Modeling a geometric locus problem with GeoGebra

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REZUMAT. În rezolvarea unor probleme de loc geometric analiza, modelarea, studiul cazurilor particulare ce ajută la intuirea soluției poate atrage din partea rezolvitorului un efort de lungă durată. În acest sens, folosirea unor programe specializate pentru modelarea unor fenomene matematice, în genere transformări geometrice în plan, este un instrument acceptat şi folosit din ce în ce mai mult. În cadrul lecțiilor de predare-învățare a matematicii, implementarea investigației desfăşurată pe platforme de geometrie dinamică precum Geogebra poate atrage definirea unor noi metode de predare-învățare "nonstandard", metode ce sunt acceptate ca metode de graniță ce folosesc instrumentul informatic în stransă legătură cu demersul matematic al metodelor clasice. Vom numi acest demers "infogeometria dinamică".

1. Methods of teaching Mathematics

Different methods of teaching mathematics have been proposed by different educators and knowledge of these methods may help in working out a better teaching strategy. It is not appropriate for a teacher to commit to one particular method. A teacher should adopt a teaching approach after considering the nature of the students, their interests and maturity and the resource available, [Teo84]. Every method has certain merits and few demerits and it is the work of the teacher to decide which method is the best for the students, [Muh07]. Some of the methods of teaching Mathematics are as follows:

- Lecture Method;
- Inductive-Deductive Method;

- Heuristic Method (Discovery/Inquiry Method);
- Analitical-Synthetical Method;
- Brain Storming;
- Think-Pair-Share;
- Learning by Doing;
- Problem Solving Approach.

All the above mentioned methods may not be equally appropriate and suitable for all levels of mathematics teaching. The teacher, after knowing about all these methods, their merits, should be able to make his/her own method by imbibing the good qualities of all methods.

The method finally adopted must: ensure maximum participation of students, proceed from concrete to abstraction and provide knowledge at the understanding level, [AGO02].

The investigation above will present an implementation of a mathematical platform GeoGebra, in order to solve a problem of geometry.

A geometrical locus will be analyzed, both in a analytical way and in a applied soft analyze. The results will be put together and this investigation could give a real result in putting together some of methods of math teaching like mentioned methods.

2. Modeling a geometric locus problem with GeoGebra

The statement of the problem:

On the fixed line *d*, is considered the fixed points A, B in the π plane and the mobile point M. In the plane π is built regular polygons of [AM] and [BM] sides, with *m*, respectively *n* number of sides, where

$m,n \in N,\ m,n \geq 3$.

The circumscribed circles to those polygons are intersected i M and P. Is required the geometric locus of P, [Daf03].

For solving this problem we distinguish the cases $m=n$ si $m\neq n$, in each case the polygons can be situated on the same side of the line *d* or on the different sides of this one. We organize the line *d* as an axis, axis of real numbers $x'x$, points A,B with abscissas *a*, *b*, constants, and point M with the variable abscissa *x*, (fig.1). The soft permit us to construct the hypotheses of the problem, construct regular polygons with a specified number of sites, permit to change easy the number of sites of the polygons and construct the geometrical locus of specified point P.

