DIFFERENTIAL TRANSFORM METHOD FOR MHD FLUID FLOW IN A POROUS CHANNEL UNDER OPTICALLY THICK LIMIT RADIATION

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ABSTRACT: *In this paper, Magnetohydrodynamic (MHD) fluid flow in a porous channel under optically thick limit radiation was examined. The non-dimensional equations are solved constructing a semi-analytical numerical technique by differential transform method (DTM). DTM is useful in this work as it can be used to accurately solve larger class of linear and nonlinear problems. The effect of velocity and temperature for different values of physical parameter such as thermal Grashof number, radiation parameter, Magnetic parameter and porosity parameter is considered. It is observed that the velocity increases with increasing values of magnetic and decreases with increasing values of Grashof number and porosity parameter. Also, the temperature decreases with increasing values of magnetic and radiation parameter but decreases with Grashof number and porosity parameter. Furthermore, there was an increase in heat transfer due to thermal conduction as the radiation increases.*

KEYWORDS: *Magnetohydrodynamics, Heat Transfer, Radiation, Optically Thick Limit, Differential Transform Method*

1. INTRODUCTION

Magnetohydrodynamics (MHD) is the combined impact of magnetic force and hydraulics behaviour of a fluid. Its application is found in the fields of astrophysics and cosmology since the matter of the universe is made up of interstellar medium, plasma, interplanetary medium. It also covers typical computational astronomy topics, such as magneto-convection, MHD turbulence and hydromagnetic generator action. In engineering, MHD is used to study the magnetic behaviour of plasmas in fusion reactors, magnetic force casting and liquid-metal cooling of nuclear reactors. Due to its numerous application of the flow through a porous medium in many fields, and thus has attracted numerous number of researchers, [4][6][7][9][15][16][18] [21] are few among many studies in the literature.

[8] studied convection flow through porous medium with inclined temperature gradient. The study of a periodic solution on oscillatory flow-through the channel in rotating porous medium was carried out in [12]. In [22], the unsteady MHD free convective visco-elastic fluid flow bounded by an infinite inclined porous plate in the presence of heat source, viscous dissipation and ohmic heating was examined.

Furthermore, High increase in scientific and technical applications on the effect of radiation flow at high temperature has more importance in many engineering processes, which is attributed to the fact that attainment of high temperature in some engineering device, can cause gas to be ionized and become electrically conducting. Hence, the need to study radiative heat transfer in nuclear power plant, gas turbine and various propulsion devices for aircraft, missile and space vehicles. For the effect of radiation on flow past different geometry, quite a lot of investigation have been made by [3][11][17][19][20].

[5] examined the radiative effect on MHD flow in a vertical channel under optically thick approximation using numerical technique. The effect of thermal radiation and rotation on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium was investigated in [17]. [19] analyzed hydromagnetic natural convection flow with radiative heat transfer past an accelerated moving vertical plate with ramped temperature through a porous medium. Hall effects on unsteady MHD natural convection flow of heat-absorbing fluid past an accelerated moving vertical plate with ramped temperature were studied in [14]. [2] investigates the free convection boundary layer flow in a rotating MHD fluid past a vertical porous medium with thermal radiation using perturbation technique.

In this paper, we considered the fluid flow of MHD in a vertical channel under an optically thick limit approximation with magnetic field parameter and radiation parameter which were solved, constructing a semi-analytical numerical technique by differential transform method (DTM), which has been used to solve larger classes of linear and nonlinear problems.

2. MATHEMATICAL FORMULATION

Here, we formulate and solve the problem of fluid flow of MHD in a vertical channel under an optically thick limit approximation by constructing a semi-analytical numerical technique using differential transform method (DTM).

