# CONSTRUCTION OF ROW-COLUMN DESIGNS FROM A CLASS OF NESTED BALANCED INCOMPLETE BLOCK DESIGNS

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***ABSTRACT:*** *Designs that with the ability to remove heterogeneity in the experimental material, have received considerable attention in the literature. By convention, the parameters of row-column designs constructed in this paper are v, b, p, q which respectively denote the number of treatments, rectangular blocks, rows and columns; and indeed, each of the rectangular blocks has nested within it, p rows and q columns. Designs of series I and II which are block designs are re-configured in a systematic manner in this paper to give block designs with the ability to remove heterogeneity in multiple directions, namely row and column; and these row and column are also nested within each of available rectangular blocks. All the designs constructed are balanced with respect to two of the three components of the resulting designs namely, rectangular block and, column. This is consequently reflected in the concurrence of treatment pairs that were obtained for the row ($\lambda_{ijr}$), column ($\lambda_{ijc}$), and rectangular block ($\lambda_{ijb}$) components of the designs respectively. Although, there were cases when some pairs of treatments do not occur together in the same block, row and column simultaneously, this does not violate connectedness property for all the designs constructed. Row-column designs constructed in this paper are for even and odd number of treatments, that is, v = 6, 7, 8, 9, 10, and 11. All the designs constructed are less restrictive with the attendant reduction in the required number of experimental units.*

***KEYWORDS****: Nested design; Heterogeneity; Row and Column Blocking Variables; Concurrence of Treatment Pairs.*

## 1. INTRODUCTION

One of the major concerns of any experimenter is to have reliable results in which the variability that is present in the experimental units or plots would be reduced considerably. In an attempt to control this variability, one of the commonly used tools is grouping of the experimental units into different homogenous groups, such that each group is referred to as a block. A block experiment is utilised for the purpose of estimating the effect of certain treatments, and be that as it may, treatments are of primary interest for comparison by the experimenter with due cognisance of suitable environment within which they are to be compared. Suitable environment is usually provided when the experimental material is grouped into relatively homogeneous blocks. These set of homogeneous blocks represents a single system of blocks with the characteristic feature of removing heterogeneity in one direction.

There are many ways of blocking the experimental units in a comparative experiment with v treatments. In basic experimental designs, treatments are allocated at random in each block, because experimental units or plots in each block are assumed to be more or less uniform. When all treatments can be accommodated within each block in a design, then the design is referred to as complete block design. Sometimes it is difficult, inconvenient or impossible to have blocks that can accommodate all treatments in each block, and as a consequence the experimenter has to contend with the use of incomplete blocks. For instance, there can be a block in which the available number of plots may not be enough to accommodate all treatments, as evident in the following illustration: an agricultural field experiment in which the size of a block of land may be too small to accommodate the required number of plots; and in a chemical engineering experiment, a number of trials that can be conducted within a specified interval of time may be limited due to available resources or practical considerations or both. The foregoing illustrations have inherent constraints that may necessitate the use of incomplete blocks.

In general, Incomplete Block Designs (IBD's) involve the allocation of a number of treatments into blocks whose size is less than the number of treatments. Incomplete block design was introduced by Yates (1936), and these are designs that are arranged in groups or blocks that are smaller than a complete replication. In order to eliminate heterogeneity to a greater extent, then it is possible to use incomplete block designs that are suitable for eliminating heterogeneity in more than one direction. Furthermore, incomplete block designs can be classified according to the number of times in which pairs of treatments occur together. It could be possible for pairs of treatments to occur together the same number of times or different number of times in the same block, with the former and the latter being commonly referred to as, balanced incomplete block designs (BIBDs) and partially balanced incomplete block designs (PBIBDs), respectively.

A balanced incomplete block designs with parameters v, b, r, k, λ is a block design with ‘v’ treatments and ‘b’ blocks of size ‘k’ each, such that every treatment occurs in exactly ‘r’ blocks and that any two distinct treatments occur together in exactly ‘λ’ blocks. More importantly, an improvement on this, for the purpose of further reduction in variability may result in designs with two systems of blocks, an example of such designs is Nested Balanced Incomplete Block Design (NBIBD). NBIBD is a design with two system of blocks, the second nested within the first; such that ignoring either system leaves a balanced incomplete block design whose blocks are those of the other system. This paper indeed adopts different NBIB designs for Series-I and Series-II by Saka and Adeleke (2015) to construct row-column designs for a number of parameter combinations.

