Movement Analysis of a Fluid through a Laval Nozzle Axial Symmetry with Fluent Program

Amado George Ştefan, Constantin Nistor
Military Technical Academy, Bucharest, Romania

ABSTRACT: The article presents an ideal fluid dynamics equations (4), the differential equation for the velocity potential of ideal fluid motion stationary (6. 8) and analyze the movement transonic nozzle Laval, the assumption of small perturbations is deduced partial differential equation of transonic motion (13). Fluent software is applied to determine the Mach number and temperature distribution, depending on the mass flow through the nozzle, the configuration of Fig. 1., 2. In Fig. 4.a, b, c, d, e, f and 7.a, b, c, d, e, f are the Mach number distributions and temperature inside the Laval nozzle, depending on mass flow. The numerical results obtained by applying the program Fluent are given in Fig. 5, 6, 8, 9.
KEYWORDS: Fluid Mechanics, Dynamics of gas, Turbo-jet

Introduction

In the classical theory of fluid dynamics is an important issue concerns the analysis of movement through a nozzle convergent - divergent (Laval nozzle). Are important research results obtained by Th. Meyer, S.A.Ceaplâghin, L. Prandtl, S.V. Falkovich, E. Carafoli [Car57], T. Oroveanu, C. Jacob, V.N. Constantinescu [CC84] P. Bradeanu [Bra73] S. Galetuse , C. Berbente, V. Stanciu, S. Danaila, O. Popa, who developed the theory of two-dimensional movements and unite through the nozzle convergent - divergent.

Analytical methods have been developed and numerical algorithms that enable effective, such as approximate methods and supplementary assumptions which depends a priori validity of the method.

The purpose of the paper is to present the form of partial differential equations in analysis of usage for the transition from subsonic to supersonic by Laval nozzle and using Fluent program to determine the variation of parameters features two-dimensional movement.

1. Fundamental equations

Ideal fluid equations of motion, L. Euler deducted. be expressed as vector form

$$\frac{\partial \overline{v}}{\partial t} + (\overline{v} \cdot \nabla)\overline{v} = \overline{f} - \frac{1}{\rho} \cdot \text{grad}(p), \qquad (1)$$

expression which is obtained by Navier-Stokes equations taking $\nu = 0$. In irrotational motion $(\text{rot}(\overline{v}) = 0)$ equation (1) becomes

$$\frac{\partial \overline{v}}{\partial t} + \frac{1}{2} \cdot \text{grad}\left(v^2\right) = \overline{f} - \frac{1}{\rho} \cdot \text{grad}(p). \qquad (2)$$

The assumptions
- mass forces derived from a potential U:

$$\overline{f} = -\text{grad}(U); \quad U = g \cdot z + U_o \ \text{ if } \ \overline{f} = \overline{g}.$$

- fluid density is a function of pressure $\rho = \rho(p)$, there is a function of P, so that $\text{grad}(P) = \frac{1}{\rho} \cdot \text{grad}(p)$, by multiplying with $d\overline{r}$ the relationship becomes:

$$P = \int \frac{dp}{\rho(p)},$$

and equation (1) takes the form:

$$\int \frac{\partial \overline{v}}{\partial t} \cdot d\overline{r} + \frac{v^2}{2} + U + \int \frac{dp}{\rho(p)} = C, \qquad (3)$$

relationship called pressure equation, C is a constant that depends on the current line or vortex line along which movement has made elementary $d\overline{r}$.

1.1. Velocity potential equation for stationary motion of ideal fluid

It is considered a potential movement (irrotational) $\overline{v} = \mathrm{grad}(\varphi)$ and φ is the velocity potential. The ideal fluid assumptions neglect to add mass forces compared to pressure forces, results system:

$$\begin{aligned} & \mathrm{div}(\rho \cdot \overline{v}) = 0, \\ & \frac{1}{2} \cdot \mathrm{grad}\left(v^2\right) + \frac{1}{\rho} \cdot \mathrm{grad}(p) = 0, \\ & h + \frac{v^2}{2} = h_0, \text{(ecuatia} - \text{entalpiei)}, \end{aligned} \qquad (4)$$

$h =$ specific enthalpy $\left(h = \frac{a^2}{\gamma - 1} \right)$ ho = specific enthalpy of stagnation.