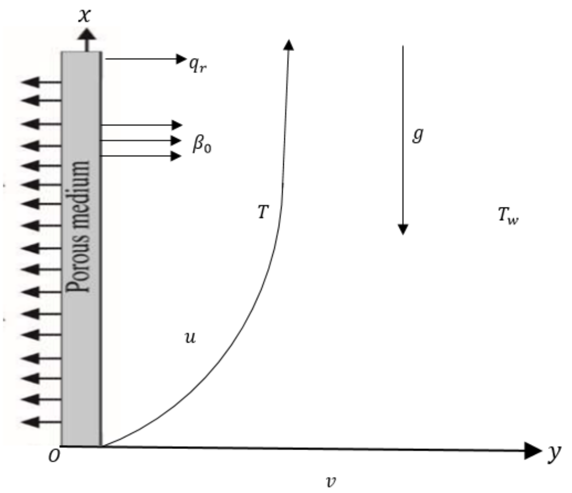

Figure 1: The schematic representation of the problem

We consider the laminar flow between vertical walls as shown in Figure 1 in which the velocity at the center is maximum and the velocity at the wall is zero, the velocity distribution being symmetric about the y-axis. We assume that the temperature of the walls are the same and are at a constant temperature gradient. The x-axis is taken along the plate in an upward direction and the y-axis is taken normal to the plate. The temperature and velocity fields are symmetric about the central line of the channel in a magnetic region. The viscosity, specific heat and the thermal conductivity are not depending on the temperature. The variation in density is included in the body force term. By usual Boussineq's and boundary layer approximation, the fluid flow of MHD in a vertical channel is governed by the following momentum and energy equation respectively:

$$\begin{cases} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} - \left(\frac{\sigma\beta_0^2}{\rho} + \frac{v}{k_p}\right)u \\ \qquad + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g\beta(T_w - T) \\ u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_c}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \end{cases} \qquad (1)$$

Where u and v are the velocity of the fluid, T is the fluid temperature, σ is the viscocity of the fluid, k_c is the thermal conductivity, β is the thermal expansion of the fluid, C_p is the specific heat capacity, k_p is the porous medium permeability, ρ is the fluid density. For optically thick fluid, using the Rosseland approximation ([SH72]), q_r the radiative heat flux is given as

$$\frac{\partial q_r}{\partial y} = -\frac{16}{3\tau}\sigma T^3\frac{\partial T}{\partial y} \qquad (2)$$

For which τ is the thermal diffusivity.
The boundary conditions are:

$$\begin{aligned} T = 0 \quad u = 0 \quad \text{on } y = 0 \\ T'' = 0 \quad u'' = 0 \quad \text{on } y = 1 \end{aligned} \qquad (3)$$

We shall introduce the following non-dimensional quantities

$$y = hY, u = \frac{\tau u}{h} T = -a\phi, M = \frac{\sigma h^2\beta_0^2}{\rho v},$$

$$F = \frac{16\alpha h a^3}{3k\tau}, Ga = \frac{g\beta h^3 a}{v\tau}, k = \frac{\rho C_p \tau}{k_c a}, K = \frac{h^3 k_p}{v^2\tau} \qquad (4)$$

Where
Ga= Thermal Grashof number,
M=Magnetic field parameter,
F=Radiation parameter
g=Gravitational acceleration,
T=Temperature,
x, y=Cartesian coordinate along the porous channel,
u, v=Velocity,
$\overline{q_r}$=radiative heat flux,
β=Thermal expansion of the fluid,
C_p=Specific heat capacity,
k_c=Thermal conductivity,
k_p= permeability parameter,
K=Porosity parameter,
σ=Viscosity of the fluid,
ρ=Fluid density,
τ=Thermal diffusivity
α=Heat transfer coefficient due to thermal condition,
U=Dimensionless velocity,
ϕ=Dimensionless temperature,
w=wall condition

The dimensionalise equations for momentum and energy equation are respectively

$$\frac{\partial^2 U}{\partial Y^2} - M_1 U = \lambda + Ga\phi \qquad (5)$$

where $\lambda = \frac{h^3}{\rho v\tau}\frac{dp}{dx}$; $M_1 = M + \frac{1}{K}$

$$\frac{\partial^2\phi}{\partial Y^2} + F\phi^3\frac{\partial\phi}{\partial Y} = -kU \qquad (6)$$

The corresponding boundary conditions are:

$$\begin{aligned} U = 0, \quad \phi = 0 \quad \text{on} Y = 0 \\ U'' = 0, \quad \phi'' = 0 \quad \text{on } Y = 1 \end{aligned} \qquad (7)$$

2.1 Method of Solution

In order to solve equations (5) – (6) with boundary conditions (7), we employ differential transform method.

Differential Transform Method (DTM) Algorithm

The differential transform method construct a semi-analytical numerical techniques that makes use of Taylor series for the solution of differential equations in the form of polynomials. There is no need for linearization or perturbations, large computational work and round-off errors are avoided. It has been used to solve accurately larger class of linear and nonlinear problems [10].