## 2. ROW-COLUMN DESIGNS

Designs with appropriate blocking structure that enhance efficient use of experimental materials to date, has received considerable attention. Generally, the notation (v, b, k) defines a block design consisting of v treatments which are made up of k units per block. These designs could be called a 1-dimensional block designs because heterogeneity is removed in one direction. Example 1, gives a 1-dimensional block design with v=6, b=4, k=3.

**Example 1:**

Table 1. Block design of size (6, 4, 3)

| Block Unit | Block number | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1 | 0 | 3 | 0 | 0 |
| 2 | 1 | 4 | 3 | 2 |
| 3 | 2 | 5 | 4 | 5 |

On the other hand a row-column design with more than one rectangular block is capable of removing heterogeneity with respect to the following: rectangular block; row; and column. A row-column design of size (v, b, p, q) has v treatments allocated to b block, each block consisting of pq units further grouped into p row and q columns. Example 2, that follows gives a row-column design with v=6, b=3, p=2 q=3. Here the size of rectangular block say, $k_1$=6; the size of the row blocking variable say, $k_2$=3; and the size of the column blocking variable say, $k_3$=2.

**Example 2: Row-column design of size (6, 3, 2, 3)**

0 1 2 | 0 3 4 | 0 3 5 | 3 4 6 | 5 2 1 | 2 4 1

Several classes of row-column designs are given by a number of authors, see Agrawal and Prasad (1982), Ipinyomi and John (1985), Ipinyomi and Adeleke (1989). Importantly. a resolvable row-column design, which is of the type in Example 2 above, is such that a systematic allocation and arrangement of treatments into sets enhances a complete replication within each set of blocks. As a consequence the p rows and q columns are nested within each rectangular block.

## 3. BALANCED INCOMPLETE BLOCK BUILDING BLOCKS FOR ROW-COLUMN DESIGNS

The methodology for design construction adopted in this paper is premised on already existing designs with two sets of blocks, such that a set of small blocks or rather sub-blocks are nested in a large block. The process involves steps that are carefully and systemically implemented, and at the same time sustaining the symmetry of the initial nested balanced incomplete block designs. Similar methodology was adopted to a set of BIBDs provided in Cochran & Cox (1957 pp. 473) by Adeleke & Ipinyomi (1999) to construct optimal row-column design of size (10,18, 2,5) for ten treatments, while other designs with fewer number of blocks were also obtained by varying design parameters. This paper however uses nested BIBD’s obtained of Saka & Adeleke (2015) to obtain a class of row column designs that are balanced with respect to the two components of the design namely column and rectangular block. Designs that are to be constructed in the section that follows immediately are for the number of treatments, v = 6, 7, 8, 9, 10, and 11. The concurrence for all the treatments are obtained for each of the design constructed in order to reveal the symmetry inherent in the three components of the designs. The reference or initial designs are nested balanced incomplete block designs and this has made the parameters of the resulting row-column designs to be restricted in some sense. This is for the purpose of ensuring without any hindrance seamless alternating between the reference NBIBD’s and the resulting row-column designs. Designs with different concurrencies are in some special situations found to be of choice to the experimenter, see for example Chigbu et al. (2003, and 2008). Indeed, two or more block designs may appear similar in all respect but for their concurrence.

## 4. CONSTRUCTION OF ROW-COLUMN DESIGNS USING NBIB DESIGNS

In this section, different designs on NBIB (Series-I and Series-II) given by Saka and Adeleke (2015) are utilised for the purpose of constructing row-column designs for a number of parameter combinations.

### 4.1 Designs of NBIB for Series-I arranged as Row-Column Designs

Design 1:
A row column design of size (7, 7, 2, 3)

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 2 3 | 2 3 4 | 3 4 5 | 4 5 6 | 5 6 7 | 6 7 1 | 7 1 2 |
| 6 5 4 | 7 6 5 | 1 7 6 | 2 1 7 | 3 2 1 | 4 3 2 | 5 4 3 |

Design 1 was obtained from nested balanced incomplete design of Series 1 with v=7 and t=3.