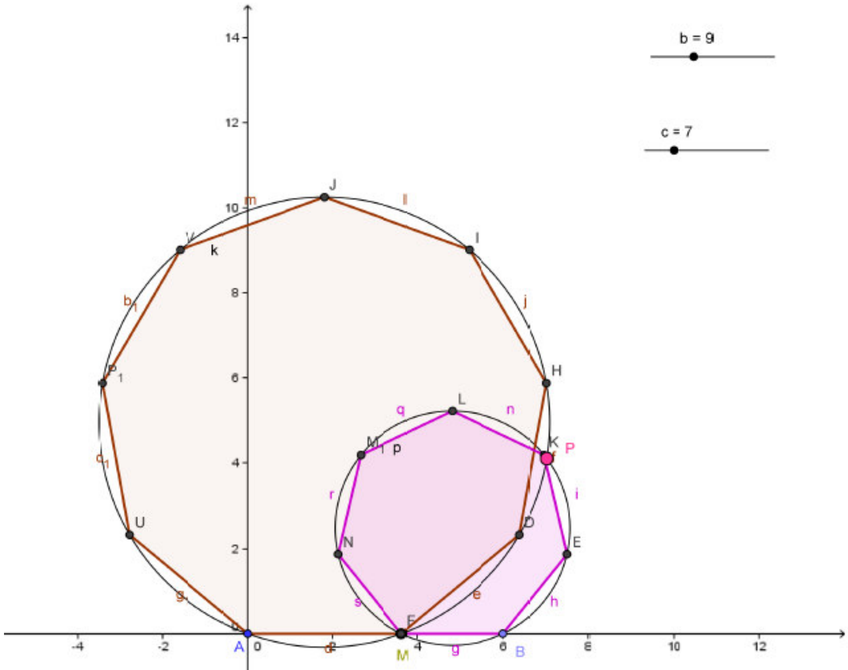

Fig.1. The hipotheses of the problem

After the investigation with GeoGebra soft, we obtain the following results, in a analytical way:

1) If $m=n$ and regular polygons are in the same half plane (the superior half plane), the geometric locus of P is the reunion between an arc with the extremes A and B, an capable arc of $360^o/n$ and the half lines X'A) si (BX.

When M browses the half line X'A), i.e. $x \in (-\infty, a)$, the circles are tangent and P=M.

When M browses [AB], i.e. $x \in [a,b]$, P describes the arc APB, capable of $360^o/n$.

When M browses (BX, i.e. $x \in (b,+\infty)$, the circles are tangent and P=M.

We find that line MP, when $x \in (a,b)$, passes through a fixed point P_0, where P_0 is the middle point of the arc that completes the arc APB to a circle, i.e. the arc AP_0B is capable of $180^o - 360^o/n$.

2) If $m=n$ and regular polygons are in different half planes, the geometric locus of P is the reunion of the arcs P'A, BP_0, which are capable of $180^o - 360^o/n$, and segment [AB].

When M browses the half line X'A), i.e. $x \in (-\infty, a)$, the point P describes the arc P_0'A.

When M browses [AB], i.e. $x \in [a,b]$, the circles are tangent and P=M.

When M browses (BX, i.e. $x \in (b,+\infty)$, the point P describes the arc BP0.

The points P_0' and P_0 aren't points of geometric locus, being limit positions of point P and they correspond to - ∞, respectively +∞.

Is ascertained that the line MP passes through fixed point P_0' when $x \in (-\infty,a)$, and through fixed point P_0 when $x \in (b,+\infty)$.

In the specific case $m = n = 4$ the arc that represents the geometric locus from (1) is a half circle and the arcs AP_0', BP_0 from (2) are quarter of circle, being arcs capable of 90^o.

3) If $m < n$ and the regular polygons are in the same half plane (superior half plane), the geometric locus of P is the reunion of three arcs: Q*A, capable of $180^o(1/m - 1/n)$ and the measure equal with $360^o/n$, AP_2B, capable of $180^o(1/m + 1/n)$, and BP*, capable of $180^o(1/m - 1/n)$ and the arc AP* measure equal with $360^o/n$.

The arcs AP* and AQ* are symmetric related to line *d*.

When M browses the half line X'A), i.e. $x \in (-\infty,a)$, the point P describes the arc Q*A.

When M browses [AB], i.e. $x \in [a,b]$, the point P describes the arc BPA.

When M browses (BX, i.e. $x \in (b,+\infty)$, the point P describes the arc BP*.

The points P* and Q* aren't points of geometric locus, being limit positions of point P, when *x* goes to and - ∞, respectively +∞.