The differential transformation of the sth derivative of the function $U(y)$ is defined as follows:

$$\bar{U}(s) = \frac{1}{s!}\left[\frac{d^sU(t)}{dt^s}\right]_{t=t_0} \qquad (8)$$

Where $U(t)$ is the original function and $\bar{U}(s)$ is the transformed function. Differential inverse transformation of $\bar{U}(s)$ is defined as follows:

$$U(y) = \sum_{s=0}^{\infty} \bar{U}(s)(y-y_0)^s \qquad (9)$$

The basic operations of the dimensional transform which are useful in the transformation of equation (5) – (7) are summarized as follows: [10]

Original Function	Transformed Function
$f(y) = \dfrac{d^nU(y)}{dy^n}$	$\bar{F}(s) = \dfrac{(s+n)!}{s!}\bar{U}(s+n)$
$f(y) = \lambda U(y)$	$\bar{F} = \lambda \bar{U}(s)$
$f(y) = \prod_{i=1}^{n} U_i(t)$	

$$\bar{F}(s) = \sum_{l_1=0}^{s}\sum_{l_2=0}^{s-l_1}\cdots\sum_{l_n=0}^{s-l_1\cdots l_{n-1}} \bar{U}(l_1)\cdots\bar{U}(l_{n-1})\bar{U}(s-l_1-\cdots-l_n)$$

Solution by Differential Transform Method (DTM)

By applying the differential transform method (DTM) procedure appropriately, the differential transform of equations (5) – (6) are obtained respectively as

$$(s+1)(s+2)\bar{U}(s+2) = \lambda + M_1\bar{U}(s) + Ga\bar{\phi}(s)$$

$$(s+1)(s+2)\bar{\phi}(s+2) = -k\bar{U}(s) - F(s+1)\sum_{l_1=0}^{s}\sum_{l_2=0}^{s-l_1}\sum_{l_3=0}^{s-l_1-l_2}\sum_{l_4=0}^{s-l_1-l_2-l_3}\bar{\phi}(l_1)\bar{\phi}(l_2)\bar{\phi}(l_3)\bar{\phi}(s+1-l_1-l_2-l_3-l_4)$$

which implies

$$\bar{U}(s+2) = \frac{\lambda + M_1\bar{U}(s) + Ga\bar{\phi}(s)}{(s+1)(s+2)} \qquad (10)$$

$$\bar{\phi}(s+2) = \frac{-k\bar{U}(s) - F(s+1)\sum_{l_1=0}^{s}\sum_{l_2=0}^{s-l_1}\sum_{l_3=0}^{s-l_1-l_2}\sum_{l_4=0}^{s-l_1-l_2-l_3}\bar{\phi}(l_1)\bar{\phi}(l_2)\bar{\phi}(l_3)\bar{\phi}(s+1-l_1-l_2-l_3-l_4)}{(s+1)(s+2)} \qquad (11)$$

The corresponding transformed boundary conditions for (7) are:

$$\bar{U}(0) = 0, \quad \bar{\phi}(0) = 0 \qquad (12)$$

and

$$\sum_{s=0}^{N} s(s+1)\bar{U}(s) = 0, \; \sum_{s=0}^{N} s(s+1)\bar{\phi}(s) = 0 \qquad (13)$$

We then solve equations (10) and (11) subject to equations (12) and (13) by rearranging the set of algebraic equations to obtain an Eigenvalue problem.

The values of $\bar{U}(0), \bar{\phi}(0)$ are unknown, so we set them respectively as

$$\bar{U}(0) = a, \quad \bar{\phi}(0) = b \qquad (14)$$

The values of $\bar{U}(2), \bar{U}(3), \ldots, \bar{U}(N)$ and $\bar{\phi}(2), \bar{\phi}(3), \ldots, \bar{\phi}(N)$ can be determined in terms of a and b by using equation (14) suitably in equations (10) and (11) for $s=0, 1, 2, \ldots$

Next, $\bar{U}(0), \bar{U}(1), \ldots, \bar{U}(N)$ and $\bar{\phi}(0), \bar{\phi}(1), \ldots, \bar{\phi}(N)$ are substituted into equation (13) to get a and b respectively.

For the inverse transformation, we use the inverse transformation rule (9) appropriately which gives the solution of $U(y)$ and $\phi(y)$.

The heat transfer coefficient due to thermal condition is given by

$$\alpha = -\left(\frac{d\phi}{dy}\right) \qquad (15)$$

$$= -(U(1) + 2U(2)y + 3U(3)y^2 + \cdots + NU(N)y^{N-1})$$

3. RESULTS AND DISCUSSION

MHD fluid flow through a vertical porous channel under an optically thick limit approximation using differential transform method (DTM) is examined. In order to observe the effects of physical parameters such as thermal Grashof number Ga, radiation parameter F, magnetic parameter M, porosity parameter K on the flow patterns, we analyse the computation of the flow field. The profile for velocity and temperature are obtained for the afore mentioned physical parameters. It should be noted that we set N to be 5.