Design 2:
A row column design of size (9, 9, 2, 4)

| | | | | | |
|---|---|---|---|---|---|
| 1 2 3 4 | 2 3 4 5 | 3 4 5 6 | 4 5 6 7 | 5 6 7 8 | 6 7 8 9 |
| 8 7 6 5 | 9 8 7 6 | 1 9 8 7 | 2 1 9 8 | 3 2 1 9 | 4 3 2 1 |
| 7 8 9 1 | 8 9 1 2 | 9 1 2 3 | | | |
| 5 4 3 2 | 6 5 4 3 | 7 6 5 4 | | | |

Design 2 was obtained from nested balanced incomplete design of Series 1 with v=9 and t=4.

Design 3:
A row column design of size (11, 11, 2, 5)

| | | | | | |
|---|---|---|---|---|---|
| 1 2 3 4 5 | 2 3 4 5 6 | 3 4 5 6 7 | 4 5 6 7 8 | 5 6 7 8 9 | 6 7 8 9 10 |
| 10 9 8 7 6 | 11 10 9 8 7 | 1 11 10 9 8 | 2 1 11 10 9 | 3 2 1 11 10 | 4 3 2 1 11 |
| 7 8 9 10 11 | 8 9 10 11 1 | 9 10 11 1 2 | 10 11 1 2 3 | 11 1 2 3 4 | |
| 5 4 3 2 1 | 6 5 4 3 2 | 7 6 5 4 3 | 8 7 6 5 4 | 9 8 7 6 5 | |

Design 3 was obtained from nested balanced incomplete design of Series 1 with v=11 and t=5.

