SOLUTION OF THIRD ORDER KORTEWEG-DE VRIES EQUATION BY HOMOTOPY PERTURBATION METHOD USING MAHGOUB TRANSFORM

Olusola Adebanwo Dehinsilu[1], Olutunde Samuel Odetunde[1], Biodun Tajudeen Efuwape[1], Sefiu Adekunle Onitilo[1], Peter Ibikunle Ogunyinka[1], Oludapo Omotola Olubanwo[1], Abiodun Adefolarin Onaneye[3], Olufemi Adeyinka Adesina[1,2]

[1] Olabisi Onabanjo University, Ago Iwoye, Nigeria, Department of Mathematical Sciences
[2] Obafemi Awolowo University, Ile-ife, Nigeria, Department of Mathematics
[3]Abraham Adesanya Polytechnic, Ijebu-Igbo, Nigeria, Department of Electrical Electronics and Computer Engineering

Corresponding author: Olusola Adebanwo Dehinsilu, shollymath24@gmail.com

ABSTRACT: *This article aims at depicting the genuity and usefulness of the amalgamated technique known as Homotopy perturbation with Mahgoub Transformation in unveiling the particular and coarse-grained solutions of the equation of Korteweg-de Vries of third order and nonlinear. Considered for illustrations were three cases with the proposed mechanism engaged to gain their coherent calculations. Acquired outcomes via the proposed mechanism were contrasted with solutions achieved by means of Adomian Polynomial with Elzaki Transformation and Elzaki transformation with Homotopy perturbation and were viewed as the equivalence of one another. Considering the snappy gathering of the results experienced by the method for the representative cases, it is therefore suggested that this mechanism is progressively effective, accommodating, productive and of high precision with lesser counts. Besides, it is of basic to acquaint the actuality that this article had incorporated another modus operandi to the existing means of disentangling equations of this kind.*
KEYWORDS: *Korteweg-de vries equation, Mahgoub transformation, Homotopy perturbation, nonlinear, partial differential equation*

1. INTRODUCTION

The equation of Korteweg-de Vries is a research-based model of waves on oversimplified water exteriors established by [15]. The equation of KdV is a nonlinear, dispersive partial differential equation for a function $U(x,t)$ with two genuine factors, space x and time t with a general equation given as

$$\frac{\partial U}{\partial t} + a\frac{\partial^3 U}{\partial x^3} + bU\frac{\partial U}{\partial x} = 0, \qquad (1)$$

with initial conditions

$$U(x,0) = f(x), \qquad (2)$$

as elucidated in [6][18]. The equation of Korteweg-de Vries assumes a key job in the investigation of engendering of low profuseness water waves in oversimplified water bodies.

A few analysts have made various methodologies towards understanding the equations of Korteweg-de Vries of third order and nonlinear. Some of which incorporated Homotopy perturbation scheme and putting into use, Elzaki Transformation in [7], schematic iteration of Homotopy in [10], graphical sketch of equation with certain solution of Korteweg-de Vries in [6], numerical responses for a linearized equation of KdV on boundless space in [22], numerical clarifications of equation of KdV engaging the fully extended premise works in [9], numerical organizations of secluded singular waves for equation of KdV through finite component systems in [12], Elzaki Transformation with Adomian polynomial style in [11], numerical fix of the equation of Korteweg-de Vries by restricted contrast and Adomian decay style in [14], fundamental immitative of the n-soliton answers in [20], line solution approach for equation of KdV in [19], seasoned iterative approach for unraveling equations of Korteweg-de Vries in [21].

