is two further ways of obtaining cycles with an Hicksian multiplier-accelerator model: one is by non-linear functions, another is by making our simple system "shock-dependent". It is something of the latter nature that Frisch, Slutsky and Kalecki proposed. Namely, given the simplest Hicksian system, we could argue that:

$$I_t = I_0 + \beta(Y_{t-1} - Y_{t-2}) + u_t$$

where $u_t$ is an exogenous shock parameter - reflecting, say, innovation in the production sector. Combining this with the standard MA structure, we obtain:

$$Y_t = c_0 + I_0 + cY_{t-1} + \beta(Y_{t-1} - Y_{t-2}) + u_t$$

or

$$Y_t - (c + \beta)Y_{t-1} + \beta Y_{t-2} = (c_0 + I_0 + u_t)$$

What Frisch-Slutsky demonstrates is that if we have parameter values c and $\beta$ so as to obtain case of damped, not explosive, oscillations, then the impulse provided by the stochastic element will be sufficient to maintain regular cycles provided random shocks are continuously or at least intermittently allowed. If we had allowed explosive oscillations, however, these shocks would not essentially affect the system. Thus, we need damped oscillations for the random shocks to yield an essentially regular cycle.

Frisch-Slutsky type of analysis has been resurrected in New Classical "Real Business Cycle" theory. Effectively, these models employ growth models of the standard Neoclassical variety (e.g. Solow-Swan or Cass-Koopmans) and add Frisch-Slutsky type of stochastic elements to a variety of elements (e.g. production, tastes, spending, etc.) -yielding then continual growth and cycles

Finally, recall that we have so far confined ourselves to real-side "income-expenditure" relationships in the Hicksian model. To expand upon the Keynesian roots of this model, we can add monetary factors to the multiplier-accelerator model so as to allow the LM side to enter the essentially IS multiplier-accelerator model. Essentially, we have, as before:

$$C_t = c_0 + cY_{t-1}$$
$$I_t = I_0 + \alpha r_{t-1} + \beta(Y_{t-1} - Y_{t-2})$$

but now, note, we have allowed past interest ($r_{t-1}$) to enter as a determinant of investment along with autonomous investment and the accelerator. It is assumed, of course, that $\alpha < 0$. For simplicity, we are ignoring the growth rate of autonomous investment. Having this, then, we obtain:

$$Y_t = c_0 + I_0 + cY_{t-1} + \alpha r_{t-1} + \beta(Y_{t-1} - Y_{t-2})$$

or

$$Y_t = c_0 + I_0 + (c + \beta)Y_{t-1} - \beta Y_{t-2} + \alpha r_{t-1}$$

for our second-order linear difference equation for the goods market.

However, this is not sufficient as we do not have anything determining the rate of interest. We shall propose the traditional Keynesian liquidity preference theory for this. Thus, money demand is defined as:

$$L_t = L_0 + \eta Y_t + \sigma r_t$$

with the first component being autonomous money demand, the second as the transactions demand for money and the last as the speculative demand. It is assumed that $\eta > 0$ and $\sigma < 0$ so money demand is positively related to income and negative related to interest.

We shall also propose a money supply rule. Namely:

$$M_t - M_{t-1} = \mu(Y_{t-1} - Y_{t-2})$$

so the change in the money supply is some negative function of past changes in income ($|\mu < 0$). The logic is simple policy: in recessions, governments expand the money supply whereas in booms, they restrict the money supply. Smyth proposes to reduce this to:

$$M_t = M_0 + \mu Y_{t-1}$$

so money supply at time t is some function of past income and some constant $M_0$. Thus, in equilibrium:

$$L_t = M_t$$

or:

$$L_0 + \eta Y_t + \sigma r_t = M_0 + \mu Y_{t-1}$$

so, solving for $r_t$:

$$r_t = [\mu Y_{t-1} + M_0 - L_0 - \eta Y_t] / \sigma$$

Lagging this once:

$$r_t = [\mu Y_{t-2} + M_0 - L_0 - \eta Y_{t-1}] / \sigma$$

which we can place in our original goods market difference equation:

