From "Nolite turbare circulos meos!" to “Don’t delete my folder”

Math. Valerian Antohe, Ph.D.
“D.P.Perpessicius” Highschool, Brăila, România

REZUMAT. Arhimede îsi desena figurile pe nisipul plajei, pe pământ bătut sau în cenusă pusă pe o pardoseală ori pe propriul să corp, uns în prealabil cu untdelemn; pe corp trasa figurile cu ajutorul unghiei. Când generalul roman Marcellus a cucerit în anul 212 î.e.n. Siracusa din Sicilia, orasul lui Arhimede, un soldat roman a dat peste acest geniu contemplându-si cercurile pe care le desenase pe nisip. "Nolite turbare circulos meos!" (nu-mi strica cercurile) i-a strigat Arhimede soldatului; dar romanul, iritat, l-a înjunghiat cu spada, omorându-l. În zilele de azi locul de exprimare a ideilor matematice a fost înlocuit de ecranul calculatorului iar platformele dedicate studiului matematicii au deschis o nouă perspectivă domeniului cercetării. O incursiune în domeniu este desigur o provocare dar şi o invitație pentru educați şi educatori.

1. Dynamic mathematics software

Archimedes drew his figures on beach sand, mud or ash on a floor or put on his body, previously anointed with oil; on his body the figures were drawn with nails. When the Roman general Marcellus conquered in 212 BC Siracusa in Sicily, the city of Archimedes, a Roman soldier came across this genius contemplating his drew circles on the sand. "Nolite turbare circulos meos!" (Do not break my circles) told Archimedes to the soldier, but this, irritated, stabed him with the sword, killing him.

Today the place of the geometric constructions is on dynamic platforms supported by specialized software. One of these platforms is GeoGebra software. As the inventor stated, GeoGebra is dynamic mathematics software for all levels of education that joins arithmetic, geometry, algebra and calculus. It offers multiple representations of objects in its graphics, algebra, and spreadsheet views that are all dynamically linked. While other interactive software (e.g. Cabri Geometry, Geometer's Sketchpad) focus on dynamic manipulations of geometrical objects, the idea behind GeoGebra is to connect geometric, algebraic, and numeric representations in an interactive way. You can do constructions with points, vectors, lines, conic sections as well as functions and change them dynamically afterwards. Furthermore, GeoGebra allows you to directly enter and manipulate equations and coordinates. Thus you can easily plot functions, work with sliders to investigate parameters, find symbolic derivatives, and use powerful commands like Root or Sequence, (www.geogebra.org).

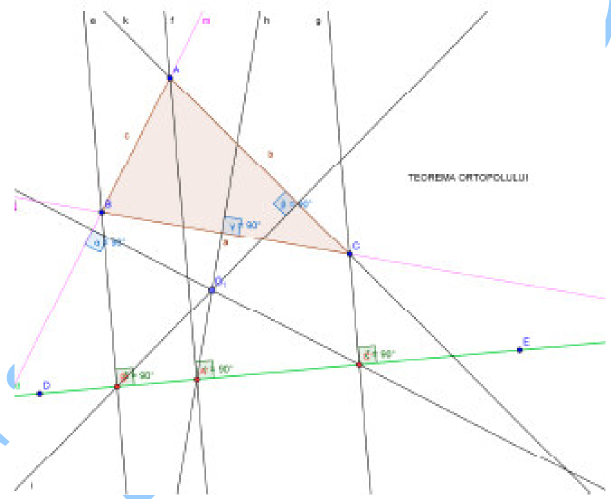

Fig.1. Theorema of ortopol

Because some of us (teachers of different subjects), see the computers in terms of "IT specialist", producers of software and not like a user, we propose this ongoing project in order to bring the computer in the context of approaching successful teaching, [AGO02]. We believe that the teacher, regardless of specialty they teach, shouldn't know what's in the magic box (called generic computer), but he'll have to know that magic box will help him to assist the student to learn, the route to knowledge becoming pleasant! There is a struggle to integrate the computer in school. This approach must be read: "the integration of educational software in education". We thus meet this desire. The computers, along with some educational software applications, have long since proved to be very helpful in teaching mathematics. An "educational software application” is something that you can use on your computer, without having advanced knowledge about computers and programming. Draw, build, unite, investigates properties, change shape and size. Do properties remain true? Why? Can you formulate the theorem from this investigation? But to prove rigorously? The computer may be, after all, an excuse! But the teaching approach can be adapted to other disciplines, too. Experience should not only be lived, but shared.

2. Modeling demonstration of some classical theorem with GeoGebra

Ceva’s theorem: In a triangle ABC, three lines AG, BD and CE intersect at a single point F if and only if EA/EB·GB/GC·DC/DA=-1.