Notwithstanding, the mechanisms employed in the past frameworks was nonlinear terms linearization which apparently was somewhat tricky regarding calculation when perspectives, for example, irreversibility are being thought of. Hence, in this article, the amalgamation of Homotopy partition with Mahgoub transformation was engaged to contain the equations of KdV with third order and nonlinear. This game-plan was conceivable without linearizing the nonlinear perspectives and subsequently Mahgoub transformation is viewed as ceaselessly fitting showed up diversely corresponding to the past techniques. Mahgoub transformation is another vital transformation gotten through the old-style Fourier integral and was shown by Mohand Mahgoub in [16]. It was utilized to offer responses for partial differential conditions in [17] and was as well used to comprehend linear standard distinctive equations with changeable coefficients in [5], linear constitutive Volterra equations in [1]. It was moreover applied in treating populace development and rot issues in [4], used to offer response for second kind linear Volterra integro-divergent equations in [3], was similarly used in lighting up first kind linear Volterra integral equations in [2], and was likewise considered in [13] and was additionally used to contain equations of Burger's in [8].

2. MAHGOUB HOMOTOPY TRANSFORM ON KDV EQUATION: A THEORETICAL APPROACH

This perspective shows the viability of the proposed method by considering the general form of the equations identical Korteweg-de Vries of nonlinear with the underlying conditions.
Consider the general form of equation of KDV given as [7]

$$DU + RU + NU = 0, \quad (3)$$

with initial condition

$$u(x,0) = f(x), \quad (4)$$

where D is the operator of linear differential with respect to t, R is a differential operator with respect to x and N is a non-linear differential operator.
Taking the Mahgoub Transformation of equation (3), we achieve

$$M[DU(x,t) + RU(x,t) + NU(x,t)] = M[0],$$

$$M[DU(x,t)] + M[RU(x,t)] + M[NU(x,t)] = M[0]. \quad (5)$$

Engaging the differentiation property of Mahgoub Transform in equation (4) produces

$$vH(x,v) - vu(x,0) = -M[RU(x,t) + NU(x,t)],$$

$$H(x,v) = u(x,0) - \frac{M}{v}[RU(x,t) + NU(x,t)] \quad (6)$$

Taking the inverse Mahgoub Transform of both sides in equation (6) gives

$$M^{-1}[H(x,v)] = M^{-1}[u(x,0)] - M^{-1}\left\{\frac{M}{v}[RU(x,t) + NU(x,t)]\right\},$$

$$U(x,t) = f(x) - \left\{M^{-1}\left\{\frac{M}{v}\left[\sum_{n=0}^{\infty} a\frac{\partial^3 U}{\partial x^3} + bU\frac{\partial U}{\partial x}\right]\right\}\right\}. \quad (7)$$

Now by applying Homotopy perturbation method to equation (7), we acquire

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = f(x) - p\left\{M^{-1}\left\{\frac{M}{v}\left[\sum_{n=0}^{\infty} aRU(x,t) + bNU(x,t)\right]\right\}\right\}. \quad (8)$$

The nonlinear term in equation (8) can be disintegrated as

$$NU(x,t) = \sum_{n=0}^{\infty} p^n H_n(u), \quad (9)$$

where $H_n(u)$ is

$$H_n(u_0, u_1, u_2, \ldots u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}(p^i U_i),$$

then equation (7) becomes

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = f(x) - p\left\{M^{-1}\left\{\frac{M}{v}\left[\sum_{n=0}^{\infty} ap^n\frac{\partial^3 U_n}{\partial x^3} + \sum_{n=0}^{\infty} bp^n H_n(u)\right]\right\}\right\}, \quad (10)$$

where $H_n(u)$ are He's polynomials. The initial barely any segments of He's polynomials are

$$H_0(u) = u_0\frac{\partial U_0}{\partial x},$$

$$H_1(u) = u_0 \frac{\partial U_1}{\partial x} + u_1 \frac{\partial U_0}{\partial x},$$

$$H_2(u) = u_0 \frac{\partial U_2}{\partial x} + u_1 \frac{\partial U_1}{\partial x} + u_2 \frac{\partial U_0}{\partial x}, \dots$$