$$Y_t = c_0 + I_0 + (c+\beta)Y_{t-1} - \beta Y_{t-2} + (\alpha/\sigma)[Y_{t-2} + M_0 - L_0 - \eta Y_{t-1}]$$

so, reorganizing:

$$Y_t - (c+\beta-\alpha\eta/\sigma)Y_{t-1} + (\beta-\alpha\mu/\sigma)Y_{t-2} = c_0 + I_0 + (\alpha/\sigma)(M_0 - L_0)$$

which is a second order difference equation. Thus, our particular solution (equilibrium) would be:

$$Y_p = [c_0 + I_0 + (\alpha/\sigma)(M_0 - L_0)]/[\sigma(1-c) + \alpha(\eta-\mu)]$$

whereas our characteristic equation would be:

$$r^2 - (c+\beta-\alpha\eta/\sigma)r + (\beta-\alpha\mu/\sigma) = 0$$

The eigenvalues to be determined are then:

$$r_1, r_2 = [(c+\beta-\alpha\eta/\sigma) \pm \sqrt{(c+\beta-\alpha\eta/\sigma)^2 - 4(\beta-\alpha\mu/\sigma}]/2$$

where roots are real if the discriminant is non-negative i.e. D > 0 or $(c+\beta-\alpha\eta/\sigma)^2 \geq 4(\beta-\alpha\mu/\sigma)$ and complex otherwise. So, for real roots:

$$(c+\beta)^2 + (\alpha\eta/\sigma)^2 - 2(c+\beta)\alpha\eta/\sigma \geq 4(\beta-\alpha\mu/\sigma)$$

As $\beta > 0$ and $\alpha\eta/\sigma < 0$, then the term on the right is positive. Of course, $(c+\beta)^2 > 0$ and $(\alpha\eta/\sigma)^2 > 0$, thus, the term $2(c+\beta)\alpha\eta/\sigma$ is of crucial significance as to whether the inequality holds or not. Namely, the greater the absolute value of $c, \beta, \alpha, \eta$ and the smaller the value of $\sigma$, the greater the possibility of oscillations. Now, by the Schur Criterion, one always obtains damped monotonic dynamics if:

(i) $1 + (c+\beta-\alpha\eta/\sigma) - (\alpha\eta/\sigma - \beta) > 0$

(ii) $1 - (c+\beta-\alpha\eta/\sigma) - (\alpha\eta/\sigma - \beta) > 0$

(iii) $1 + (\alpha\eta/\sigma - \beta) > 0$

Now, the first Schur Criterion (i) can be rewritten as $1 + c - \alpha\eta/\sigma - \alpha\mu/\sigma + 2\beta > 0$ thus given that $\alpha, \mu, \sigma < 0$ and $c, \eta, \beta > 0$, then obviously $\alpha\mu/\sigma > 0$ and $\alpha\eta/\sigma < 0$. Thus, the necessary condition for stability following (i) is that:

$$c > \alpha(\mu+\eta)/\sigma - 2\beta - 1$$

Now, the second part of the Schur criterion (ii) can be rewritten $1 - c + \alpha\eta/\sigma - \alpha\mu/\sigma > 0$ where, as (1-c) > 0 and $\alpha\eta/\sigma > 0$ and $\alpha\mu/\sigma < 0$, then this is obviously fulfilled. Finally, the third part of the Schur criterion (iii) can be rewritten as $\beta < 1 - \alpha\mu/\sigma$ which may or may not be fulfilled. Now, recall that in the original Hicks model, the condition for stability was that $\beta < 1$. Thus, this new condition, that $\beta < 1 - \alpha\mu/\sigma$, makes the region of instability bigger whereas the region of stability is reduced. We should note, heuristically, the the condition that $1 > \beta + c$ implies the IS curve is downward sloping. Whereas if $\beta + c - 1 > \alpha\eta/\sigma$ implies the IS curve is steeper than the LM curve.

The analysis of the consequent MA dynamics follows conventionally - which we shall skip over here and merely refer to Smyth for details. Other types of models which include money, growth and cycles along simple Keynesian relationships include Laidler and "Keynes-Wicksell" monetary growth models.

## Bibliography

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