Turn $\mathrm{grad}(p)$ in the form:

$$\mathrm{grad}(p) = \frac{\partial p}{\partial x} \cdot \overline{i} + \frac{\partial p}{\partial y} \cdot \overline{j} + \frac{\partial p}{\partial z} \cdot \overline{k} = \frac{dp}{d\rho} \cdot \left(\frac{d\rho}{dx} \cdot \overline{i} + \frac{d\rho}{dy} \cdot \overline{j} + \frac{d\rho}{dz} \cdot \overline{k} \right) = a^2 \cdot \mathrm{grad}(\rho) \qquad (5)$$

and inserts into (4). Are obtained

$$\mathrm{grad}(\rho) = \frac{1}{a^2} \cdot \mathrm{grad}(p) = -\frac{\rho}{2 \cdot a^2} \cdot \mathrm{grad}\left(v^2\right)$$

$$\mathrm{div}(\rho \cdot \overline{v}) = \rho \cdot \mathrm{div}(\overline{v}) + \overline{v} \cdot \mathrm{grad}(p) = \rho \cdot \left(\mathrm{div}(\overline{v}) - \frac{\overline{v}}{2 \cdot a^2} \cdot \mathrm{grad}\left(v^2\right) \right)$$

the resulting

$$\operatorname{div}(\overline{v}) = \frac{\overline{v}}{2 \cdot a^2} \cdot \operatorname{grad}(v^2), \tag{6}$$

called the potential equation, which a represents the local speed of sound

$$a^2 = \frac{dp}{d\rho} = \gamma \cdot R \cdot T = a_0^2 - \frac{\gamma - 1}{2} \cdot v^2, \tag{7}$$

$a_0^2 = \gamma \cdot R \cdot T_0$, $T_0 =$ stagnation temperature, R = gas constant. Taking into account that $\overline{v} = \operatorname{grad}(\varphi)$ and $\operatorname{div}(\overline{v}) = \operatorname{div}(\operatorname{grad}(\varphi)) = \Delta\varphi$, the equation (6) is transcribed as

$$\Delta\varphi = \frac{\operatorname{grad}(\varphi)}{2 \cdot \left(a_0^2 - \frac{\gamma - 1}{2} \cdot \operatorname{grad}^2(\varphi)\right)} \cdot \operatorname{grad}(\operatorname{grad}^2 \varphi). \tag{8}$$

Noting $\overline{v} = v_x \overline{i} + v_y \overline{j} + v_z \overline{k}$, equation (6) becomes:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \frac{1}{2 \cdot a^2} \cdot (v_x \overline{i} + v_y \overline{j} + v_z \overline{k}) \times$$
$$\times \left[\frac{\partial}{\partial x}(v_x^2 + v_y^2 + v_z^2) + \frac{\partial}{\partial y}(v_x^2 + v_y^2 + v_z^2) + \frac{\partial}{\partial z}(v_x^2 + v_y^2 + v_z^2) \right]$$

from which we deduce

$$\left(1 - \frac{v_x^2}{a^2}\right) \cdot \frac{\partial v_x}{\partial x} + \left(1 - \frac{v_y^2}{a^2}\right) \cdot \frac{\partial v_y}{\partial y} + \left(1 - \frac{v_z^2}{a^2}\right) \cdot \frac{\partial v_z}{\partial z} - \frac{v_x . v_y}{a^2} \cdot \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) - \tag{9}$$
$$- \frac{v_x \cdot v_z}{a^2} \cdot \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) - \frac{v_y \cdot v_z}{a^2} \cdot \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) = 0$$

called fundamental equation of ideal compressible fluid dynamics.