4) If $m < n$ and the regular polygons are in different half planes, the geometric locus of P is the reunion of three arcs: T*A, capable of

$180^o(1/m - 1/n)$ and the measure equal with $360^o/n$, APB, capable of $180^o(1/m + 1/n)$ with limits A and B, and BS*, capable of $180^o(1/m - 1/n)$ and the arc BS* measure equal with $360^o/m$.

When M browses the half line X'A), i.e. $x \in (-\infty,a)$, the point P describes the arc T*A.

When M browses [AB], i.e. $x \in [a,b]$, the point P describes the arc APB.

When M browses (BX, i.e. $x \in (b,+\infty)$, the point P describes the arc BS*.

The arcs BT* and BS* are symmetric related to line *d*.

The points T* and S* aren't points of geometric locus, being limit positions of point P, when *x* goes to and - ∞, respectively +∞.

In the specific case $m = 3$, $n = 4$ the arc APB, representing the geometric locus from (3), is a half circle and the arcs AT* and BS* from (4) are situated on the circle with diameter [AB] , being arcs capable of 90^o.

<u>Observation 1</u>

The circle with diameter [AB], or a part of it, becomes a geometric locus only when $m = n = 4$ or $m=3$, $n=4$, because the equation

$1/m + 1/n = 1 / 2$ has only these solutions in **N,** (fig.2).

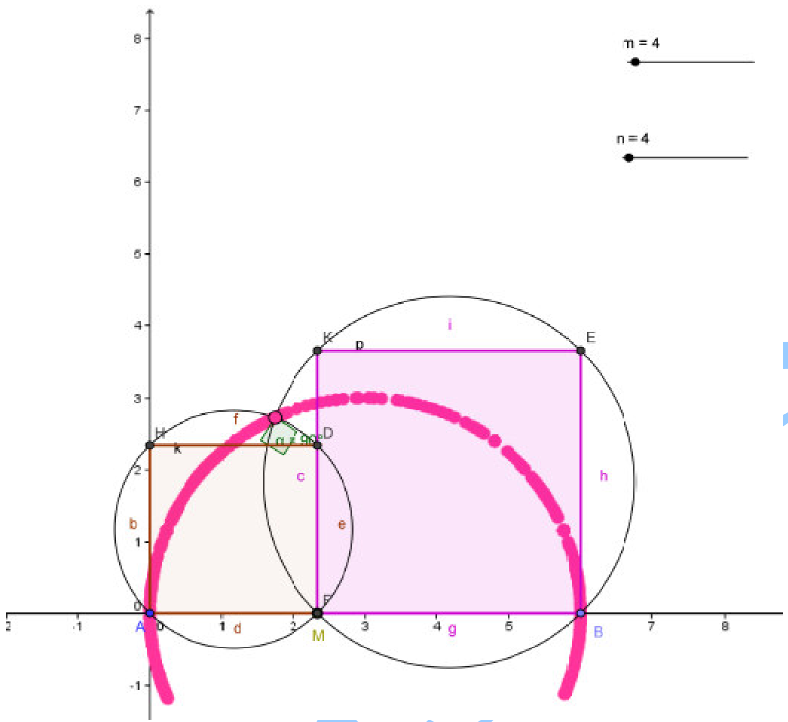

Fig. 2. Number of sites $m = n = 4$

Observation 2

If m=3, n=4 and m=4, n=6 the geometric loci are the same, the corresponding arcs being capable of 105°, 75°, 165° and 15°. The difference is that for m=3, n=4 the limit points P*, T* are situated on the arc capable of 75°, and for m=4, n=6 these are situated on the arc capable of 105°. These cases result from the solving in **N** of the equation

$1/m + 1/n = 1 - 1/p - 1/q$.

On observe that arcs are capable of 105°, 75°, 165° and 15°, (fig.3).

Observation 3

A part of geometric locus can be the same for different pairs of regular polygons with different number of sides.

