

GeoGebra software in teaching-learning process

Valerian Antohe

Pedagogical High School “D.P.Perpessicius”, Brăila, România

ABSTRACT: The development of computer technologies can be seen as a miracle in the modern civilization. What new reality of the IT surrounding environment have brought to us now was unthinkable several decades ago. The level of information and its flow on different communication channels, its impact on the individual are more and more difficult to quantify. A great challenge for educational systems is to create some harmonious co-existence between face book, wikis, virtual realities, virtual learning and traditional methods. The examples presented will illustrate the impact of GeoGebra software in this sense.

KEYWORDS: virtual learning, teaching, tactics, strategies.

Introduction

Since the school year 1998 a new curriculum was implemented in the Romanian secondary school. This reduces the number of mathematics hours per week and some teaching topics too. For example, problems of plane geometry, study of geometrical locus problems, study of some real function in terms of graphics representation, series of real numbers may be just some topics that have undergone serious changes. According to our national curriculum every school can prepare his own curriculum, accepted by students, proposed by teachers. In this sense computer based math education was implemented to teaching process and for applying such ideas of modern teaching. GeoGebra software can help the teachers to implement ICT projects into their teaching. In this sense, some math topics that are not declared compulsory in curriculum can be studied. Hence, the aim of this study is to find out strategies in order to use GeoGebra as a method of teaching and analyze possible effects of these strategies.

1. Methodology, development tactics and strategies

It is a challenge to use the computer in learning processes. This implies the integration of educational software in education. The educational software and its applications are very helpful in teaching-learning methods for maths. An "educational software application" is something that you can use, without having advanced knowledge about computers and programming. One accept that investigating properties of a geometrical or algebraically domain, changing shape and size in order to establish if properties remain true and formulation of the theorem from this investigation, should be a real route for any mathematical provocation. [Ant2009].

Most of the teachers and school managers complain about the difficulties in working with the educational software provided by the Romanian Ministry of Education and about the lack of resources allocated for upgrades, for software acquisition from the free market and for appropriate training, [Ist07]. For these entire reasons one accept that the presence of open source software involved in education will become a special opportunity and GeoGebra software is one of the best example in this sense, [HL10].

The starting moment of the GeoGebra software operation is considered the moment in which this software was created and launched in the universal informational environment (It took place in the 1990s under the umbrella of "open source" systems) and was meant to be a dynamic structure, focused on the following goals:

- to provide free access to a dynamic educational software;
- to offer free teacher training courses;
- to support and develop materials both for students and teachers with a view to improving education in the fields of mathematics, sciences and technology;
- to contribute and to promote collaboration between practitioners and researchers in order to create a national users community.

Is the education system ready to accomplish the new tasks? The question is older than the latest developments.

Using the computer in education presents the advantage of facilitating the passage from a passive accumulation of information by the students to them learning through discovery. As a result, they learn how to learn, thus developing abilities and cognitive strategies that they will use and adapt in other diverse situations. This induces a great flexibility of learning and stimulates students to involve themselves in the educational process and become partners of the teacher. In the next section, some examples of good practice will be presented.

2. Educational software induce a great flexibility in teaching

There are some requirements for students and for teachers in order to develop activities in the mathematics curriculum with the help of the GeoGebra and these requirements are generally accepted and known in GeoGebra community.

Maths activities could be classified as: activities dedicated to reach competence in using GeoGebra software, activities dedicated to visualize mathematical results and theorems, activities dedicated to establish research in maths, finding new theorem and generalize results, special activities which are focused to discover "other open problems" in order to be rigorously demonstrated in other future activities.

2.1. Activities dedicated to reach competence in using GeoGebra software

Many activities of the first stage, using GeoGebra, have been developed around the GeoGebra community and a lot of these could be finding on GeoGebra wikis. Since the main objective of this lesson is to teach students to use the GeoGebra software, the maths involved with the lesson is a review of geometry and measurement previously covered.

2.2. Activities dedicated to visualize mathematical results and theorems

Speaking about the second issue, visualize results and maths theorem, one could give some examples:

Ceva's Theorem is a very popular theorem in elementary geometry. Given a triangle ABC , and points D , E , and F that lie on lines BC , CA , and AB respectively, the theorem states that lines AD , BE and CF are concurrent if and only if $\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$. The proof is classical and will be not described.

This proof could be used in the GeoGebra context because the software can visualize the area of any triangle considered in the proof, (Fig.1).