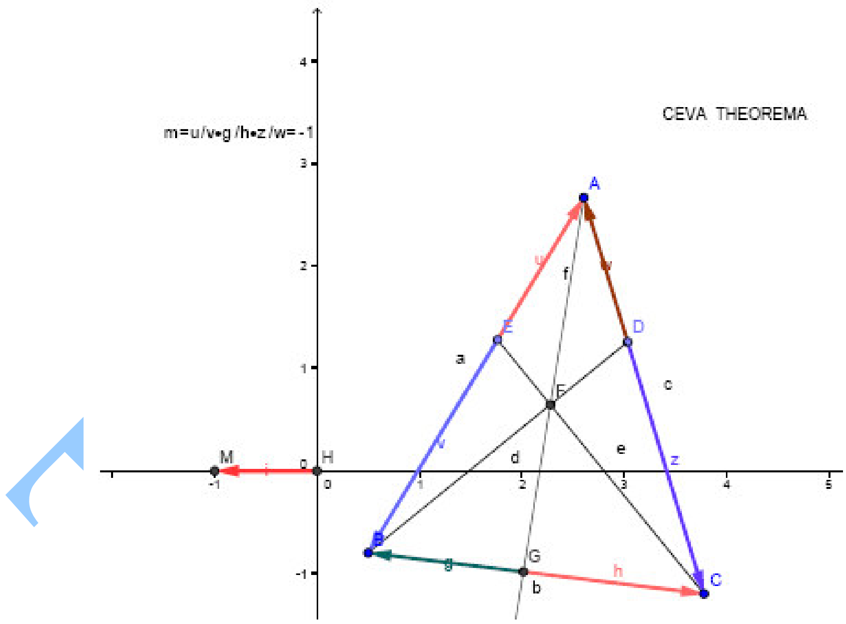

Fig.2. The vector HM represent the value of EA/EB· GB/GC· DC/DA. If the one of the point will be moved HM remain the same

Menelaus's theorem: If a line meets the sides BC, CA, and AB of a triangle in the points D, E, and F then the products of the ratios AE/EC·CD/DB·BF/FA=1, [Nic93].

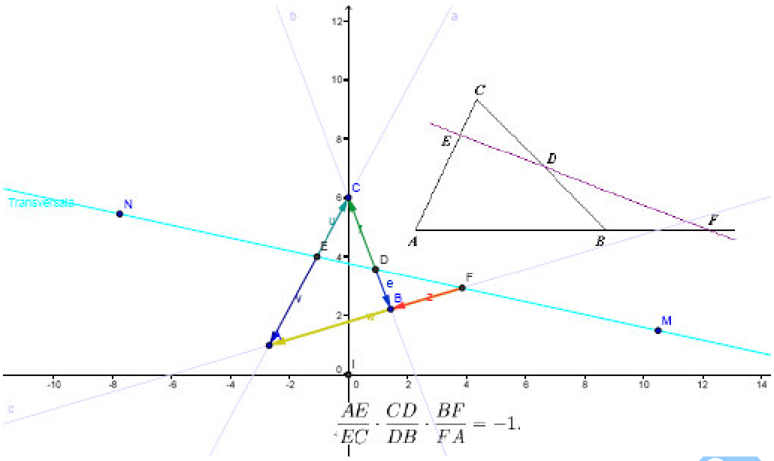

Fig.3. On geogebra applet one can drag the point N or M to move the transversal line (MN).

3. Modeling mathematical demonstration with GeoGebra

3.1. Show that:

$$\frac{4a^2+2a+1}{a}-\frac{12a}{4a^2+2a+1}\geq 4\,,\qquad \forall a>0.$$

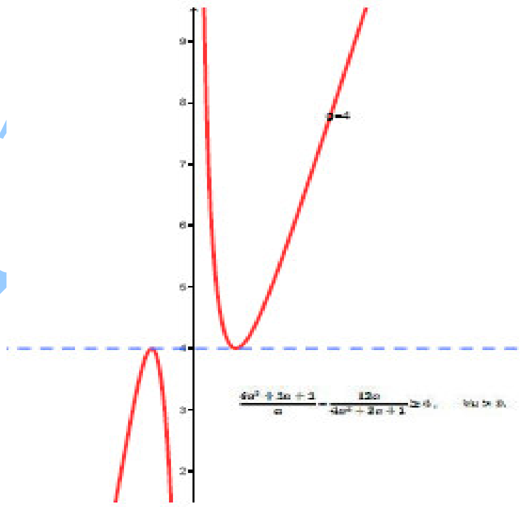

Fig.4.One observe that the graph of function is up to horizontal line of y=ct=4

Geogebra can construct other complex contexts like conic through five points and put together the 2D construction and the analytic equation:

