EFFICIENCY OF AUDU & ADEWARA (2017) TWO-PHASE FACTOR-TYPE ESTIMATORS WITH TWO AUXILIARY VARIABLES IN SAMPLE SURVEY

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ABSTRACT: In this paper, efficiency of Audu & Adewara ([AA17]) two-phase factor-type estimators with two auxiliary variables for estimating finite population mean were examined using simulation. These estimators were obtained by incorporating some known functions of auxiliary variables X and Z in some existing factor-type estimators. Bias and Mean square error (MSE) of these estimators, $\bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ were obtained using tailor's series expansion. Audu & Adewara ([AA17]) revealed that although $\bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$, have minimum MSE and high PRE than all other related existing factor-type estimators considered using three life dataset but of these two, which is more efficient and most preferred. The simulation results obtained in this study revealed that $\bar{y}_{FTAA}^{\beta_2(d)}$ is more efficient than $\bar{y}_{FTAA}^{\beta_1(d)}$ and hence, most preferred.
***KEYWORDS*:** Factor-type estimator, Two-phase sampling, Mean square error (MSE), Efficiency.

1. INTRODUCTION

It has been established that when the population mean $\bar{X}$ of auxiliary variables X is not known but X is correlated to another auxiliary variable Z whose population mean $\bar{Z}$ is known, then information on Z can be used to improved the efficiency of estimator at hand ([CS12]). In this study, information on secondary auxiliary variable were incorporated into the work of Shukla ([Shu02]) in addition to power transformation with sole aim of improving Shukla ([Shu02]) factor-type estimator.
Shukla ([Shu02]) suggested a factor-type estimator for population mean under two-phase sampling as:

$$\bar{y}_{FTd} = \bar{y}_2 \frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2} \tag{1}$$

$$Bias(\bar{y}_{FTd})_I = \bar{Y}P\theta_3\left[\rho_{xy}C_xC_y - \psi_4 C_x^2\right] \tag{2}$$

$$Bias(\bar{y}_{FTd})_{II} = \bar{Y}P\left[(\theta_1\psi_3 - \theta_2\psi_4)C_x^2 + \theta_2\rho_{xy}C_xC_y\right] \tag{3}$$

$$MSE(\bar{y}_{FTd})_I = \bar{Y}^2\left[\theta_2C_y^2 + \theta_3P^2C_x^2 + 2\theta_3P\rho_{xy}C_xC_y\right] \tag{4}$$

$$MSE(\bar{y}_{FTd})_{II} = \bar{Y}^2\left[\theta_2C_y^2 + \theta_4P^2C_x^2 + 2\theta_2P\rho_{xy}C_xC_y\right] \tag{5}$$

Where:

$$\theta_1 = \frac{1}{n_1} - \frac{1}{N},\ \theta_2 = \frac{1}{n_2} - \frac{1}{N},\ \theta_3 = \frac{1}{n_2} - \frac{1}{n_1},\ \theta_4 = \theta_1 + \theta_2$$

$$A = (d-1)(d-2),\qquad B = (d-1)(d-4),\qquad C = (d-2)(d-3)(d-4),$$

$$\psi_1 = \frac{A+C}{A+fB+C},\ \psi_2 = \frac{fB}{A+fB+C},\ \psi_3 = \frac{A+fB}{A+fB+C},\ \psi_4 = \frac{C}{A+fB+C},\ P = \psi_3 - \psi_1 = \psi_2 - \psi_4$$

d is an unknown positive real number to be estimated i.e $d \in \Re^+$

In his work, it was observed that factor-type estimator $\bar{y}_{FTd}$ was more efficient than classical ratio estimator $\bar{y}_r^d$, if $-2C_{yx} < P < 0$ under case I and if $-2C_{yx}\left(1+\delta\right)^{-1} < P < 0$ under case II where $\delta = \theta_1\theta_2^{-1}$.

2. AUDU & ADEWARA ([AA17]) ESTIMATORS

Consider a preliminary large sample S_1 of size n_1 drawn from population Ω of size N by SRSWOR and secondary sample S_2 of size $n(n < n_1)$ drawn either of the following manners:
Case I: as a subset from the preliminary sample i.e. $S_2 \subset S_1$.
Case II: as an independent sample from population i.e. $S_2 \subset \Omega$.
Motivated by Choudhury and Singh ([CS12]) and Khan et. al ([KSS12]), Audu & Adewara ([AA17]) suggested the following estimators under two-phase sampling as:

$$\bar{y}_{FTAA}^{\beta_1(d)} = \bar{y}_2\left[\frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2}\right]\left[\frac{a_z\bar{Z}+b_z}{a_z\bar{z}_1+b_z}\right]^{\beta_1} \quad (6)$$

$$\bar{y}_{FTAA}^{\beta_2(d)} = \bar{y}_2\left[\frac{(A+C)\bar{x}_1 + fB\bar{x}_2}{(A+fB)\bar{x}_1 + C\bar{x}_2}\right]\left[\frac{a_z\bar{z}_1+b_z}{a_z\bar{z}_2+b_z}\right]^{\beta_2} \quad (7)$$

where $a_x > 0$, $b_x > 0$, $a_z > 0$ and $b_z > 0$ are assumed to be known as either real numbers or (linear or non-linear) functions, $0<\beta_1<1$ and $0<\beta_2<1$

