

MONTE CARLOS STUDY ON POWER RATES OF SOME HETEROSCEDASTICITY DETECTION METHODS IN LINEAR REGRESSION MODEL WITHOUT MULTICOLLINEARITY PROBLEM

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ABSTRACT: Violation of constant error variance assumption in any linear regression model leads to the problem of heteroscedasticity. In practices, variance of error terms are unequal and unknown in nature, but there is need to determine the presence or absence of this problem that do exist in unknown error term as a preliminary diagnosis on the set of data we are to analyze or perform hypothesis testing on. Although, there are several forms of heteroscedasticity and several detection methods of heteroscedasticity, but for any researcher to arrive at a reasonable and correct decision, best and consistent performed methods of heteroscedasticity detection under any forms or structured of heteroscedasticity must be determined. This paper consider seven heteroscedasticity structures that were originally proposed by different authors for developing statistical tools for heteroscedasticity detection in linear regression model. Nine heteroscedasticity detection methods were considered in order to determine some heteroscedasticity methods that are best to be used for the determination of unequal variance presence among the error terms in a linear regression model when there exist no forms of correlation between the exogenous variables of the model via the power of the test. In this work, Monte Carlo experiment was conducted one thousands (1000) times on a linear regression model with three predictor variable that exhibit no degree of multicollinearity ($\rho = 0$) and seven sample sizes ($n = 15, 20, 30, 50, 100$ and 250). The parameters of the model were specified to be $\beta_0 = 4$, $\beta_1 = 0.4$, $\beta_2 = 1.5$, $\beta_3 = 3.6$ and the various tests were examined at 0.1, 0.05 and 0.01 levels of significance. The Confidence interval criterion (C.I) was used to determine the performances of the methods. The study concluded that when power of a test is considered to determine the preferred methods of heteroscedasticity detection in a model where there exist no multicollinearity between the exogenous variables at significance level of $\alpha = 0.1$, BG test or GFQ test are the preferred methods, when testing under significance level of $\alpha = 0.05$, the preferred method to used is GFQ method, while at $\alpha = 0.01$, the preferred method to use in testing for the presence of heteroscedasticity is either BG method or NVST method.

KEYWORDS: Regression model, heteroscedasticity, heteroscedasticity detection methods, heteroscedasticity structures, significance levels, Confidence Interval and Power rates.

1. INTRODUCTION

Heteroscedasticity cause serious problems in econometrics data. The consequences of using Ordinary Least Square (OLS) estimator when there is heteroscedasticity also affects the population parameters that leads to unbiasedness but inefficient, biased variance estimates and invalid hypothesis. Given this fact, the detection of heteroscedasticity in a linear regression model needs to be identified. [1] opined that the effect of multicollinearity on type I error rates of the ordinary least square estimator is trivial in which the error rates exhibit no or little significance difference from the pre-selected level of significance. In evaluating a computationally simple asymptotic test there are heteroscedasticity detection methods proposed by Spearman [15], Rao [14], Goldfeld and Quandt [8], Breusch and Godfery [2], Park [13], Glejser [7], Breusch and Pagan [3], Harrison and Mc Cabe [10] and White [17]. These tests, originally designed for structural form and methods of detecting heteroscedasticity with various sample sizes under appropriate assumptions. These were used to test for the presence or absence of heteroscedasticity in linear regression models without multicollinearity problem. This paper attempts to determine the preferred heteroscedasticity detection methods via power rate among some methods of detecting heteroscedasticity in linear regression model when there is no multicollinearity problem in the linear regression model.