On comparing the coefficients of like powers of p, it procures

$$p^0: U_0(x,t) = f(x),$$

$$p^1: U_1(x,t) = -M^{-1}\left\{\frac{M}{v}[Ra + bH_0(u)]\right\},$$

$$p^2: U_2(x,t) = -M^{-1}\left\{\frac{M}{v}[Ra + bH_1(u)]\right\},$$

$$p^3: U_3(x,t) = -M^{-1}\left\{\frac{M}{v}[Ra + bH_2(u)]\right\},$$

$$p^4: U_4(x,t) = -M^{-1}\left\{\frac{M}{v}[Ra + bH_3(u)]\right\} \dots$$

Hence, the approximate solution of equation (3) becomes

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + \cdots \qquad (11)$$

Equation (11) is presently the approximate solution of KdV equation (3) which converges to the exact solution.

3. APPLICATION

The adequacy of the merge of transformation of Mahgoub with Homotopy perturbation will be shown by understanding three instances of equations of Korteweg-de Vries of third order and nonlinear utilizing the proposed style.

Example 1 [7]:

Consider the equation $\frac{\partial U}{\partial t} + a\frac{\partial^3 U}{\partial x^3} + bU\frac{\partial U}{\partial x} = 0$, where $a = b = 1, i.e.$

$$\frac{\partial U}{\partial t} + \frac{\partial^3 U}{\partial x^3} + U\frac{\partial U}{\partial x} = 0, \qquad (12)$$

with initial condition

$$U(x,0) = (1-x). \qquad (13)$$

Solution:

Take Mahgoub transform of both sides in equation (12) to earn

$$M\left[\frac{\partial U}{\partial t} + \frac{\partial^3 U}{\partial x^3} + U\frac{\partial U}{\partial x}\right] = M[0],$$

using the differential property of Mahgoub transform, we acquire

$$vH(x,v) - vu(x,0) + M\left[\frac{\partial^3 U}{\partial x^3}\right] + M\left[U\frac{\partial U}{\partial x}\right] = 0,$$

$$vH(x,v) - vu(x,0) = -M\left[\frac{\partial^3 U}{\partial x^3} + U\frac{\partial U}{\partial x}\right],$$

$$H(x,v) = u(x,0) - \frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + U\frac{\partial U}{\partial x}\right]. \qquad (14)$$

Taking the inverse Mahgoub transform of equation (14), we achieve

$$M^{-1}[H(x,v)] = M^{-1}[u(x,0)] - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + U\frac{\partial U}{\partial x}\right]\right\},$$

$$U(x,t) = M^{-1}[1-x] - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + U\frac{\partial U}{\partial x}\right]\right\},$$

$$U(x,t) = (1-x) - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + U\frac{\partial U}{\partial x}\right]\right\}, \qquad (15)$$

Now applying homotopy perturbation method to equation (15), we obtain

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = (1-x) - p\left\{M^{-1}\left[\frac{M}{v}\left[\sum_{n=0}^{\infty} p^n H_n(u) + \sum_{n=0}^{\infty} p^n \frac{\partial^3 U}{\partial x^3}\right]\right]\right\}, \qquad (16)$$

where H_n are He's polynomial to be determined. Now the He's polynomials are

$$H_0(u) = u_0 \frac{\partial U_0}{\partial x},$$

$$H_1(u) = u_0 \frac{\partial U_1}{\partial x} + u_1 \frac{\partial U_0}{\partial x},$$

$$H_2(u) = u_0 \frac{\partial U_2}{\partial x} + u_1 \frac{\partial U_1}{\partial x} + u_2 \frac{\partial U_0}{\partial x}, \dots$$