In two dimensions, the relationship (9) becomes

$$\left(1 - \frac{v_x^2}{a^2}\right) \cdot \frac{\partial v_x}{\partial x} + \left(1 - \frac{v_y^2}{a^2}\right) \cdot \frac{\partial v_y}{\partial y} - \frac{v_x \cdot v_y}{a^2} \cdot \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) = 0, \tag{10}$$

or considering

$$v_x = \partial\varphi / \partial x, \quad v_y = \partial\varphi / \partial y,$$

$$\left(1 - \frac{v_x^2}{a^2}\right) \cdot \frac{\partial^2 \varphi}{\partial x^2} + \left(1 - \frac{v_y^2}{a^2}\right) \cdot \frac{\partial^2 \varphi}{\partial y^2} - \frac{2 \cdot v_x \cdot v_y}{a^2} \cdot \frac{\partial^2 \varphi}{\partial x \partial y} = 0 \qquad (11)$$

equation known as Molenbrock - Ceaplâghin – Steichen.

1.2. Movement in two dimensional transonic Laval nozzle

Velocity potential is adopted as $\varphi = v_\infty \cdot x + \varphi'$.

The small perturbation hypothesis and the reference Mach number, M_∞, with values corresponding to sonic parameters, is adopted

$$v_x = v_\infty \cdot \left(1 + v_x'\right) = v_\infty \cdot \left(1 + \frac{\partial \varphi'}{\partial x}\right),$$

$$v_y = v_\infty \cdot v_y' = v_\infty \cdot \frac{\partial \varphi'}{\partial x},$$

$$\frac{a_\infty^2}{\gamma - 1} + \frac{v_\infty^2}{2} = \frac{a^2}{\gamma - 1} + \frac{\left(v_x^2 + v_y^2\right)}{2},$$

$$\left(1 - \frac{v_x^2}{a^2}\right) = \left(\frac{a^2 - v_x^2}{a^2}\right) \approx \left(\frac{a^2 - v_x^2}{a_\infty^2}\right), \quad \left(1 - \frac{v_y^2}{a^2}\right) = \left(\frac{a^2 - v_y^2}{a^2}\right) \approx \left(\frac{a^2 - v_y^2}{a_\infty^2}\right),$$

$$M_\infty = \frac{v_\infty}{a_\infty},$$

and noting

$$\varphi'_x = \frac{\partial \varphi'}{\partial x}, \ \varphi'_y = \frac{\partial \varphi'}{\partial y}, \ \varphi'_{xx} = \frac{\partial^2 \varphi'}{\partial x^2}, \ \varphi'_{yy} = \frac{\partial^2 \varphi'}{\partial y^2}, \ \varphi'_{xy} = \frac{\partial^2 \varphi'}{\partial x \partial y}$$

relation (11) becomes

$$\left[1-M_{\infty}^{2}+(\gamma+1)\cdot M_{\infty}^{2}\cdot\varphi_{x}^{'}\right]\cdot\varphi_{xx}^{'}+2\cdot M_{\infty}^{2}\cdot\varphi_{y}^{'}\cdot\varphi_{xy}^{'}+\left[(\gamma+1)\cdot M_{\infty}^{2}\cdot\varphi_{x}^{'}-1\right]\cdot\varphi_{yy}^{'}=0 \tag{12}$$

An order of magnitude analysis of terms allows us to stop them so that those important to obtain a simple expression for all modes of motion.