2. EXISTING HETEROSCEDASTICITY DETECTION METHODS UNDER PREVIEW

There are several existing methods for detecting the existence of heteroscedasticity in linear regression model. This paper considered nine existing methods of heteroscedasticity detection, the methods are:

2.1. Breusch-Pagan test(BP): Breusch and Pagan [3] developed a test used in examining the presence of heteroscedasticity in a linear regression model. The variance of the error term was tested from a regression and is dependent on the value of the independent variables. Breusch-Pagan illustrates this test by considering the following:

In linear regression model, multiple regressions assess relationship between one dependent variable and a set of independent variables. Ordinary Least Squares (OLS) Estimator is most popularly used to estimate the parameters of regression model. The estimator has some very attractive statistical properties which have made it one of the most powerful and popular estimators of regression model. A common violation in the assumption of classical linear regression model is the presence of heteroscedasticity. Heteroscedasticity is a situation that arises when the variance of the error term is not constant. The performance of OLS estimator is inefficient in the presence of heteroscedasticity even though it is still unbiased. It does not have minimum variance any longer Gujarati [9]. In literature, there are various methods existing in detecting heteroscedasticity. Among them is the Breusch-Pagan test. [3] developed a test used in examining the presence of heteroscedasticity in a linear regression model. The variance of the error term was tested from a regression and is dependent on the value of the independent variables. Breusch-pagan illustrates this test by considering the following regression model

$$Y = \beta_0 + \beta_1 X_1 + u \quad (1)$$

Suppose that we estimate the regression model and obtain from this fitted model a set of value for the residuals \hat{u} , OLS constrains these so that their mean is 0 and give the assumption that their variance does depend on the independent variables, an estimate of this variance can be obtained from the average of the squared values of the residuals. If the assumption is not held to be true, a simple model might be that the variance is linearly related to independent variable. Such a model can be examined by regressing the squared residuals on the independent variable, using an auxiliary regression equation of the form.

$$z \hat{u}^2 = \gamma_0 + \gamma_1 X + v \quad (2)$$

Breusch-pagan test is a chi-square test; the test statistic is distributed with $n\chi^2$ with k degrees of freedom. If the test statistic has a p-value below an appropriate threshold e.g $p < 0.05$ then the null hypothesis of homoscedasticity is rejected and heteroscedasticity assumed. The procedure under the classical assumptions, OLS is the best linear unbiased estimate (BLUE), i.e, it is unbiased and efficient.

It remains unbiased under heteroscedasticity, but efficiency is lost. Before deciding upon an estimation method, one may conduct the Breusch-pagan test to examine the presence of heteroscedasticity. The Breusch-pagan test is based on model of the type

$$\sigma^2 = h(z_i' \gamma) \quad (3)$$

For the variance of the observations where $z_i = (1, z_{2i}, \dots, z_{pi})$ explain the difference in the variances. The null hypothesis is equivalent to the (p-1) parameter restriction: $\gamma_2 = \dots = \gamma_p$

For Breusch-pagan test largrange multiplier (LM) yields the test statistic.

$$LM = \left(\frac{\partial l}{\partial \theta} \right)^T \left(-E \left[\frac{\partial^2 l}{\partial \theta \partial \theta^1} \right] \right)^{-1} \left(\frac{\partial l}{\partial \theta} \right) \quad (4)$$

This test is analogous to follow the simple three-step procedure:

Step1: Apply OLS in the model

$$Y = X\beta + \varepsilon \quad (5)$$

And compute the regression residuals.

Step2: Perform the auxiliary regression

$$\varepsilon_i^2 = \gamma_1 + \gamma_2 z_{2i} + \dots + \gamma_p z_{pi} + \eta_i \quad (6)$$

Where z could be partly replaced by independent variable X

Step 3: The test statistic is the result of the coefficient of determination of the auxiliary regression in step 2 and sample size n with $LM = nR^2$. The test statistic is asymptotically distributed as χ_{p-1}^2 under the null hypothesis of homoscedasticity. Finally, we assume to reject the null hypothesis and to highlight the presence of heteroscedasticity when LM-statistic is higher than the critical value.