No.	Name	Definition	Algebra
1	Point A		A = (-1.9, 2.88)
2	Point B		B = (-0.8, 4)
3	Point C		C = (2.5, 1.76)
4	Point D		D = (0.44, 1.02)
5	Point E		E = (-1.6, 1.18)
6	Ellipse c	Conic through A, B, C, D, E	c: 48.36x² - 72.3x y + 117.34y² + 109.01x - 639.44y = -505.27

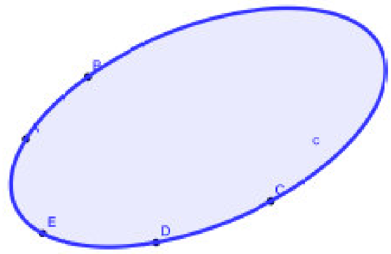

Fig.5. Conic through five points

3.2. Let ABC be a triangle and M be an inner point of the triangle so that AM = BC. Shoe that

$$max\left\{\frac{BM}{AC}\cdot\frac{CM}{AB}\right\} \geq \sqrt{2}-1$$

and this is the best possible constant, [Teo84].

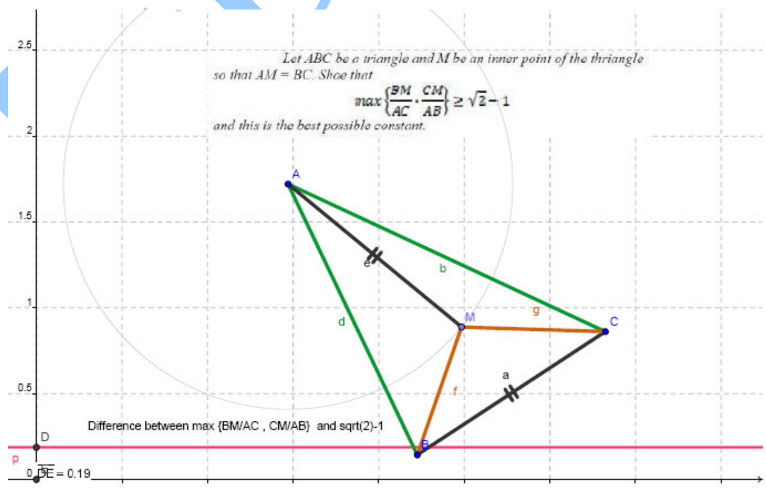

Fig.5. The difference between left member and the right member of the inequality it is represented by the horizontal line of equation y>0

3.3. Let $M \in (AB)$. On the same side of line AB equilateral triangles ACM and BDM are erected. Set $\{P\} = BC \cap DM, \{N\} = AD \cap CM, \{Q\} = AD \cap BC$, AM = a, MB = b and MP = x . Prove that:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{x} \quad and \quad \frac{PQ}{QC} + \frac{NQ}{QD} \geq 1$$

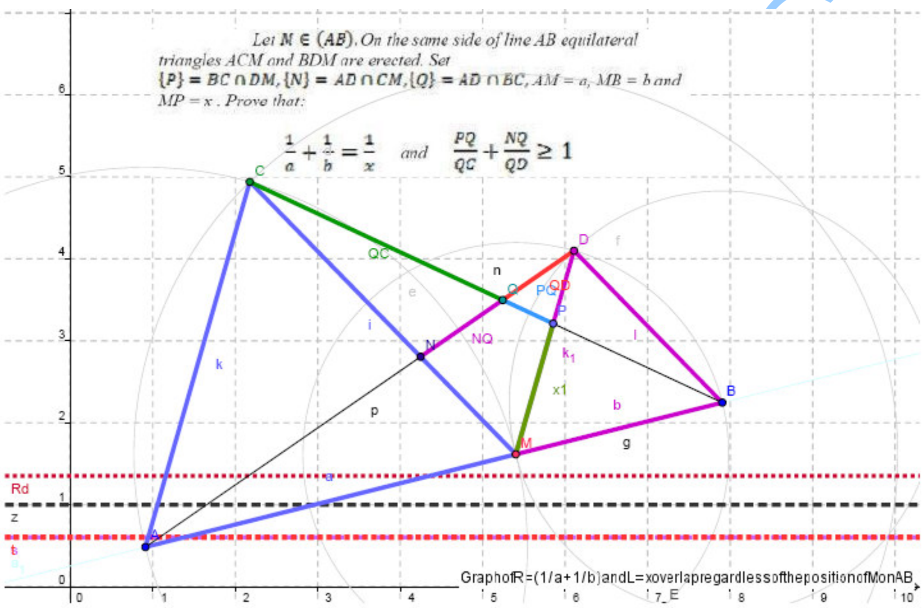

Fig.6. Inequalities are represented an position of horizontal line which represent the values of each members

3.4. Prove that in any acute-angled triangle the following inequality holds:

$$\frac{a+c}{cosB} + \frac{b+a}{cosC} + \frac{c+b}{cosA} \geq 4(a+b+c).$$

After the construction, the relation remains the same if any of the coin of the triangle is moved in plane.