Thus, of the partial differential equation (12), it follows:

$\left(1-M_{\infty}^{2}\right)\cdot\varphi_{xx}^{'}+\varphi_{yy}^{'}=0$, subsonic motion equation, (12.a)

$\left(M_{\infty}^{2}-1\right)\cdot\varphi_{xx}^{'}-\varphi_{yy}^{'}+M_{\infty}^{2}\cdot(\gamma+1)\cdot\varphi_{x}^{'}\cdot\varphi_{xx}^{'}=0$

transonic motion equation, (12.b)

$\left(M_{\infty}^{2}-1\right)\cdot\varphi_{xx}^{'}-\varphi_{yy}^{'}=0$. supersonic motion equation. (12.c)

If transonic regime, in the minimum nozzle section shall be adopted $M_{\infty}=1,\ \ v_{\infty}=a_{c}$, so that equation (12.b) becomes

$$(\gamma+1)\cdot\varphi_{x}^{'}\cdot\varphi_{xx}^{'}-\varphi_{yy}^{'}=0. \tag{13}$$

2. Analysis of transonic movement in the vicinity of section minimum Laval nozzle with Fluent program

Fluent software is used to determine the Mach number and temperature variation in Laval nozzle according to the mass flow.

Laval nozzle is rotational symmetry, by flowing air evacuation being made in the free atmosphere.

2.1. The geometry of the field, the nozzle and boundary conditions

Minimum section diameter is 792.04 mm for angle of the first voucher to 8°. The origin of the coordinate system was chosen on the nozzle axis, the input section. Minimum diameter section is to share $x=1109.37$ mm and output section to share $x=1393.81$ mm, from the origin.

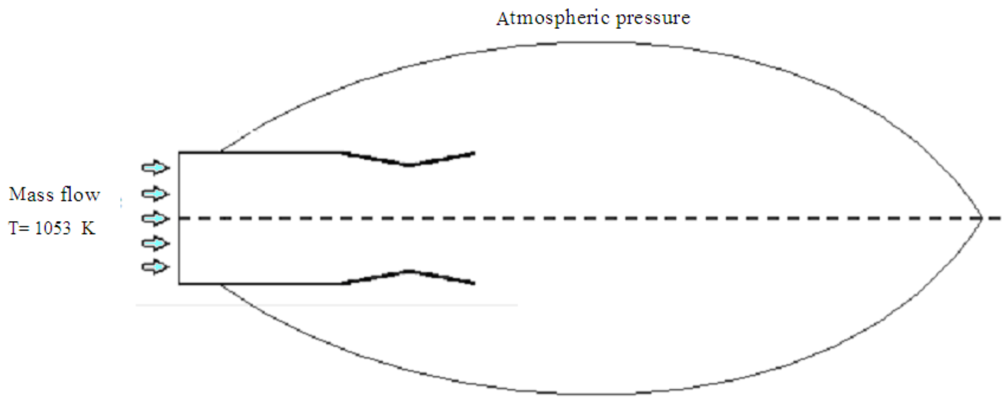

Figure 1. Domain geometry and boundary conditions

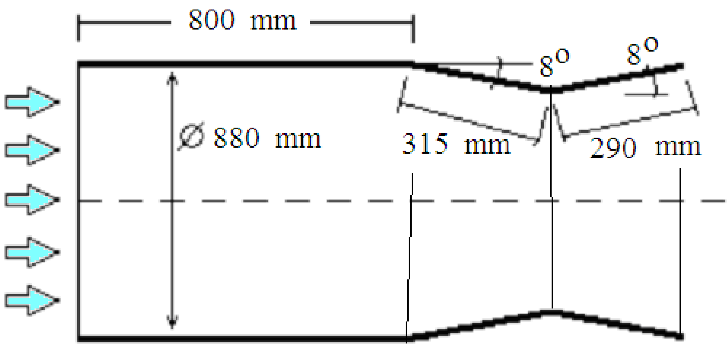

Figure 2. Nozzle geometry

For sharing field quadrilateral cells were used in the walls and minimum section using geometric progressions to ensure finer division.