2.2. Park Test(PT): Park [Par66] propose a LM test, the test assumes the proportionality between error variance and the square of the regressors. According to Gujarati, the Park LM test formulizes the graphical method by suggesting that σ^2 is a particular function of the explanatory variable X_i . Park illustrates this test by regressing the natural log of squared residuals against the independent variable, if the independent variable has a significant coefficient, the data is likely to be heteroscedasticity in nature.

In order to obtain the error term \hat{u}_i , we run a regression equation

$$Y_i = \beta_1 + \beta_2 X_{2i} + \dots + \beta_k X_{ki} + u_i \quad (7)$$

$$Var(u_i) = \sigma_i^2$$

by running the auxiliary regression we obtain the model below

$$\sigma_i^2 = \sigma^2 X_i^\beta e^v \quad (8)$$

We need to find the log

$$\ln \sigma_i^2 = \ln \sigma^2 + \beta \ln X_i + v_i \quad (9)$$

Where v_i the stochastic disturbance term, since σ_i^2 is not known, Park suggest using $\hat{\mu}_i^2$ as a proxy and run the following regression

$$\ln \mu_i^2 = \ln \sigma^2 + \beta \ln X_i + v_i = \alpha + \beta \ln X_i + v_i \quad (10)$$

If β turns out to be statistically significant, we there say that heteroscedasticity is present in the data and if it turns out to be insignificant, we may accept the assumption of homoscedasticity.

2.3. Spearman's Rank Correlation test(ST):

Spearman [15] used Spearman's Rank correlation and assumes that the variance of the disturbance term is either increasing or decreasing as X increases and there will be a correlation between the absolute size of the residuals and the size of X in an OLS regression. The data on X and the residuals are both ranked. The rank correlation coefficient is define

$$r_{X,e} = 1 - \left[\frac{6 \sum_i d_i^2}{n(n^2-1)} \right]; -1 \leq r \leq 1 \quad (11)$$

where d_i is the difference between the rank of X and the rank of ε in observations. i and n is the number of individual ranked. Under the assumption that the population correlation coefficient is 0, the rank correlation coefficient has a normal distribution with 0 mean and variance $1 / ((n - 1))$ in large sample. The appropriate test statistic is $r_{X,e}(\sqrt{n - 1})$ and the null hypothesis of homoscedasticity will be rejected at the 5% level if its absolute value is greater than 1.96 and at 1% level if its absolute values are greater than 2.58, using two tailed tests. The test can be performing with any of the levels if the explanatory variable is more than one. The preceding rank correlation coefficient can be used for heteroscedasticity. We assume the following model

$$Y_i = \beta_1 + \beta_2 X_i + u_i ; i = 1, 2, \dots, n \quad (12)$$

the following steps are involved in spearman's rank correlation test

Run the above regression and obtain the residual \hat{u}_i and take their absolute values $|\hat{u}_i|$

Arrange $|\hat{u}_i|$ and X_i in either increasing order or decreasing order and run Spearman's Rank Correlation coefficient by using formula

$$r_{X,\varepsilon} = 1 - 6 \left[\frac{\sum d_i^2}{n(n^2-1)} \right] \quad (13)$$

If there is a systematic relationship between u_i and X_i , the rank correlation coefficient between the two should be statistically significant in which heteroscedasticity can be suspected. Given the null hypothesis that the true population rank correlation coefficient is zero and that $n > 8$, it can be shown that

$$t = \frac{r_s \sqrt{n-2}}{\sqrt{1-r_s^2}} \sim t_{n-2} \text{ follows student's } t\text{-distribution}$$

with $(n-2)$ degree of freedom. Therefore, in an application the rank correlation coefficient is significant on the basis of the t-test, we do not reject the hypothesis that there is heteroscedasticity in the problem. We therefore reject the null hypothesis of heteroscedasticity whenever $t_o \geq t_{1-\alpha,(n-2)}$. If there are more than one explanatory variable, rank correlation can be computed between $|\varepsilon_i|$ and each of the explanatory variable separately and be tested using t_o .