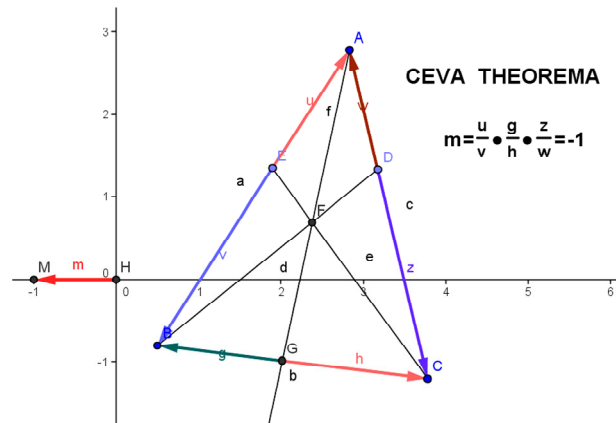


Figure 1. The Ceva's Theorem file.ggb project realized with the GeoGebra software

A lot of web pages contain different proofs of this theorem, and one of this is contained in figure 2.

Let ABC be a triangle and lines (or cevians) AD, BE and CF joining vertices with opposite sides intersect at a single point P. Prove that:

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

Proof:

1. $\triangle ABE$: FPC is a transversal:
 $\frac{AF}{FB} \cdot \frac{BP}{PE} \cdot \frac{EC}{CA} = -1$ (Menelaus' Theorem)
2. $\triangle BCE$: DPA is a transversal:
 $\frac{BD}{DC} \cdot \frac{CA}{AE} \cdot \frac{EP}{PB} = -1$ (Menelaus' Theorem)
3. Multiplying (2) x (4):
 $\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$

Q.E.D.

See also:
Nagel Point
Gergonne Point

Please press the Up arrow key to play again

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Figure 2. Virtual learning on dynamic geometry images, <http://agutie.homestead.com/files/ceva.htm>, Antonio Gutierrez 2005

The exponential and logarithmic function

The equation $f(x) = f^{-1}(x)$ has for exponential and logarithmic function the form $a^x = \log_a x$ for $a \in (0, 1) \cup (1, \infty)$. GeoGebra allows showing cases which are not typical for mathematical textbook, [Gun10].

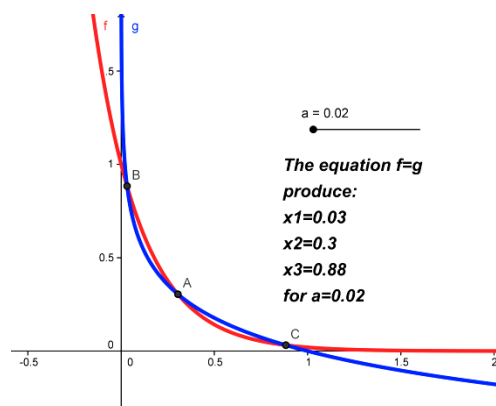


Figure 3. The equation $f(x) = f^{-1}(x)$, The equation $f(x) = a^x$, $a=0.02$

These examples illustrate how it is possible to use the method of generating problems with generator problem, concretely with solving equation $f(x) = f^{-1}(x)$ for different types of function. The benefit of this purpose consists in connection between teaching mathematical analysis and both analytical and synthetic geometry at school. It is very important that the students' knowledge be not isolated. A teacher has a possibility to explain the notions of inverse function, graphs of different function. Students can use the GeoGebra software by finding the numerical solutions for the equation $f(x) = f^{-1}(x)$ and this will visualize mathematical results, (Fig.3).

2.3. Activities dedicated to establish research in math, finding new theorem and generalize results

In this section, some of maths activities will be projected, considering that each maths teacher could adopt his own strategy because for most of all secondary maths teachers the ability to use an educational soft could be considered like a provocation.

The applet illustrates two theorems discovered three hundred years ago. Let P be a point in the plane of an equilateral $\triangle ABC$. Both theorems are concerned with the relative lengths of segments AB , BP and CP , joining P the vertices of $\triangle ABC$.

Theorem (F. van Schooten, 1615 – 1660)

For P on the circumcircle of an equilateral $\triangle ABC$, the longest of the three segments (PA , PB or PC) it is equals the sum of the shorter two, (Fig. 4).

Theorem (D. Pompeiu, 1873 – 1954)

For P not on the circumcircle of $\triangle ABC$, there exists a triangle with sides PA , PB and PC . (The triangle in question is known as *Pompeiu's triangle*), (Fig.5).

Ptolemy's theorem Let a convex quadrilateral $ACPB$ be inscribed in a circle. Then the sum of the products of the two pairs of opposite sides equals the product of its two diagonals. In other words, $AD \cdot BC + AB \cdot CD = AC \cdot BD$, (Fig.6).