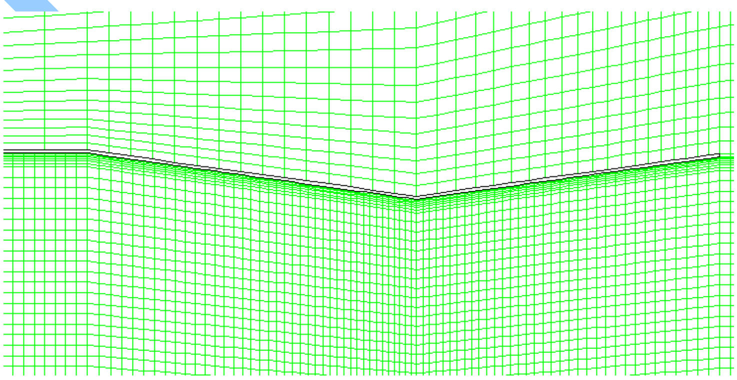

Figure 3. In dividing cells of the domain, detail of mesh in the wall

The fluid is considered compressible and viscous, laminar-turbulent flow with turbulent Spalart-Allmaras model. Mass flow of air entry into the field was varied from 70 to 95 kg /s, input temperature of 1053 ° K is considered.

2.2. Variation of Mach number in Laval nozzle according to the mass flow

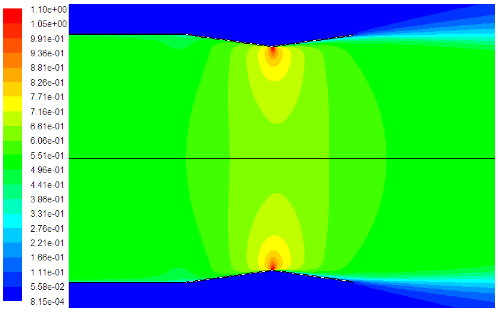

Figure 4.a. Mass flow 70 kg/s

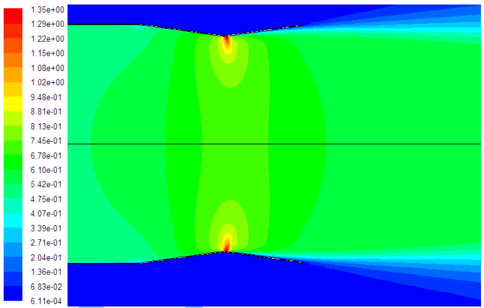

Figure 4.b. Mass flow 75 kg/s