2.4. Glejser test(GLJ):

Glejser [7] developed a test similar to the Park test, after obtaining the residual (\hat{u}_i) from the OLS regression. Glejser suggest that regressing the absolute value of the estimated residuals on the explanatory variables that is thought to be closely associated with the heteroscedastic variance and attempts to determine whether as the independent variable increase in size, the variance of the observed dependent variable increases. This is done by regressing the error term of the predicted model against the independent variable. A high t-statistic (or low prob-value) for the estimate coefficient of the independent variable(s) would indicate the presence of heteroscedasticity.

Glejser illustrates this test by considering the following steps:

Step1: Estimate original regression with OLS and find the sample residual ε_i

Step2: Regress the absolute value $|\varepsilon_i|$ on the explanatory variable that is associated with heteroscedasticity

Step3: Select the equation with the highest R^2 and lowest standard errors to represent heteroscedasticity

Step4: Perform a t-test on the equation selected from step3 on Y_i . If Y_i is statistically significant, reject the null hypothesis of homoscedasticity.

2.5. Goldfeld-Quandt test(GFQ):

Goldfeld and Quandt [8] developed an alternative test to LM test, applying this test requires to perform a sequence of intermediate stages. First step involves to arrange the observations either is ascending or in descending order. Another step aims to divide the ordered sequence into two equal sub-sequences by omitting an arbitrary number P of the central observation. Consequently, the two equal sub-sequences will summarize each of them a number of $\frac{(n-p)}{2}$ observations.

We then compute two different OLS regression the first one for the lowest values of X_i and the second for the highest values of X_i , in addition, obtain the

residual sum of squares (RSS) for each regression equation, RSS_1 for the lowest values of X_i and RSS_2 for the highest values of X_i . An F-statistic is calculated based on the following formula:

$$F = \frac{RSS_1}{RSS_2} \quad (14)$$

The F-statistics is distributed with $\frac{(N-P-2K)}{2}$ degrees of freedom for both numerator and denominator. Subsequently, compare the value obtained for the F-statistic with the tabulated values of F-critical for the specified number of degrees of freedom and a certain confidence level. If F-statistic is higher than F-critical, the null hypothesis of homoscedasticity is rejected and the presence of heteroscedasticity is confirmed.

2.6. Breusch-Godfrey test(BG): Breusch and Godfrey [2] developed a LM test of the null hypothesis of no heteroscedasticity against heteroscedasticity of the form $\sigma_t^2 = \sigma^2 h(z_t' \alpha)$, where z_t is a vector of independent variables. This vector contains the regressors from the original least square regression. The test is performed by completing an auxiliary regression of the squared residuals from the original equation on $(1, z_t)$. The test statistic follows a chi-square distribution with degrees of freedom equal to the number of z under the null hypothesis of no heteroscedasticity.

2.7. White's test(WT): White [17] proposed a statistical test that establishes whether the variance of the error in a regression model is constant. This test is generally, unrestricted and widely used for detecting heteroscedasticity in the residual from a least square regression. Particularly, White test is a test of heteroscedasticity in OLS residual. The null hypothesis is that there is no heteroscedasticity. The procedure for running the test is shows as follows: Given the model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \quad (15)$$

Estimate equation (8) and obtained the residual \hat{u}_i we then run the following auxiliary regression

$$\hat{u}_i^2 = b_1 + b_2 X_{2i} + b_3 X_{3i} + b_4 X_{2i}^2 + b_5 X_{3i}^2 + b_6 X_{2i} X_{3i} + v_i \quad (16)$$

The null hypothesis of homoscedasticity is $H_0: b_1 = b_2 = \dots = b_m = 0$ where H_0 highlights the fact that the variance of the residual is homoscedasticity i . e, $var(\varepsilon_i) = Var(Y_i) = \sigma^2$.