Rotate triangle ACP around A through the angle CAB so that C is mapped onto a point, say C' , on AB . P is mapped to P' .

In case $\triangle ABC$ is equilateral, my come up with the configuration in the proof of van Schooten's theorem. When $\triangle ABC$ is equilateral the rotation places P' on the extension of BP . The same is true when $\triangle ABC$ is isosceles. In the general case, we may only claim that $C'P' \parallel BP$. (The reason is the same in all three cases. Since the quadrilateral $ABPC$ is cyclic $\angle ACP + \angle ABP = 180^\circ$. After the rotation, $\angle AC'P' = \angle ABP''$, where P'' is the intersection of AP' and BP .) Triangles ABP'' and $AC'P'$ are similar, implying $AC' / C'P' = AB / BP''$ so that $BP'' = AB \cdot C'P' / AC'$.

Triangles ABC and $AP''P$ are also similar because, in addition to $\angle PAP'' = \angle CAB$, $\angle AP''P = \angle AP'C' = \angle APC = \angle ABC$ (the latter as the two inscribed angles subtended by the same arc.) This gives $AC/AP = BC/PP''$ so that $PP'' = BC \cdot AP / AC$. But $PP'' = BP + BP'' = BP + AB \cdot CP / AC$. We see that

$BP + AB \cdot CP / AC = BC \cdot AP / AC$. Multiplying by AC gives Ptolemy's identity.

Ptolemy's theorem offers one approach to proving theorems.

A proof below is more elementary.

Rotate the plane 60° around B so as to make vertex C overlap vertex A . Let A' and P' be the images of A and P , respectively. We'll show that APP' is Pompeiu's triangle. By the construction, $\triangle BPP'$ is equilateral, so that $PP' = BP$. In addition, AP' is the image under the above rotation of CP , so that $AP' = CP$. The remaining side in $\triangle APP'$ is just AP , and we are done with Pompeiu's theorem.

Assuming A , P and P' are collinear (i.e., when Pompeiu's triangle is degenerate), $\angle APB$ is either 60° or 120° . (This is because it is either equal to $\angle BPP'$, which is 60° , or is supplementary to it.) But then the quadrilateral $ABCP$ is cyclic, proving van Schooten's theorem and its converse.

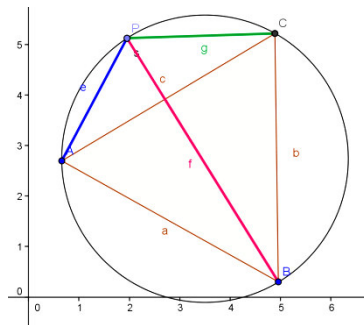


Figure 4. Theorem F. van Schooten, $PB=PA+PC$

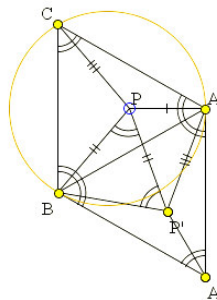


Figure 5. Theorem D. Pompeiu

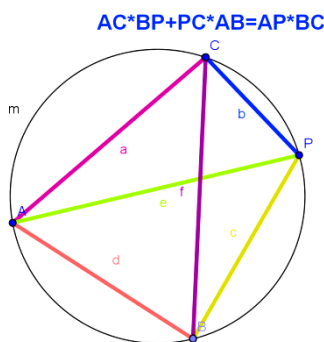


Figure 6. Ptolemy's theorem, $AC \cdot BP + AB \cdot CP = AP \cdot BC$

The F. van Schooten's theorem can be extended to triangles that are not equilateral.

Theorem 1: Given $\triangle ABC$ and a point P on its circumcircle, a, b, c denote the sides of ABC and d_a, d_b, d_c the distances from P to sides $a = BC, b = CA, c = AB$. We are going to show that one of the ratios $a/d_a, b/d_b, c/d_c$ is the sum of the other two.

The proof depends on the following:

Lemma:

In $\triangle ABC$, $h = b \cdot c / (2R)$, where h is the altitude from A and R is the circumradius. (For the proof see Metric Relation in a Triangle).

To continue with the theorem, for $\triangle PBC$, lemma gives: $d_a = PB \cdot PC / (2R)$, and similarly for triangles PCA and PAB . Next we obtain:

$$(1) \quad \begin{aligned} a/d_a &= a / (PB \cdot PC) \cdot (2R) = a \cdot PA \cdot (2R) / (PA \cdot PB \cdot PC), \\ b/d_b &= b / (PC \cdot PA) \cdot (2R) = b \cdot PB \cdot (2R) / (PA \cdot PB \cdot PC), \\ c/d_c &= c / (PA \cdot PB) \cdot (2R) = c \cdot PC \cdot (2R) / (PA \cdot PB \cdot PC). \end{aligned}$$

$\triangle ABC$ divides its circumcircle into three arcs, say (BC) , (CA) , (AB) . If, for instance, P is on arc (BC) then by Ptolemy's theorem, $a \cdot PA = b \cdot PB + c \cdot PC$.