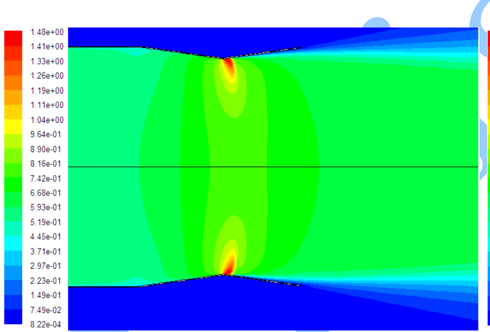

Figure 4.c. Mass flow 80 kg/s

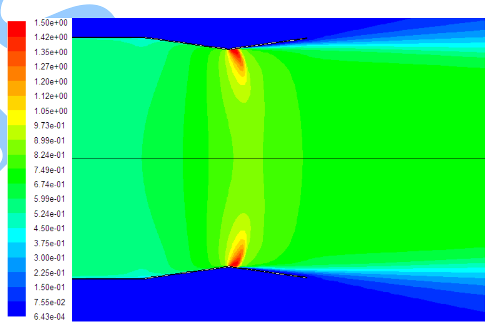

Figure 4.d. Mass flow 85 kg/s

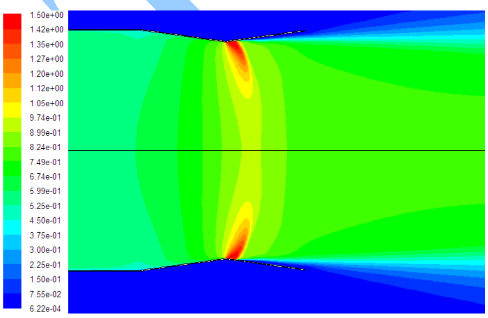

Figure 4.e. Mass flow 90 kg/s

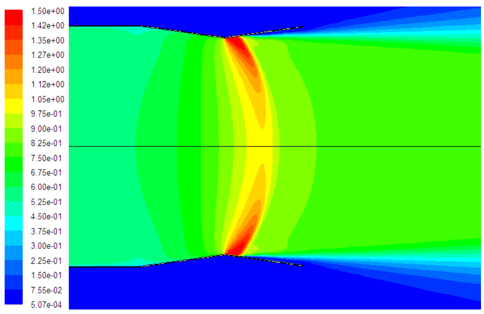

Figure 4.f. Mass flow 95 kg/s

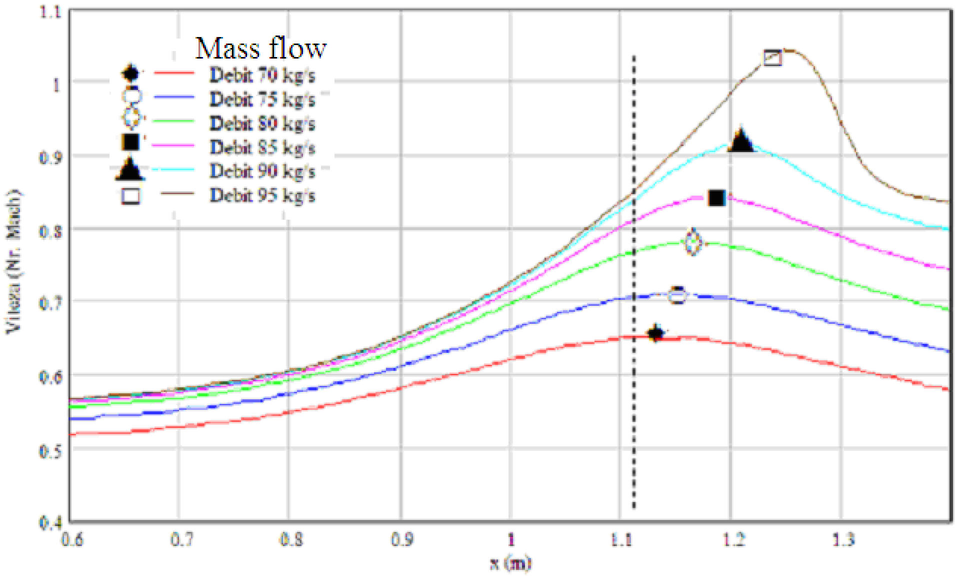

Figure 5. Variation of Mach number on the axis Laval nozzle depending on the mass flow

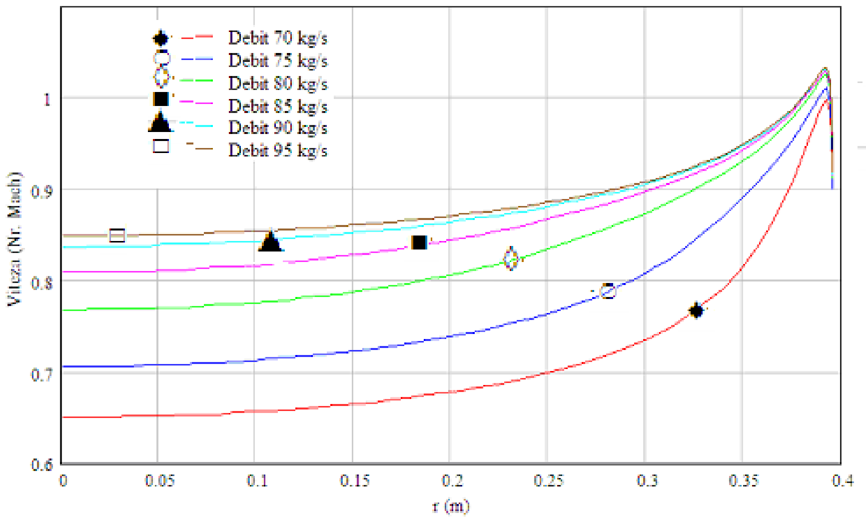

Figure 6. Mach number variation in the minimum section Laval nozzle according to the mass flow