The alternative hypothesis is H_1 , it aims at the fact that the variance of the residual is heteroscedasticity $var(\varepsilon_i) = Var(Y_i) = \sigma_i^2$ that is at least one of the b_i 's is different from zero, the null hypothesis is rejected. The LM-statistic equal to nR^2 , this follows

a χ^2 distribution characterized by $m-1$, where n is the number of observation established to determine the auxiliary regression and R^2 is the coefficient of determination. Finally, we assume to reject the null hypothesis and to highlight the presence of heteroscedasticity when LM-statistic is higher than the critical value.

2.8. Harrison McCabe test(HM): Harrison and McCabe [10] proposes a test to check the heteroscedasticity of the residuals. The breakpoint in the variances is set by default to the half of the sample. The p-value is estimated-using simulation. If the binary quality measure is false, then the homoscedasticity hypothesis can be rejected with respect to the given level.

2.9. Non-Constant Variation Score test(NVST): Rao [14], Cox and Hinkley [5] develop a test of null hypothesis $H_0: E(\varepsilon^2/X_1, X_2, \dots, X_k) = \sigma^2$ against an alternative (H_1) hypothesis with a general functional form. We recall the central issue is whether $E(\varepsilon^2) = \sigma^2 w_i$ is related to X and X_i . Then, a simple strategy is to use OLS residuals to estimate disturbance and check the relationship between ε_i^2 and X_i and X_i^2 . Suppose that the relationship between ε^2 and X is linear

$$\varepsilon^2 = X\alpha + v \quad (17)$$

Then, we test $H_0: \alpha = 0$ against $H_1: \alpha \neq 0$ and base the test on how the squared OLS residual ε correlate with X .

3. MATERIALS AND METHOD

Consider the multiple linear regression model of the form:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_p X_{pt} + u_t \quad (18)$$

where, $u_t \sim N(0, \sigma_t^2)$; u_t is the error term and σ_t^2 is the heteroscedasticity variance that is considered. Y_t is the dependent variable, X_{pt} is the explanatory variables that contain multicollinearity and β_p is the regression coefficient of the model.

A Monte Carlo Experiment was performed 1000 times, in generating the data for the simulation study. On error variance containing heteroscedasticity structures considered:

$$\sigma_t^2 = \sigma^2 (X_{2t}^2)^2 \quad (19)$$

$$\sigma_t^2 = \sigma^2 (X_{2t}^2) \quad (20)$$

$$\sigma_t^2 = \sigma^2 (X_{2t}) \quad (21)$$

$$\sigma_t^2 = \sigma^2 [E(y_t)^2] \quad (22)$$

$$\sigma_t^2 = \sigma^2 [E(y_t)] \quad (23)$$

$$\sigma_t^2 = \sigma^2(1 + X_{2t}^2) \quad (24)$$

$$\sigma_t^2 = \sigma^2[\exp(\beta_0 + \delta\beta_1X_{1t} + \delta\beta_2X_{2t} + \delta\beta_3X_{3t})] \quad (25)$$

where $\delta = 0$ and 0.2 .

3.1. Generation of error term

The error term ε_i was generated to be normally distributed with mean zero and variance σ^2 , that is, $\varepsilon_i \sim N(0, \sigma^2)$.

The error term containing different explanatory variables, heteroscedasticity structure and dependent variable were generated.

3.2. Generation of explanatory variables

The procedure used by Mansson, Shukur and Kibira [12], Lukman and Ayinde [11] and Durogade [6], was adopted to generate explanatory variables in this study. This is given as:

$$X_{ti} = (1 - \rho^2)^{0.5} * z_{ti} + \rho z_{tp} \quad (26)$$

where $t = 1, 2, \dots, n$ and $i = 1, 2, \dots, p$, where z_{ti} is the independent standard normal distribution with mean zero and unit variance. ρ is the correlation between any two explanatory variables in this study multicollinearity level is zero i.e. $\rho = 0$, and p is the number of explanatory variables.