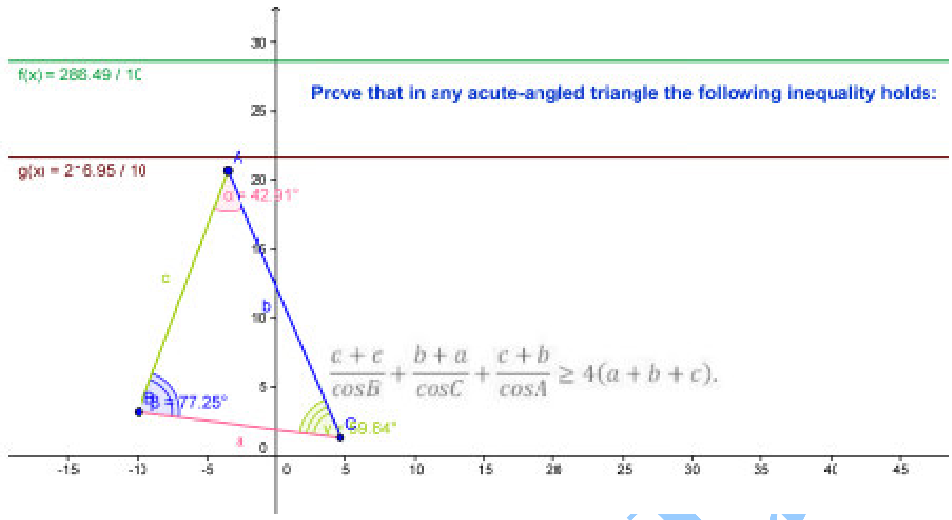

Fig.7. The relation remains the same if any of the coin of the triangle is moved in plane

4. The transition to 3D in modeling with GeoGebra

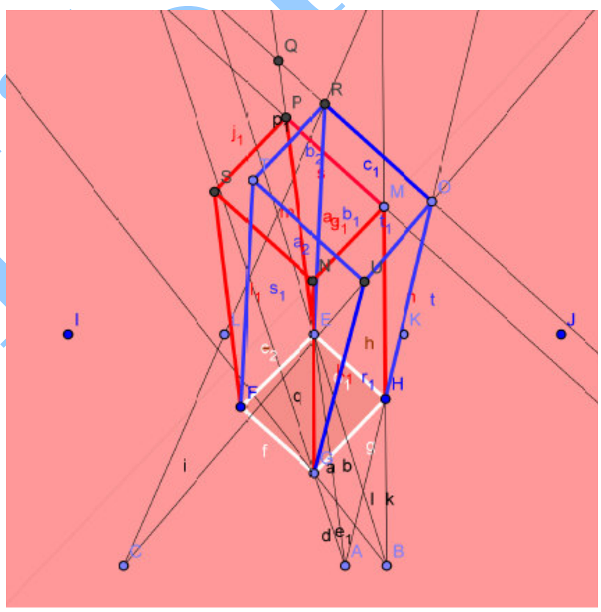

Fig.8. Anaglifa that represent a paralelipiped that must be seen useing glasses with two colours, red and blue

Anaglifa that represent a paralelipiped that must be seen useing glasses with two colours, red and blue, [Rot72]. This investigation in realise 3D construction with GeoGebra software could be involved in some future work for mathematicians and an ideea for developers of the platform.

Conclusions

A step by step construction, which represents the visual interpretation of a mathematical context, follows the next steps: construct geometric figures based on their definitions, apply geometric transformations, understand the relationship between Euclidean construction and proof, and create demonstrations that involve animation and action buttons, find out geometrically and algebraically connections in a rigorous proof, [Cri00].

One appreciate the pedagogical implications of exploring geometry in a dynamic environment, both as an investigation tool and as a demonstration tool, the connection between math educators and specialists in informatics is one of the best and a provocation too.

References

[AGO02] **V. Antohe, C.Gherghinoiu, M. Obeadă,** - *Metodica predării matematicii. Jocul didactic matematic*, 122-125, Brăila, Editura Ex Libris, 2002.

[Cri00] **C. Crişan,** – "*Euclid şi Computerul?" Noi modalităţi de explorare a geometriei euclidiene folosind aplicaţia The Geometer's Sketchpad*, 48-53, Focşani, Editura EMPRO, 2000.

[Nic93] **L. Nicolescu,** – *Metode de rezolvare a problemelor de geometrie*, Bucureşti, Editura Universităţii Bucureşti, 1993.

[Rot72] **G.Rotariu,** – *Anaglife geometrice*, Bucureşti, Editura didactică şi pedagogică, 1972.

[Teo84] **N. Teodorescu,** – *Probleme din Gazeta Matematică*, Ediţie selectivă şi metodologică, Bucureşti, Editura tehnică, 1984.