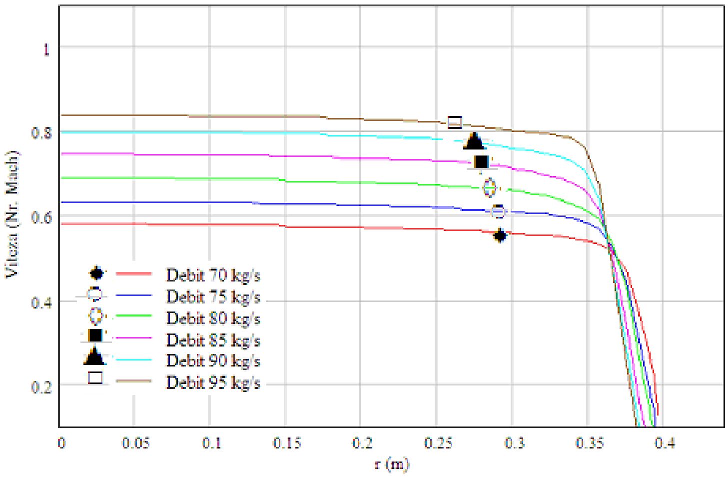

Figure 7. Mach number variation in the exit section Laval nozzle according to the mass flow

2.3. Variation of temperature in Laval nozzle according to the mass flow

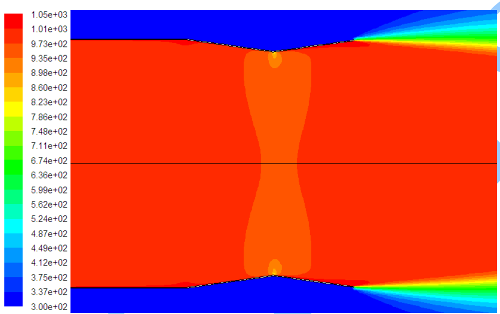

Figure 8.a. Mass flow 70 kg/s

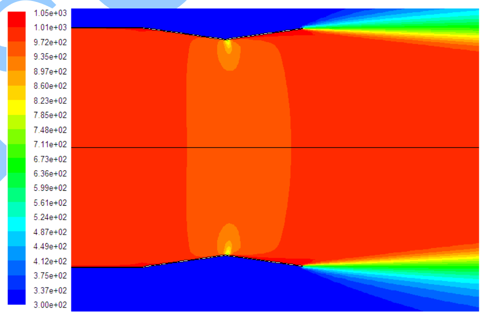

Figure 8.b. Mass flow 75 kg/s

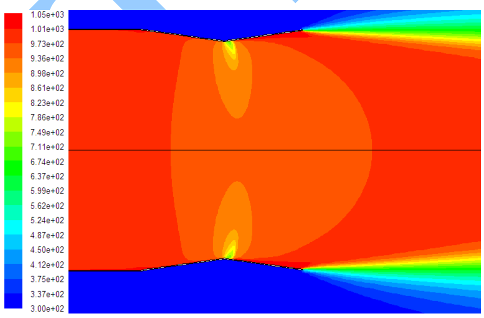

Figure 8.c. Mass flow 80 kg/s

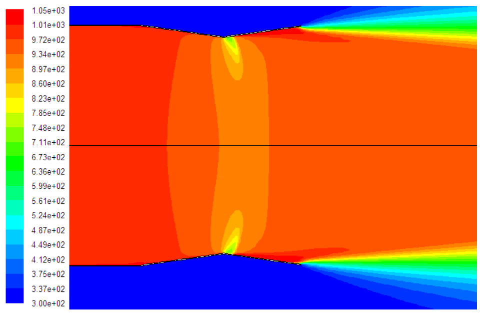

Figure 8.d. Mass flow 85 kg/s

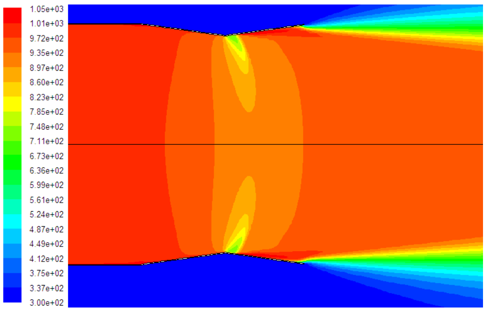

Figure 8.e. Mass flow 90 kg/s

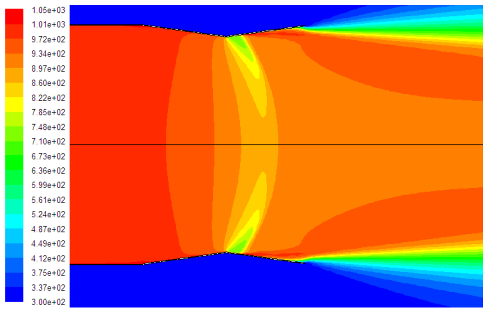