Besides the examples presented in observations 1 and 2, more illustrate next:

The equation $1/m + 1/n = 1/p + 1/q \neq 1/2$, where $m,n,p,q \in \mathbf{N}-\{0,1,2\}$, has the solution $m=2k+1$, $n=2k+2$, $p=k+1$, $q=(2k+1)(2k+2)$, $k \in \mathbf{N}$, $k \geq 2$.

For $k=2$ we obtain the pairs of polygons with $m=5$; $n=6$ and $p=3$; $q=30$. For $k=3$ we obtain $m=7$; $n=8$ and $p=4$; $q=56$,(fig.4).

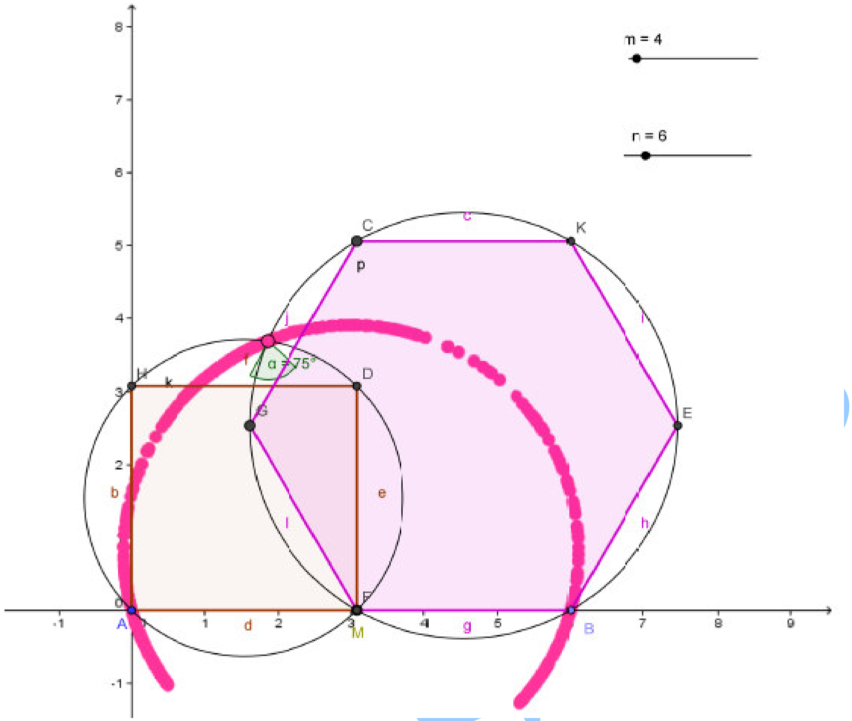

Fig. 3. Arc capable of 75° when m=4 and n=6.

Observation4

The arcs APB and BP_0 from cases 1), respectively 2) are on the same circle.The arcs APB and BS*, BP* and APB, AQ* AND at* from cases 3) and, respectively 4) are on the same circle.

Observation 5

The case $m > n$ is treated similar with the case $m < n$.

Observation 6

If the plane π is variable, the line remaining fixed, the geometric locus of P is obtained by rotating the geometric locus presented in cases 1), 2), 3), 4) around the axis *d*.

Corollary

The geometric loci presented in cases 1), 2), 3), 4) represent geometric images of the set of real numbers, browsed in the direction indicated by arrows, (fig.5).