The effect of velocity and temperature for different values of Grashof number *Ga=0.5, 1, 2* is presented in Figure 2 and 3 respectively. It is observed that the velocity decreases with increasing values of Grashof number while that of temperature increases as the value of Grashof number Ga increases.

The effect of velocity and temperature for different values of Magnetic parameter *M = 5, 10, 15* is presented in Figure 4 and 5 respectively. It is observed that the velocity increases as the magnetic parameter increases while the temperature decreases with increasing values of Magnetic parameter (M).

The effect of velocity and temperature for different values of porous permeability *(K=0.1, 0.2, 0.4)* is presented in Figure 6 and 7 respectively. It is observed that the velocity decreases with increasing values of porosity parameter while that of temperature increases as the value of porosity parameter *K* increases.

The effect of temperature for different values of radiation (*F=5, 10, 15, 20)* is presented in Figure 8. It is observed that the temperature decreases with increasing values of radiation *(F)*.

Figure 9 display the heat transfer due to thermal conduction for various values of radiation parameter. It is observed that there is an increase in the heat transfer due to thermal conduction as the radiation increases.

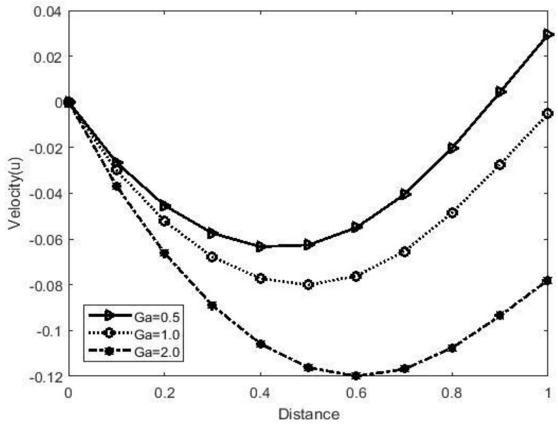

Figure 2: Velocity Profile for different values of Grashof number $\boldsymbol{Ga}$. $\boldsymbol{M = F = 5, K = 0.1}$

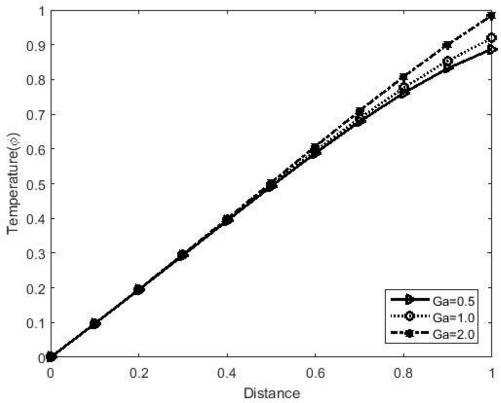

Figure 3: Temperature Profile for various values of Grashof number $\boldsymbol{Ga}$. $\boldsymbol{M = F = 5, K = 0.1}$

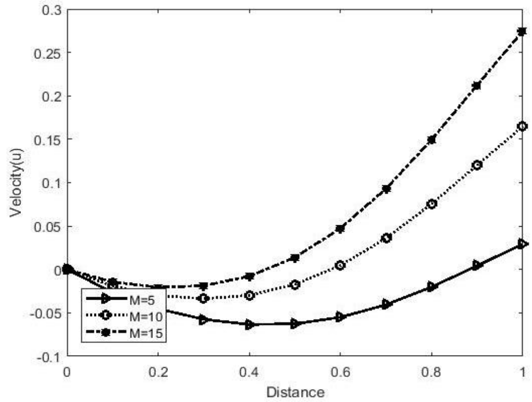

Figure 4: Velocity Profile for different values of Magnetic Parameter $\boldsymbol{M}$. $\boldsymbol{F = 5, Ga = 0.5, K = 0.1}$

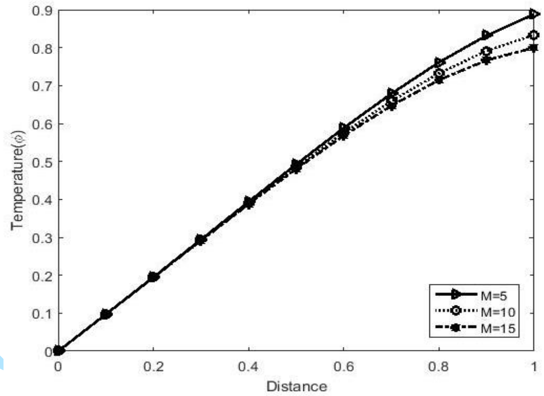

Figure 5: Temperature Profile for different values of Magnetic Parameter $\boldsymbol{M}$. $\boldsymbol{F = 5, Ga = 0.5, K = 0.1}$