On comparing the coefficients of like powers of p on both sides, we get

$$p^0: U_0(x,t) = f(x) = 1 - x,$$

$$H_0(u) = u_0 \frac{\partial U_0}{\partial x} = (1-x)(-1) = -(1-x).$$

$$\begin{aligned} p^1: U_1(x,t) &= -M^{-1}\left\{\frac{M}{v}\left[H_0(u) + \frac{\partial^3 U_0}{\partial x^3}\right]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[-(1-x)+0]\right\} \\ &= M^{-1}\left\{\frac{M}{v}[(1-x)]\right\} \\ &= M^{-1}\left\{\frac{(1-x)}{v}\right\} \\ &= (1-x)M^{-1}\left\{\frac{1}{v}\right\} = (1-x)t. \end{aligned}$$

$$\begin{aligned} H_1(u) &= u_0 \frac{\partial U_1}{\partial x} + u_1 \frac{\partial U_0}{\partial x} \\ &= (1-x)(-t) + t(1-x)(-1) \\ &= -2t(1-x). \end{aligned}$$

$$\begin{aligned} p^2: U_2(x,t) &= -M^{-1}\left\{\frac{M}{v}\left[H_1(u) + \frac{\partial^3 U_1}{\partial x^3}\right]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[-2t(1-x)+0]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[-2t(1-x)]\right\} \\ &= M^{-1}\left\{\frac{2(1-x)}{v}M[t]\right\} \\ &= 2(1-x)M^{-1}\left\{\frac{1}{v}.\frac{1}{v}\right\} \\ &= 2(1-x)M^{-1}\left\{\frac{1}{v^2}\right\} = 2(1-x)\frac{t^2}{2!} \\ &= t^2(1-x). \end{aligned}$$

$$\begin{aligned} H_2(u) &= u_0 \frac{\partial U_2}{\partial x} + u_1 \frac{\partial U_1}{\partial x} + u_2 \frac{\partial U_0}{\partial x} \\ &= (1-x)(-t^2) + (1-x)t(-t) \\ &\quad + t^2(1-x)(-1) \\ &= -3t^2(1-x). \end{aligned}$$

$$\begin{aligned} p^3: U_3(x,t) &= -M^{-1}\left\{\frac{M}{v}\left[H_2(u) + \frac{\partial^3 U_2}{\partial x^3}\right]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[-3t^2(1-x)+0]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[-3t^2(1-x)]\right\} \\ &= M^{-1}\left\{\frac{3(1-x)}{v}M[t^2]\right\} \\ &= 3(1-x)M^{-1}\left\{\frac{1}{v}.\frac{2}{v^2}\right\} \\ &= 6(1-x)M^{-1}\left\{\frac{1}{v^3}\right\} = 6(1-x)\frac{t^3}{3!} \\ &= t^3(1-x). \end{aligned}$$

Since

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + \cdots,$$

Then

$$U(x,t) = (1-x) + (1-x)t + (1-x)t^2 + (1-x)t^3 + \cdots,$$
$$U(x,t) = (1-x)[1 + t + t^2 + t^3 + \cdots].$$

Upon convergence,

$$U(x,t) = (1-x)\left[\frac{1}{1-t}\right] = \left[\frac{1-x}{1-t}\right]. \quad (17)$$

Equation (17) is the approximate solution of equation (12) and this solution agrees with the solution obtained from [7].

Example 2 [10]:

Consider the identical KdV equation

$$\frac{\partial U}{\partial t} + \frac{\partial^3 U}{\partial x^3} + 6U\frac{\partial U}{\partial x} = 0, \quad (18)$$

with initial condition

$$U(x,0) = x. \quad (19)$$

Solution:

Taking the Mahgoub transform of both sides in equation (18), we obtain

$$M\left[\frac{\partial U}{\partial t} + \frac{\partial^3 U}{\partial x^3} + 6U\frac{\partial U}{\partial x}\right] = M[0],$$

using the differential property of Mahgoub transform, we posses

$$vH(x,v) - vu(x,0) + M\left[\frac{\partial^3 U}{\partial x^3}\right] + M\left[6U\frac{\partial U}{\partial x}\right] = 0,$$

$$vH(x,v) - vu(x,0) = -M\left[\frac{\partial^3 U}{\partial x^3} + 6U\frac{\partial U}{\partial x}\right],$$

$$H(x,v) = u(x,0) - \frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + 6U\frac{\partial U}{\partial x}\right]. \quad (20)$$