3.3. Generation of Dependant variables

The dependent variables was generated, equation (18) was used in conducting the Monte Carlo experiments. The true values of the model parameters were fixed as follows: $\beta_0 = 4$, $\beta_1 = 0.4$, $\beta_2 = 1.5$, $\beta_3 = 3.6$. The sample sizes varied from 15, 20, 30, 40, 50, 100 and 250. At a specified value of n , p , the fixed X 's are first generated; followed by the u_t , and the values of Y_t were then determined. Then Y_t and X 's were then treated as real life data set while the methods were applied.

3.4. Criterion For Comparison

At a particular α level a confidence interval was set for 10%, 5% and 1%, the number of times $\hat{\alpha}$ falls in between, the set confidence interval was counted over the sample size and heteroscedasticity structures.

The heteroscedasticity test with highest number of count is chosen to be the best test with interm of power of the test.

$$\hat{\alpha} = \frac{r}{R} \quad (27)$$

Where r is the number of times $\hat{\alpha}$ falls in between the confidence interval set at a particular significance level. While R is the number of times the experiment was carried out. The heteroscedasticity test with highest number of count is chosen to be the best.

3.5. Power Of A Statistical Test

Statistical Power of a test can be define as the probability of rejecting null hypothesis given that null hypothesis is false.

Type II error is an error committed in hypothesis testing when the null hypothesis that is wrong is been accepted. It is denoted by β , where $\beta = \text{prob}[\text{accepting } H_0/H_0 \text{ is false}]$ i.e the probability of accepting the null hypothesis (H_0) when it is false.

Power of a test is the probability of rejecting the null hypothesis when the null hypothesis is false. It is denoted as $1 - \beta$, where $1 - \beta = \text{prob}[\text{rejecting } H_0/H_1 \text{ is true}]$.

If we consider the heteroscedasticity structure in equation (20) to (25), in all the structures, the null hypothesis will results to equal error variances(i.e. homoscedasticity) except equation (25), which will give unequal error variances when $\delta = 2$.i.e.

$$\sigma_t^2 = \sigma^2[\exp(\beta_0 + 2\beta\beta_1X_{1t} + 2\beta_2X_{2t} + 2\beta_3X_{3t})], \text{ when } \delta = 2.$$

The null hypothesis (H_0) stated that there is homoscedasticity while the alternative hypothesis (H_1) stated that there is heteroscedasticity. under this study, α is the probability of rejecting null hypothesis when it is not correct. while power is the probability of rejecting the null hypothesis (H_0) when the alternative hypothesis (H_1) is correct. In considering the Null hypothesis (H_0), rejection of null hypothesis (H_0) given that the alternative hypothesis (H_1) is true, this indicates a correct decision which is the power of the test. We then use this to determine the performances of the chosen methods by consider their performances under different significance levels (10%, 5% and 1%).

3.6. Estimated significance levels to be used to determine the power rate

The hypothesis about the methods of detecting heteroscedasticity under different forms of heteroscedasticity structures was tested at (10%, 5% and 1%) levels of significance to examine the power rate on the variance of each error terms. Intervals was then set for the significance level as follows; The interval set for $\alpha = 0.1$ is (0.09 to 0.14), the interval set for $\alpha = 0.05$ is (0.045 to 0.054), and the interval set for $\alpha = 0.01$ is (0.009 to 0.014). these intervals was set to know the number of times each power rate level falls between the range set for the confidence interval of each method of detecting heteroscedasticity in order to reject the hypothesis or not.

3.7. Sample Sizes

The sample sizes used for this research work were varied from 15, 20, 30, 40, 50, 100 and 250.

These Sample sizes were classified as small ($15 \leq n \leq 30$), medium ($40 \leq n \leq 50$) and large ($100 \leq n \leq 250$).