Therefore,

$$(2) \quad a/d_a = b/d_b + c/d_c,$$

as needed. It follows from (1) that say, when $a = b = c$, $PA = PB + PC$ is equivalent to $a/d_a = b/d_b + c/d_c$. However, while the latter holds for all triangles, the former holds only for the equilateral ones.

Observe that, as it follows from the proof, the ratio in the left-hand side of (2), (i.e., the largest ratio possible) corresponds to the arc that contains P .

The provocation of using GeoGebra software could be here the construction of a convex quadrilateral $ACPB$, inscribed in a circle. We need this construction in order to develop dynamic construction for Ptolemy's theorem and prove that $AD \cdot BC + AB \cdot BD = AC \cdot BD$. Four points must be on a circle in order to form a convex quadrilateral $ABCD$ and, important, these four points must be in this order. We have two proposed construction in two GeoGebra file. First consider like a start point the construction of diagonals using two variable angles, first give the position of the first diagonal towards the abscises axis. The second diagonal will be rotated around a fixed point on the circle, (Fig.7).

The next construction will consider first a fixed diagonal of the convex quadrilateral $ABCD$ and one coin will be mobile on one arc of the circle.

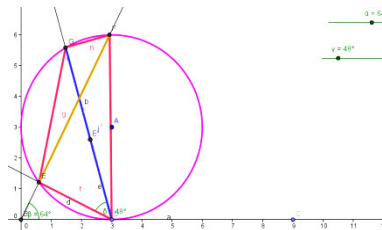


Figure 7. Construction of the convex quadrilateral $ABCD$

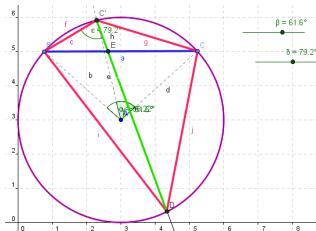


Figure 8. Construction of the convex quadrilateral $ABCD$ start from the fixed diagonal BC

Theorem 1 admits a further extension to polygons with more than three vertices.

Theorem 2: Given point P on the circumcircle of a cyclic convex polygon $X_1X_2X_3 \dots X_{n-1}X_n$ ($n > 2$), we use following notations (i and j can be interchanged):

- a_{ij} = length of side X_iX_j ;
- d_{ij} = distance from P to sidelines X_iX_j ;
- R_{ij} = ratio a_{ij}/d_{ij} .

Without loss of generality we may assume that P is on the arc X_1X_2 . Then the set of ratios $R_{12}, R_{23}, R_{34}, \dots, R_{(n-1)n}, R_{n1}$ verify the relation:

$$R_{12} = R_{23} + R_{34} + \dots + R_{(n-1)n} + R_{n1}.$$

The theorem admits a demonstration by induction method.

For using GeoGebra software this theorem produce an open problem if we want to construct a dynamical imagines of a convex polygon with n sides inscribed in a circle. Could we fix one of the vertices and others $n-1$ vertices to be mobile on the circle, preserving the relation of order between vertices so that the polygon to remain convex? Ideas to realize this for $n=4$ was established in the examples above and theorem 2 is visualized for $n=5$ in figure 9.

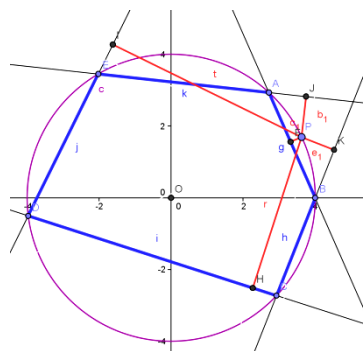


Figure 9. Theorem 2 is visualized for $n=5$

Conclusions

The teachers who use GeoGebra must be more specific, more "open minded", willing to allow for experimentation, and give more guidance at the start of any GeoGebra experiment. Flexibility of learning means flexibility of relation between teachers and students. This implies new situation of learning and a special development of new methods of teaching. The adequately integrating such approaches into projects can lead to the development of 'meta-mathematical' skills, which are particularly useful in technological instruction and technology applications. On the social side, the impact of GeoGebra may be important in shifting perceptions about mathematics as a solitary activity to the more harmonic view of mathematics as social activity.

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