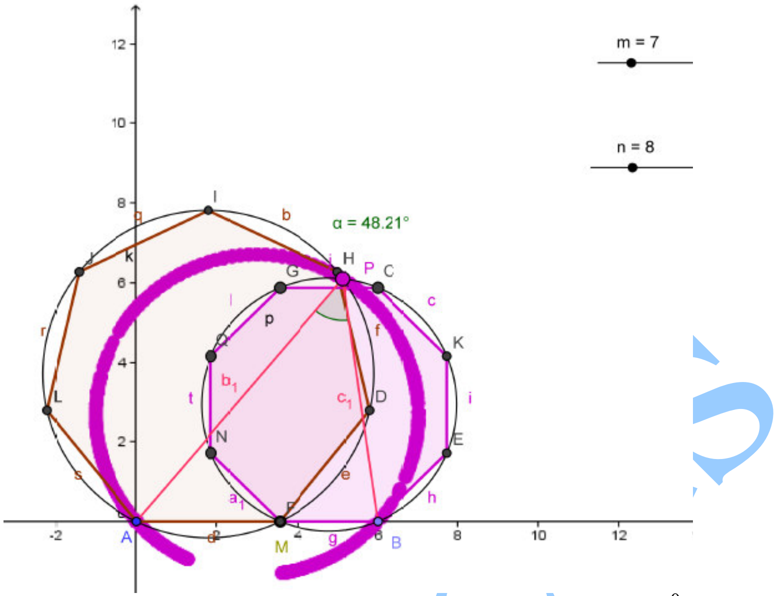

Fig. 4. m=7 and n=8 and arc APB capable of 48.21^0

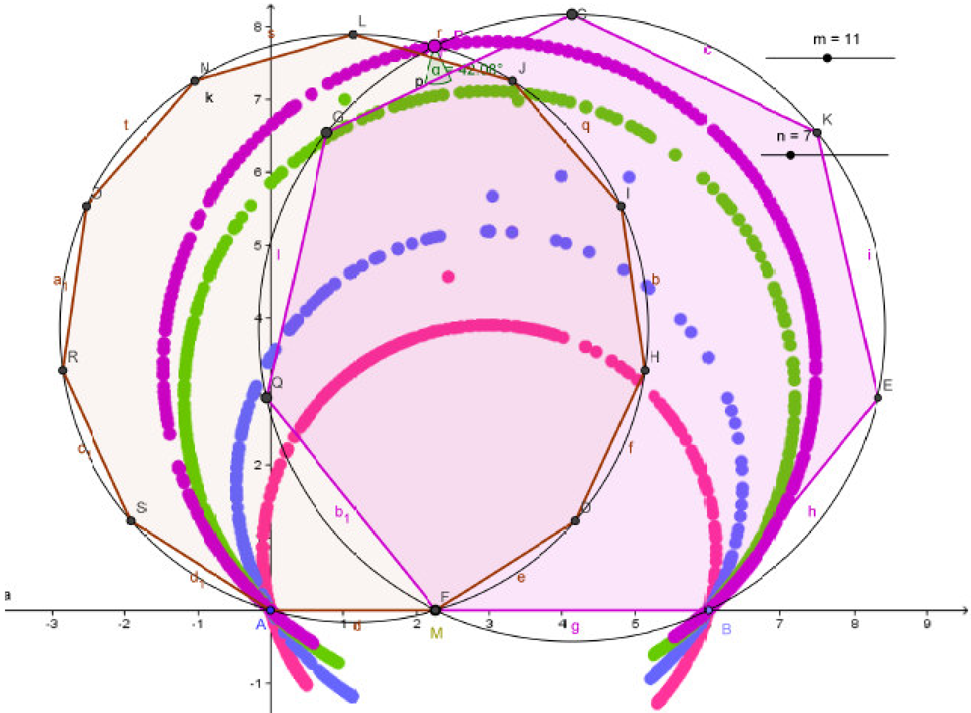

Fig.5. Geometric images of the set of real numbers

Conclusions

A step by step construction, which represent the visual interpretation of a mathematical context, a problem of a geometric locus, will follows the next steps: construct geometric figures based on problem hypotheses, apply geometric transformations, (move the point M along AB line) understand the relationship between Euclidean construction and proof, create demonstrations that involve animation and action buttons, find out geometrically and algebraically connections in a rigorous proofs, [Cri00].

One appreciate the pedagogical implications of exploring geometry in a dynamic environment, both as an investigation tool and as a demonstration tool, the connection between math educators and specialists in informatics is one of the best and a provocation too. The term "Dynamic info-geometry" could be a new method of math teaching and this paper is a start of future investigation in applied math application.

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