Table 2: Concurrence of treatment pairs for designs with v = 7, 9, 11

| Design 1: Design of size (7,7, 2, 3) | | | | Design 2: Design of size (9,9,2,4) | | | | Design 3: Design of size (11,11, 2, 5) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijc}$ | $\lambda_{ijr}$ | Treatment pairs | $\lambda_{ijb}$ | $\lambda_{ijc}$ | $\lambda_{ijr}$ | Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijc}$ | $\lambda_{ijr}$ |
| 12 | 5 | 1 | 4 | 12 | 7 | 1 | 6 | 12 | 9 | 1 | 8 |
| 13 | 5 | 1 | 2 | 13 | 7 | 1 | 4 | 13 | 9 | 1 | 6 |
| 14 | 5 | 1 | 0 | 14 | 7 | 1 | 2 | 14 | 9 | 1 | 4 |
| 15 | 5 | 1 | 0 | 15 | 7 | 1 | 0 | 15 | 9 | 1 | 2 |
| 16 | 5 | 1 | 2 | 16 | 7 | 1 | 0 | 16 | 9 | 1 | 0 |
| 17 | 5 | 1 | 3 | 17 | 7 | 1 | 2 | 17 | 9 | 1 | 0 |
| 23 | 5 | 1 | 3 | 18 | 7 | 1 | 4 | 18 | 9 | 1 | 2 |
| 24 | 5 | 1 | 2 | 19 | 7 | 1 | 6 | 19 | 9 | 1 | 4 |
| 25 | 5 | 1 | 0 | 23 | 7 | 1 | 6 | 110 | 9 | 1 | 6 |
| 26 | 5 | 1 | 0 | 24 | 7 | 1 | 4 | 111 | 9 | 1 | 8 |
| 27 | 5 | 1 | 2 | 25 | 7 | 1 | 2 | 23 | 9 | 1 | 8 |
| 34 | 5 | 1 | 4 | 26 | 7 | 1 | 0 | 24 | 9 | 1 | 6 |
| 35 | 5 | 1 | 2 | 27 | 7 | 1 | 0 | 25 | 9 | 1 | 4 |
| 36 | 5 | 1 | 0 | 28 | 7 | 1 | 2 | 26 | 9 | 1 | 2 |
| 37 | 5 | 1 | 0 | 29 | 7 | 1 | 2 | 27 | 9 | 1 | 0 |
| 45 | 5 | 1 | 4 | 34 | 7 | 1 | 6 | 28 | 9 | 1 | 0 |
| 46 | 5 | 1 | 2 | 35 | 7 | 1 | 4 | 29 | 9 | 1 | 2 |
| 47 | 5 | 1 | 0 | 36 | 7 | 1 | 2 | 210 | 9 | 1 | 4 |
| 56 | 5 | 1 | 4 | 37 | 7 | 1 | 0 | 211 | 9 | 1 | 6 |
| 57 | 5 | 1 | 2 | 38 | 7 | 1 | 0 | 34 | 9 | 1 | 8 |
| 67 | 5 | 1 | 4 | 39 | 7 | 1 | 2 | 35 | 9 | 1 | 6 |
| | | | | 45 | 7 | 1 | 6 | 36 | 9 | 1 | 4 |
| | | | | 46 | 7 | 1 | 4 | 37 | 9 | 1 | 2 |
| | | | | 47 | 7 | 1 | 2 | 38 | 9 | 1 | 0 |
| | | | | 48 | 7 | 1 | 0 | 39 | 9 | 1 | 0 |
| | | | | 49 | 7 | 1 | 0 | 310 | 9 | 1 | 2 |
| | | | | 56 | 7 | 1 | 6 | 311 | 9 | 1 | 4 |
| | | | | 57 | 7 | 1 | 4 | 45 | 9 | 1 | 8 |
| | | | | 58 | 7 | 1 | 2 | 46 | 9 | 1 | 6 |
| | | | | 59 | 7 | 1 | 0 | 47 | 9 | 1 | 4 |
| | | | | 67 | 7 | 1 | 6 | 48 | 9 | 1 | 2 |
| | | | | 68 | 7 | 1 | 4 | 49 | 9 | 1 | 0 |
| | | | | 69 | 7 | 1 | 2 | 510 | 9 | 1 | 0 |
| | | | | 78 | 7 | 1 | 6 | 511 | 9 | 1 | 0 |
| | | | | 79 | 7 | 1 | 4 | 67 | 9 | 1 | 8 |
| | | | | 89 | 7 | 1 | 6 | 68 | 9 | 1 | 6 |
| | | | | | | | | 69 | 9 | 1 | 4 |
| | | | | | | | | 610 | 9 | 1 | 2 |
| | | | | | | | | 611 | 9 | 1 | 0 |
| | | | | | | | | 78 | 9 | 1 | 8 |
| | | | | | | | | 79 | 9 | 1 | 6 |
| | | | | | | | | 710 | 9 | 1 | 4 |
| | | | | | | | | 711 | 9 | 1 | 2 |
| | | | | | | | | 89 | 9 | 1 | 8 |
| Design 1: Design of size (7,7,2,3) | | | | Design 2: Design of size (9,9,2,4) | | | | Design 3: Design of size (11,11, 2, 5) | | | |
| Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijc}$ | $\lambda_{ijr}$ | Treatment pairs | $\lambda_{ijb}$ | $\lambda_{ijc}$ | $\lambda_{ijr}$ | Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijc}$ | $\lambda_{ijr}$ |
| | | | | | | | | 810 | 9 | 1 | 6 |
| | | | | | | | | 811 | 9 | 1 | 4 |
| | | | | | | | | 910 | 9 | 1 | 8 |
| | | | | | | | | 911 | 9 | 1 | 6 |
| | | | | | | | | 1011 | 9 | 1 | 8 |

**Key:** $\lambda_{ijb}$ $\lambda_{ijc}$ $\lambda_{ijr}$ denote number of times that treatments i and j occur together in the same block, column, and row respectively.

### 4.2 Designs of NBIB for Series-II arranged as Row-Column Designs

Design 4:
A row column design of size (6, 5, 2, 3)

| | | | | |
|---|---|---|---|---|
| 1 2 3 | 2 3 4 | 3 4 5 | 4 5 1 | 5 1 2 |
| 6 5 4 | 6 1 5 | 6 2 1 | 6 3 2 | 6 4 3 |

Design 4 was obtained from nested balanced incomplete design of Series II with v=6 and t=3.

Design 5:
A row column design of size (8, 7, 4, 2)

| | | | | | |
|---|---|---|---|---|---|
| 1 2 3 4 | 2 3 4 5 | 3 4 5 6 | 4 5 6 7 | 5 6 7 1 | 6 7 1 2 |
| 8 7 6 5 | 8 1 7 6 | 8 2 1 7 | 8 3 2 1 | 8 4 3 2 | 8 5 4 3 |
| 7 1 2 3 | | | | | |
| 8 6 5 4 | | | | | |

Design 5 was obtained from nested balanced incomplete design of Series II with v=8 and t=4.