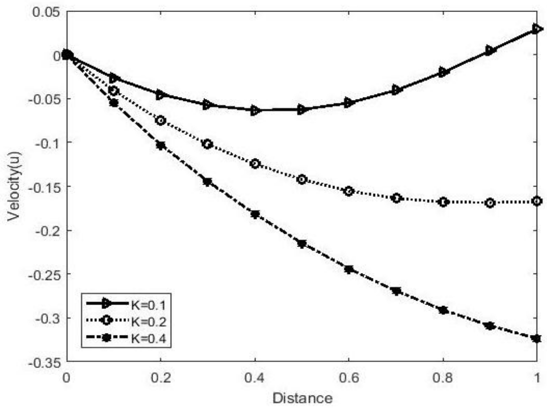

Figure 6: Velocity Profile for different values of Porosity parameter $\boldsymbol{K}$. $\boldsymbol{F = 5, Ga = 0.5, M = 5}$

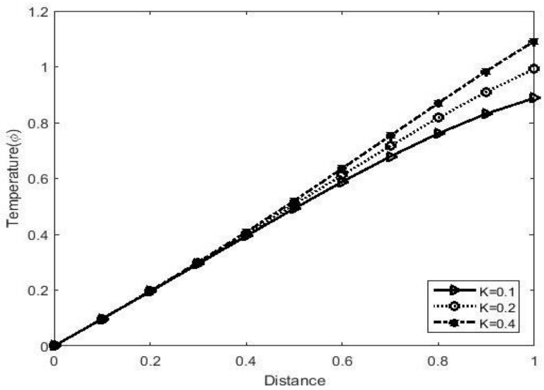

Figure 7: Temperature Profile for different values of Porosity parameter $\boldsymbol{K}$. $\boldsymbol{F = 5, Ga = 0.5, M = 5}$

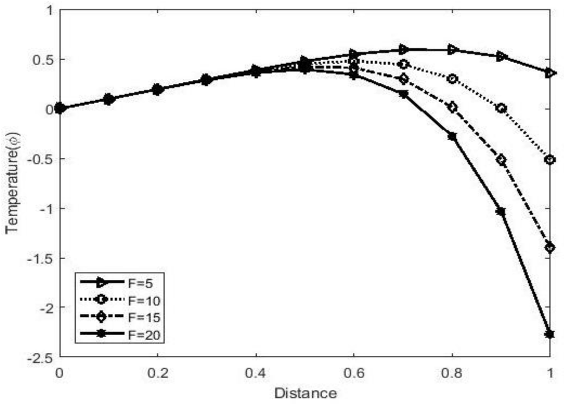

Figure 8: Temperature Profile for various values of Radiation Parameter F. $Ga = 0.5, M = 5, K = 0.1$

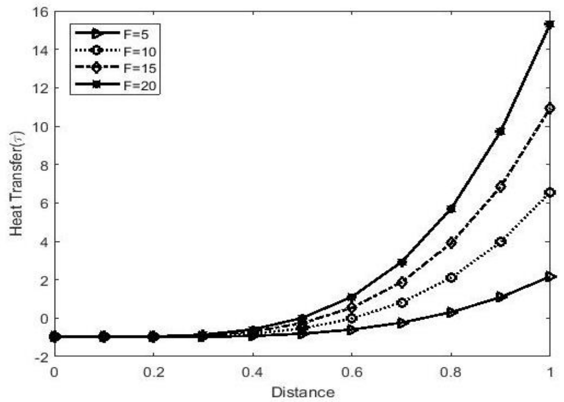

Figure 9: Heat Transfer Profile due to Thermal Conduction for different values of Radiation F

CONCLUSIONS

MHD fluid flow through a vertical porous channel under an optically thick limit approximation is examined and solved using differential transform method (DTM). The effect of velocity and temperature for different values of physical parameter such as thermal Grashof number Ga, radiation parameter F, magnetic parameter M, porosity parameter K was considered. It is observed that the velocity increases with an increasing values of magnetic and decreases with an increasing values of Grashof number and porosity parameter. Also, the temperature decreases with an increasing values of magnetic and radiation but decreases with Grashof number and porosity parameter.

Furthermore, there was an increase in the heat transfer due to thermal conduction as the radiation increases. The results of this study are in good agreement with the existing results in the literature.

In essense, the study is of interest in improving the efficiency and effectiveness of hydromagnetic materials used in nuclear power plant, gas turbine and various propulsion devices for aircraft, missile and space vehicles.

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