Figure 8.f. Mass flow 95 kg/s

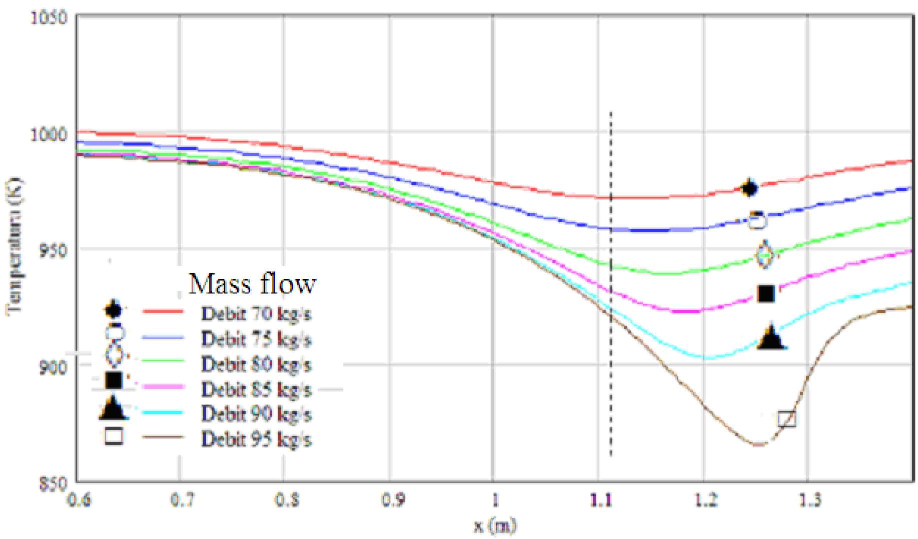

Figure 9. Variation of temperature on the axis Laval nozzle depending on the mass flow

Conclusion

The obtained data from Fluent simulation can be used for pick another coordinate system (fig. 10).

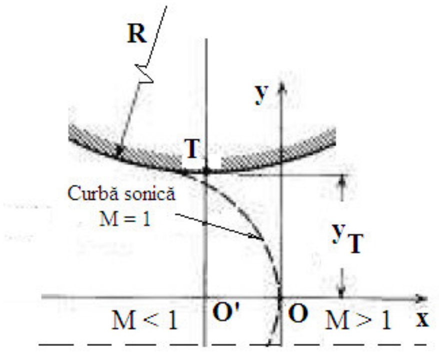

Figure 10. New coordinate system used for analytical solution

For the velocity potential $\varphi(x, y)$, near the minimum nozzle section may be adopted development series

$$\varphi(x, y) = \sum_{k=0}^{\infty} f_{2k} \cdot y^{2k} . \tag{14}$$

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