Taking the inverse Mahgoub transform of equation (20), we acquire

$$M^{-1}[H(x,v)] = M^{-1}[u(x,0)] - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + 6U\frac{\partial U}{\partial x}\right]\right\},$$

$$U(x,t) = M^{-1}[x] - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + 6U\frac{\partial U}{\partial x}\right]\right\},$$

$$U(x,t) = (x) - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} + 6U\frac{\partial U}{\partial x}\right]\right\}, \quad (21)$$

Now applying Homotopy perturbation method to equation (21), we achieve

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = (x) - p\left\{M^{-1}\left[\frac{M}{v}\left[\sum_{n=0}^{\infty} 6p^n H_n(u) + \sum_{n=0}^{\infty} p^n \frac{\partial^3 U}{\partial x^3}\right]\right]\right\}, \quad (22)$$

where H_n are He's polynomial to be determined. Now the He's polynomials are

$$H_0(u) = u_0\frac{\partial U_0}{\partial x},$$

$$H_1(u) = u_0\frac{\partial U_1}{\partial x} + u_1\frac{\partial U_0}{\partial x},$$

$$H_2(u) = u_0\frac{\partial U_2}{\partial x} + u_1\frac{\partial U_1}{\partial x} + u_2\frac{\partial U_0}{\partial x}, \ldots$$

On comparing the coefficients of like powers of p on both sides, we get

$$p^0: U_0(x,t) = f(x) = x,$$

$$H_0(u) = u_0\frac{\partial U_0}{\partial x} = (x)(1) = x.$$

$$\begin{aligned} p^1: U_1(x,t) &= -M^{-1}\left\{\frac{M}{v}\left[6H_0(u) + \frac{\partial^3 U_0}{\partial x^3}\right]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[6x+0]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[(6x)]\right\} \\ &= -M^{-1}\left\{\frac{(6x)}{v}\right\} = -(6x)M^{-1}\left\{\frac{1}{v}\right\} \\ &= -6xt. \end{aligned}$$

$$\begin{aligned} H_1(u) &= u_0\frac{\partial U_1}{\partial x} + u_1\frac{\partial U_0}{\partial x} \\ &= (x)(-6t) + (-6xt)(1) \\ &= -12xt. \end{aligned}$$

$$\begin{aligned} p^2: U_2(x,t) &= -M^{-1}\left\{\frac{M}{v}\left[6H_1(u) + \frac{\partial^3 U_1}{\partial x^3}\right]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[6(-12xt)+0]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[-72xt]\right\} \\ &= M^{-1}\left\{\frac{72x}{v}M[t]\right\} \\ &= 72xM^{-1}\left\{\frac{1}{v}.\frac{1}{v}\right\} = 72xM^{-1}\left\{\frac{1}{v^2}\right\} \\ &= 72x\frac{t^2}{2!} \\ &= 36xt^2. \end{aligned}$$

$$\begin{aligned} H_2(u) &= u_0\frac{\partial U_2}{\partial x} + u_1\frac{\partial U_1}{\partial x} + u_2\frac{\partial U_0}{\partial x} \\ &= x(36t^2) + (-6xt)(-6t) + 36xt^2(1) = 108xt^2. \end{aligned}$$

$$\begin{aligned} p^3: U_3(x,t) &= -M^{-1}\left\{\frac{M}{v}\left[6H_2(u) + \frac{\partial^3 U_2}{\partial x^3}\right]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[6(108xt^2)+0]\right\} \\ &= -M^{-1}\left\{\frac{M}{v}[648xt^2]\right\} \\ &= -M^{-1}\left\{\frac{648x}{v}M[t^2]\right\} \\ &= -648xM^{-1}\left\{\frac{1}{v}.\frac{2}{v^2}\right\} \\ &= -1296xM^{-1}\left\{\frac{1}{v^3}\right\} = -1296x\frac{t^3}{3!} \\ &= -216xt^3. \end{aligned}$$

Since

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + \cdots,$$

then

$$U(x,t) = (x) - (6x)t + (36x)t^2 - (216x)t^3 + \cdots,$$

$$U(x,t) = x[1 - 6t + 36t^2 - 216t^3 + \cdots] = x[1 - 6t + (6t)^2 - (6t)^3 + \cdots],$$

in closed form, we have

$$U(x,t) = x\left[\frac{1}{1+6t}\right] = \left[\frac{x}{1+6t}\right]. \qquad (23)$$

Equation (23) which is the solution to equation (18) gives the same result as in [10].