Design 6:
A row column design of size (10, 9, 5, 2)

| | | | | | |
|---|---|---|---|---|---|
| 1 2 3 4 5<br>10 9 8 7 6 | 2 3 4 5 6<br>10 1 9 8 7 | 3 4 5 6 7<br>10 2 1 9 8 | 4 5 6 7 8<br>10 3 2 1 9 | 5 6 7 8 9<br>10 4 3 2 1 | 6 7 8 9 1<br>10 4 5 1 2 |
| 7 8 9 1 2<br>10 6 5 4 3 | 8 9 1 2 3<br>10 7 6 5 4 | 9 1 2 3 4<br>10 8 7 6 5 | | | |

Design 6 was obtained from nested balanced incomplete design of Series II with v=10 and t=5.

Table 3: Concurrence of treatment pairs for designs with v = 6, 8, 10

| Design 4: Design of size (6,5,2,3) | | | | Design 5: Design of size (8,7,2,4) | | | | Design 6: Design of size (10,9,2,5) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijr}$ | $\lambda_{ijc}$ | Treatment pairs | $\lambda_{ijb}$ | $\lambda_{ijr}$ | $\lambda_{ijc}$ | Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijr}$ | $\lambda_{ijc}$ |
| 12 | 5 | 1 | 3 | 12 | 7 | 1 | 5 | 12 | 9 | 1 | 8 |
| 13 | 5 | 1 | 1 | 13 | 7 | 1 | 3 | 13 | 9 | 1 | 5 |
| 14 | 5 | 1 | 1 | 14 | 7 | 1 | 1 | 14 | 9 | 1 | 4 |
| 15 | 5 | 1 | 3 | 15 | 7 | 1 | 1 | 15 | 9 | 1 | 2 |
| 16 | 5 | 1 | 2 | 16 | 7 | 1 | 3 | 16 | 9 | 1 | 0 |
| 23 | 5 | 1 | 3 | 17 | 7 | 1 | 5 | 17 | 9 | 1 | 2 |
| 24 | 5 | 1 | 1 | 18 | 7 | 1 | 3 | 18 | 9 | 1 | 5 |
| 25 | 5 | 1 | 1 | 23 | 7 | 1 | 5 | 19 | 9 | 1 | 6 |
| 26 | 5 | 1 | 2 | 24 | 7 | 1 | 3 | 110 | 9 | 1 | 5 |
| 34 | 5 | 1 | 3 | 25 | 7 | 1 | 1 | 23 | 9 | 1 | 6 |
| 35 | 5 | 1 | 1 | 26 | 7 | 1 | 1 | 24 | 9 | 1 | 5 |
| 36 | 5 | 1 | 2 | 27 | 7 | 1 | 3 | 25 | 9 | 1 | 3 |
| 45 | 5 | 1 | 3 | 28 | 7 | 1 | 3 | 26 | 9 | 1 | 0 |
| 46 | 5 | 1 | 2 | 34 | 7 | 1 | 5 | 27 | 9 | 1 | 1 |
| 56 | 5 | 1 | 2 | 35 | 7 | 1 | 3 | 28 | 9 | 1 | 3 |
| | | | | 36 | 7 | 1 | 1 | 29 | 9 | 1 | 5 |
| | | | | 37 | 7 | 1 | 1 | 210 | 9 | 1 | 4 |
| | | | | 38 | 7 | 1 | 3 | 34 | 9 | 1 | 6 |
| | | | | 45 | 7 | 1 | 5 | 35 | 9 | 1 | 4 |
| | | | | 46 | 7 | 1 | 3 | 36 | 9 | 1 | 3 |
| | | | | 47 | 7 | 1 | 1 | 37 | 9 | 1 | 1 |
| | | | | 48 | 7 | 1 | 3 | 38 | 9 | 1 | 1 |
| | | | | 56 | 7 | 1 | 5 | 39 | 9 | 1 | 3 |
| | | | | 57 | 7 | 1 | 3 | 310 | 9 | 1 | 3 |
| | | | | 58 | 7 | 1 | 3 | 45 | 9 | 1 | 7 |
| | | | | 67 | 7 | 1 | 5 | 46 | 9 | 1 | 5 |
| | | | | 68 | 7 | 1 | 3 | 47 | 9 | 1 | 2 |
| | | | | 78 | 7 | 1 | 3 | 48 | 9 | 1 | 0 |
| | | | | | | | | 49 | 9 | 1 | 1 |
| Design 4: Design of size (6,5,2,3) | | | | Design 5: Design of size (8,7,2,4) | | | | Design 6: Design of size (10,9,2,5) | | | |
| Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijr}$ | $\lambda_{ijc}$ | Treatment pairs | $\lambda_{ijb}$ | $\lambda_{ijr}$ | $\lambda_{ijc}$ | Treatment Pairs | $\lambda_{ijb}$ | $\lambda_{ijr}$ | $\lambda_{ijc}$ |
| | | | | | | | | 410 | 9 | 1 | 4 |
| | | | | | | | | 56 | 9 | 1 | 7 |
| | | | | | | | | 57 | 9 | 1 | 5 |
| | | | | | | | | 58 | 9 | 1 | 3 |
| | | | | | | | | 59 | 9 | 1 | 1 |
| | | | | | | | | 510 | 9 | 1 | 4 |
| | | | | | | | | 67 | 9 | 1 | 7 |
| | | | | | | | | 68 | 9 | 1 | 5 |
| | | | | | | | | 69 | 9 | 1 | 3 |
| | | | | | | | | 610 | 9 | 1 | 4 |
| | | | | | | | | 78 | 9 | 1 | 7 |
| | | | | | | | | 79 | 9 | 1 | 5 |
| | | | | | | | | 710 | 9 | 1 | 4 |
| | | | | | | | | 89 | 9 | 1 | 7 |
| | | | | | | | | 810 | 9 | 1 | 4 |
| | | | | | | | | 910 | 9 | 1 | 4 |