3.8. Procedures for determination of the preferred heteroscedasticity method

- At a particular α level a confidence interval was set for 10%, 5% and 1%,
- Test was carried out based on earlier stated null and alternative hypothesis,
- The particular significance confidence interval α falls was determine,
- The number of times α falls in between, the set confidence interval was counted over the sample size and heteroscedasticity structures.

$$\hat{\alpha} \text{ was determined as: } \hat{\alpha} = \frac{r}{R}$$

Where r is the number of times $\hat{\alpha}$ falls in between the confidence interval set at a particular significance level. While R is the number of times the experiment was carried out.

The heteroscedasticity test with highest number of count is chosen to be the best with respect to power rate of test.

4. RESULTS AND DISCUSSION

Results obtained from the simulation experiment shows that the number of times the estimated $\hat{\alpha}$ which is the probability of taking correct decision (power of the test) of each methods fall in between the set confidence interval for $\alpha = 10\%$, 5% and 1% was counted over the sample sizes and heteroscedasticity structures for each heteroscedasticity method of detection to obtain the results in Table 1.1.

Table 1.1: The number of counts for each method of detecting heteroscedasticity structures that has the highest power rate value at all sample sizes when there is no presence of multicollinearity in the model.

$\hat{\phi}$	Methods	Sample size(n)							
		15	20	30	40	50	100	250	TOTAL
0.1	BPG	0	1	1	0	0	1	1	4
	PT	0	0	0	0	0	0	0	0
	ST	0	0	0	0	0	0	0	0
	NVST	0	0	0	0	0	0	0	0
	GLJ	0	0	0	0	0	0	0	0
	GFQ	1	1	1	1	1	1	1	7
	BG	1	1	1	1	1	1	1	7
	HM	1	1	0	1	1	1	1	6
	WT	0	0	0	0	0	0	1	1
0.05	BPG	0	0	1	0	0	1	1	3
	PT	0	0	0	0	0	0	0	0
	ST	0	0	0	0	0	0	0	0
	NVST	0	0	0	0	0	0	1	1
	GLJ	0	0	0	0	0	0	0	0
	GFQ	0	0	1	1	0	1	1	4
	BG	0	0	0	0	0	0	1	1
	HM	0	0	0	0	0	1	1	2
	WT	0	0	0	0	0	0	0	0

$\hat{\phi}$	Methods	Sample size(n)							
		15	20	30	40	50	100	250	TOTAL
0.01	BPG	0	0	0	0	0	0	1	1
	PT	0	0	0	0	0	0	0	0
	ST	0	0	0	0	0	0	0	0
	NVST	0	0	0	1	0	0	1	2
	GLJ	0	0	0	0	0	0	0	0
	GFQ	0	1	1	0	0	0	0	2
	BG	1	0	0	0	1	1	0	3
	HM	0	0	1	1	0	0	0	2
	WT	0	0	0	0	0	0	0	0

Source: Simulated data

CONCLUSION

This paper revealed the power rate exhibited by each of the nine (9) heteroscedasticity detection methods under consideration in a linear regression model considered on some heteroscedasticity structures and several sample sizes that was classified as small ($15 \leq n \leq 30$), medium ($40 \leq n \leq 50$) and large ($100 \leq n \leq 250$) through a Monte Carlo study.

The results in this paper show that when power rate of a test was considered on each method of heteroscedasticity detection in the model, BG test or GFQ test are consistently the most preferred methods of heteroscedasticity detection when there exist no problem of multicollinearity between the exogenous variables of the regression model.

The results shows that with low sample size, BG and GFQ are the best except that GFQ out performed BG when $\alpha = 0.05$, It also reveals that with medium sample size, BG and GFQ are consistently best except that HM and NVST compete well with BG at $\alpha = 0.05$ and $\alpha = 0.01$. With large sample size, BG and GFQ are consistently best except that GFQ and HM compete with BG and BPG at $\alpha = 0.05$ and NVST compete well with BG at $\alpha = 0.01$.

Inconclusion, with power rate of a test on linear regression model with no multicollinerity problem, BG and GFQ methods of heteroscedasticity are consistently best tests for heteroscedasticity detection.

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