Example 3 [10]:

Consider the homogenous KdV equation

$$\frac{\partial U}{\partial t} - 6U\frac{\partial U}{\partial x} + \frac{\partial^3 U}{\partial x^3} = 0, \qquad (24)$$

with initial condition

$$U(x,0) = 6x. \qquad (25)$$

Solution:

Taking the Mahgoub transform of both sides in equation (24), we obtain

$$M\left[\frac{\partial U}{\partial t} - 6U\frac{\partial U}{\partial x} + \frac{\partial^3 U}{\partial x^3}\right] = M[0],$$

using the differential property of Mahgoub transform, we get

$$vH(x,v) - vu(x,0) + M\left[\frac{\partial^3 U}{\partial x^3}\right] - M\left[6U\frac{\partial U}{\partial x}\right] = 0,$$

$$vH(x,v) - vu(x,0) = -M\left[\frac{\partial^3 U}{\partial x^3} - 6U\frac{\partial U}{\partial x}\right],$$

$$H(x,v) = u(x,0) - \frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} - 6U\frac{\partial U}{\partial x}\right]. \qquad (26)$$

Taking the inverse Mahgoub transform of equation (26), we get

$$M^{-1}[H(x,v)] = M^{-1}[u(x,0)] - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} - 6U\frac{\partial U}{\partial x}\right]\right\},$$

$$U(x,t) = M^{-1}[6x] - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} - 6U\frac{\partial U}{\partial x}\right]\right\},$$

$$U(x,t) = (6x) - M^{-1}\left\{\frac{1}{v}M\left[\frac{\partial^3 U}{\partial x^3} - 6U\frac{\partial U}{\partial x}\right]\right\}, \qquad (27)$$

Now applying homotopy perturbation method to equation (27), we obtain

$$\sum_{n=0}^{\infty} p^n U_n(x,t) = (6x) - p\left\{M^{-1}\left[\frac{M}{v}\left[\sum_{n=0}^{\infty} -6p^n H_n(u) + \sum_{n=0}^{\infty} p^n \frac{\partial^3 U}{\partial x^3}\right]\right]\right\}, \qquad (28)$$

where H_n are He's polynomial to be determined. Now the He's polynomials are

$$H_0(u) = u_0\frac{\partial U_0}{\partial x},$$

$$H_1(u) = u_0\frac{\partial U_1}{\partial x} + u_1\frac{\partial U_0}{\partial x},$$

$$H_2(u) = u_0\frac{\partial U_2}{\partial x} + u_1\frac{\partial U_1}{\partial x} + u_2\frac{\partial U_0}{\partial x}, \ldots$$

On comparing the coefficients of like powers of p on both sides, we get

$$p^0: U_0(x,t) = f(x) = 6x,$$

$$H_0(u) = u_0\frac{\partial U_0}{\partial x} = (6x)(6) = 36x.$$

$$p^1: U_1(x,t) = -M^{-1}\left\{\frac{M}{v}\left[-6H_0(u) + \frac{\partial^3 U_0}{\partial x^3}\right]\right\} = -M^{-1}\left\{\frac{M}{v}[-6(36x) + 0]\right\} = -M^{-1}\left\{\frac{M}{v}[(-216x)]\right\} = -M^{-1}\left\{\frac{(-216x)}{v}\right\} = (216x)M^{-1}\left\{\frac{1}{v}\right\} = 216xt.$$