**Key:** $\lambda_{ijb}$ $\lambda_{ijc}$ $\lambda_{ijr}$ denote number of times that treatments i and j occur together in the same block, column, and row respectively.

## 5. DISCUSSION OF DESIGNS CONSTRUCTED

The method of construction adopted in Saka and Adeleke (2015), is such that sub-blocks within each large block are also balanced incomplete block designs.

The procedure is rather restrictive and this property is also inherent in the resulting row-column designs that have been constructed in the preceding section of this paper. The designs are balanced with respect to block and row components, while the column component partially balanced format. In particular, the values of $\lambda$, number of times each pair of treatments occurs in the same block (row or column), for:

Design 1, with the size (7, 7, 2, 3) are, $\lambda_{ijb} = 5$, $\lambda_{ijc} = 1$, $\lambda_{ijr} = 0, 2, 3, 4$;

Design 2, with the size (9, 9, 2, 4) are, $\lambda_{ijb} = 7$, $\lambda_{ijc} = 1$, $\lambda_{ijr} = 0, 2, 4, 6$;

Design 3, with the size (11, 11, 2, 5) are, $\lambda_{ijb} = 9$, $\lambda_{ijc} = 1$, $\lambda_{ijr} = 0, 2, 4, 6, 8$;

Design 4, with the size (6, 5, 2, 3) are, $\lambda_{ijb} = 5$, $\lambda_{ijc} = 1$, $\lambda_{ijr} = 1, 2, 3$;

Design 5, with the size (8, 7, 2, 4) are, $\lambda_{ijb} = 7$, $\lambda_{ijc} = 1$, $\lambda_{ijr} = 1, 3, 5$;

and Design 6, with the size (10, 9, 2, 5) are, $\lambda_{ijb} = 9$, $\lambda_{ijc} = 1$, $\lambda_{ijr} = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

All the foregoing designs are binary, however Designs 1, 2, and 3 have incomplete rectangular block, while Designs 4, 5, and 6 have complete rectangular blocks.

## CONCLUSION

The concern of this paper for the re-configuration of the series I and II designs of Saka and Adeleke (2015) to obtain row-column designs has been achieved. Infact, the row-column designs constructed does not sacrifice the features of the initial designs in the sense of the balance inherent therein, since seamless alternating between the initial designs and their corresponding row-column designs is maitained. Experimenters that are with the aim of contending with experimental material that exhibits heterogeneity in two directions can adopt any of the row-column designs constructed in this paper. The concurrence for all the treatments pairs as shown in the tables1 and 2 above, is a confirmation of the fact that all the designs constructed are balanced with respect to the rectangular block and row components, provided the number of treatments, v satisfies, $6 \leq v \leq 11$.

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