$$H_1(u) = u_0 \frac{\partial U_1}{\partial x} + u_1 \frac{\partial U_0}{\partial x} = (6x)(216t) + (216xt)(6) = 1296xt + 1296xt = 2592xt.$$

$$p^2: U_2(x,t) = -M^{-1}\left\{\frac{M}{v}\left[-6H_1(u) + \frac{\partial^3 U_1}{\partial x^3}\right]\right\} = -M^{-1}\left\{\frac{M}{v}[-6(2592xt) + 0]\right\} = -M^{-1}\left\{\frac{M}{v}[-15552xt]\right\} = M^{-1}\left\{\frac{15552x}{v}M[t]\right\} = 15552xM^{-1}\left\{\frac{1}{v}.\frac{1}{v}\right\} = 15552xM^{-1}\left\{\frac{1}{v^2}\right\} = 15552x\frac{t^2}{2!} = 7776xt^2.$$

$$H_2(u) = u_0 \frac{\partial U_2}{\partial x} + u_1 \frac{\partial U_1}{\partial x} + u_2 \frac{\partial U_0}{\partial x} = 6x(7776xt^2) + (216xt)(216t) + 7776xt^2(6) = 139968xt^2.$$

$$p^3: U_3(x,t) = -M^{-1}\left\{\frac{M}{v}\left[-6H_2(u) + \frac{\partial^3 U_2}{\partial x^3}\right]\right\} = -M^{-1}\left\{\frac{M}{v}[-6(139968xt^2) + 0]\right\} = -M^{-1}\left\{\frac{M}{v}[-839808xt^2]\right\} = M^{-1}\left\{\frac{839808x}{v}M[t^2]\right\} = 839808xM^{-1}\left\{\frac{1}{v}.\frac{2}{v^2}\right\} = 2x(839808)M^{-1}\left\{\frac{1}{v^3}\right\} = 2x(839808)\frac{t^3}{3!} = 279936xt^3.$$

Since

$$U(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t) + U_3(x,t) + U_4(x,t) + \cdots,$$

then

$$U(x,t) = (6x) + (216x)t + (7776x)t^2 + (279936x)t^3 + \cdots,$$

$$U(x,t) = 6x[1 + 36t + (36t)^2 + (36t)^3 + \cdots].$$

In the closed form, the solution is given as

$$U(x,t) = 6x\left[\frac{1}{1-36t}\right] = \left[\frac{6x}{1-36t}\right]. \qquad (29)$$

Equation (29) which is the solution to equation (24) gives the same result as in [10].

CONCLUSIONS

In this paper, equations of Korteweg-de Vries (KdV) of third order and nonlinear were solved by the amalgamation of Mahgoub Transformation with Homotopy Perturbation style. The exhibition of the proposed method was tried and defended with three representative cases. The outcomes obtained indicated that the Mahgoub Homotopy Transformation method is a ground-breaking and effective system that gives increasingly reasonable solutions since it yielded results which conform to the exact approximations of the illustrative models. The fast assembly of the solutions coupled with minimal computational procedures presented the curiosity of this article. This technique which was legitimized to be effective in solving third order Korteweg-de Vries (KdV) equation can be applied to different general cases similar to equation (3).

REFERENCES

[1] **S. Aggarwal, N. Sharma, R. Chauhan** – *A New Application of Mahgoub Transform for Solving Linear Volterra Integral Equations*, Asian Resonance, 7(2), 46-48, 2018.

[2] **S. Aggarwal, N. Sharma, R. Chauhan** – *Application of Mahgoub Transform for solving linear Volterra integral equations of first kind*, Global Journal of Engineering Science and Researches, 5(9), 154-161, 2018.

[3] **S. Aggarwal, N. Sharma, R. Chauhan** – *Solution of Linear Volterra integrodifferential equations of the second kind using Mahgoub Transform*, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(5), 173-176, 2018.

[4] **S. Aggarwal, M. Pandey, N. Asthana, D. P. Singh, A. Kumar** - *Application of Mahgoub transform for solving population growth and decay problems*, Journal of Computer and Mathematical Sciences, 9(10), 1490-1496, 2018.

[5] **S. Aggarwal, N. Sharma, R. Chauhan, A. R. Gupta, A. Khandelwal** – *A New Application of Mahgoub Transform for Solving Linear Ordinary Differential Equations with Variable Co-efficient*, Journal of Computer and Mathematical Sciences, 9(6), 520-525, 2018.

[6] **K. Brauer** – *The Korteweg-de vries equation: history, exact solutions and graphical representation*, University of Osnabrück/Germany1, 2000.

[7] **S. S. Chavan, M. M. Panchal** – *Solution of third order Korteweg-de Vries equation by homotopy perturbation method using elzaki transform*, International Journal for research in applied science and engineering technology, 2(7), 366-369, 2014.

[8] **O. A. Dehinsilu, O. S. Odetunde, O. O. Olubanwo** – *Analytical Solution of One-Dimensional Burger's Equation with The Help of Mahgoub Transform Method*, Annals. Computer Science Series, 17(2), 305-314, 2019.

[9] **İ. Dağ, Y. Dereli** – *Numerical solutions of KdV equation using radial basis functions*, Applied Mathematical Modelling, 32(4), 535-546, 2008.

[10] **M. H. Eljaily, M. E. Tarig** – *Homotopy perturbation transform method for solving Korteweg-devries (Kdv) equation*, Pure and Applied Mathematics Journal, 4(6), 264-268, 2015.

[11] **O. E. Ige, M. Heilio, R. A. Oderinu** – *Adomian polynomial and Elzaki transform method of solving third order Korteweg-De Vries equations*, Global Journal of Pure and Applied Mathematics, 15(3), 261-277, 2019.

[12] **S. Kapoor, S. Rawat, S. Dhawan** – *Numerical investigation of separated solitary waves solution for KdV equation through finite element technique*, International Journal of Computer Applications, 40(14), 27-33, 2012.

[13] **S. B. Kiwne, S. M. Sonawane** – *Mahgoub transform fundamental properties and applications*, International Journal of Computer and Mathematical Sciences, 7(2), 500-511, 2018.

[14] **O. Kolebaje, O. Oyewande** – *Numerical solution of the korteweg de vries equation by finite difference and adomian decomposition method*, International Journal of Basic and Applied Sciences, 1(3), 321-335, 2012.

[15] **D. J. Korteweg, G. De Vries** – *On the change of form of Long Waves Advancing in a Regular Canal and on a New Type of Long Stationary Waves*, Philosophical Magazine, 39(5), 422-443, 1895.

[16] **M. A. Mahgoub** – *The New Integral Transform "Mahgoub Transform"*, Advances in Theoretical and Applied Mathematics, 11(4), 391-398, 2016.

[17] **M. A. Mahgoub, A. A. Alshikh** – *An Application of New Integral Transform "Mahgoub Transform" to Partial Differential Equation*, Mathematical Theory and Modeling, 7(1), 7-9, 2017.

[18] **P. S. Riseborough** – *The Korteweg-de Vries equation: its place in the development of nonlinear physics*, Philosophical Magazine, 91(6), 997-1006, 2011.

[19] **W. E. Schiesser** – *Method of lines solution of the Korteweg-de Vries equation*, Computers & Mathematics with Applications, 28(10-12), 147-154, 1994.

[20] **B. R. Seymour, E. Varley**– *A Simple Derivation of the N-Soliton Solutions to the Korteweg-de Vries Equation*, SIAM Journal on Applied Mathematics, 58(3), 904-911, 1998.

[21] **S. M. Yassein, A. A. Aswhad** – *Efficient Iterative Method for Solving Korteweg-de Vries Equations*, Iraqi Journal of Science, 60(7), 1575-1583, 2019.

[22] **C. Zheng, X. Wen, H. Han** – *Numerical solution to a linearized KdV equation on unbounded domain*, Numerical Methods for Partial Differential Equations: An International Journal, 24(2), 383-399, 2008.