3. PROPERTIES OF AUDU & ADEWARA ([AA17]) FACTOR-TYPE ESTIMATORS

(a). Under Case I ([AA17])

$$Bias\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_I = \bar{Y}\left[\theta_3PC_x^2\left(\rho_{xy}C_xC_y-\psi_4\right)+\theta_1C_z^2\left(\frac{\beta_1(\beta_1+1)}{2}\delta_z^2-\beta_1\delta_z\rho_{yz}C_yC_z\right)\right] \quad (8)$$

$$Bias\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y}\left[\theta_3P\psi_3C_x^2+P\theta_2C_x^2\left(C_{yx}-\psi_4\right)+\beta_1\delta_z\theta_3C_z^2\left(\delta_z\left(\beta_1+1\right)/2+PC_{xz}\right)\right] \quad (9)$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_I = \bar{Y}^2\left[\theta_2C_y^2+\theta_3C_x^2P\left(P+2C_{yx}\right)+\theta_1\beta_1\delta_zC_z^2\left(\beta_1\delta_z-2C_{yz}\right)\right] \quad (10)$$

and

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y}^2\left[\theta_2C_y^2+C_x^2P\left\{\theta_2\left(P+2C_{yx}\right)+\theta_3P\right\}+\theta_3\beta_1\delta_zC_z^2\left(\beta_1\delta_z+2PC_{xz}\right)\right] \quad (11)$$

(b). Under Case II ([AA17])

$$Bias\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_I = \bar{Y}\left[\theta_2PC_{yz}C_z^2-\theta_1PC_{yx}C_x^2-\theta_3\psi_4PC_x^2-\theta_3\beta_2\delta_zC_{yz}C_z^2\right.$$
$$\left.+\theta_2\frac{\beta_2(\beta_2+1)}{2}\delta_z^2C_z^2+\theta_1\delta_z^2C_z^2\left(\frac{\beta_2(\beta_2-1)}{2}-\beta_2^2\right)\right] \quad (12)$$

$$Bias\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y}\left[\frac{\beta_2(\beta_2+1)}{2}\delta_z^2\theta_2C_z^2+\frac{\beta_2(\beta_2-1)}{2}\delta_z^2\theta_1C_z^2-\beta_2\delta_zP\theta_2C_{xz}C_z^2\right.$$
$$\left.+\psi_3P\theta_1C_x^2-\psi_4P\theta_2C_x^2+P\theta_2C_x^2C_{yx}-\beta_2\delta_z\theta_2C_{yz}C_z^2\right] \quad (13)$$

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_I = \bar{Y}^2\left[\delta_2C_y^2+\delta_3C_x^2P\left(P-2C_{yx}\right)+\delta_3\beta_2\delta_zC_z^2\left(\beta_1\delta_z-2C_{yz}-2PC_{xz}\right)\right] \quad (14)$$

and

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y}^2\Big[\theta_2 C_y^2 + \theta_2 C_x^2 P\left(P + 2C_{yx}\right) + \theta_1\beta_2\delta_z C_z^2\left(\beta_2\delta_z - 2PC_{xz}\right)$$
$$\theta_2\delta_z\beta_2 C_z^2\left(\delta_z\beta_2 - 2C_{yz} - 2PC_{xz}\right) + \theta_1 P^2 C_x^2\Big] \quad (15)$$

4. EFFICIENCY COMPARISON

$\bar{y}_{FTAA}^{\beta_2(d)}$ is said to be more efficient than $\bar{y}_{FTAA}^{\beta_1(d)}$ whenever:

(a). For case I:

$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{II} = \bar{Y}^2\left[\theta_2 C_y^2 + C_x^2 P\left\{\theta_2\left(P + 2C_{yx}\right) + \theta_3 P\right\} + \theta_3\beta_1\delta_z C_z^2\left(\beta_1\delta_z + 2PC_{xz}\right)\right] <$$
$$MSE\left(\bar{y}_{FTAA}^{\beta_1(d)}\right)_{I} = \bar{Y}^2\left[\theta_2 C_y^2 + \theta_3 C_x^2 P\left(P + 2C_{yx}\right) + \theta_1\beta_1\delta_z C_z^2\left(\beta_1\delta_z - 2C_{yz}\right)\right]$$

and

(b). For case II:

$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{II} = \bar{Y}^2\Big[\theta_2 C_y^2 + \theta_2 C_x^2 P\left(P + 2C_{yx}\right) + \theta_1\beta_2\delta_z C_z^2\left(\beta_2\delta_z - 2PC_{xz}\right)$$
$$\theta_2\delta_z\beta_2 C_z^2\left(\delta_z\beta_2 - 2C_{yz} - 2PC_{xz}\right) + \theta_1 P^2 C_x^2\Big] <$$
$$MSE\left(\bar{y}_{FTAA}^{\beta_2(d)}\right)_{I} = \bar{Y}^2\left[\delta_2 C_y^2 + \delta_3 C_x^2 P\left(P - 2C_{yx}\right) + \delta_3\beta_2\delta_z C_z^2\left(\beta_1\delta_z - 2C_{yz} - 2PC_{xz}\right)\right]$$

5. EMPIRICAL STUDY USING SIMULATED DATA

In order to justify the results obtained in section three, a numerical simulation study is conducted to investigate the efficiency of these Audu & Adewara ([AA17]) factor-type estimators. The correlation coefficients between the study and auxiliary variables are assumed to be $\rho_{xy} = \pm 0.2$, $\rho_{xy} = \pm 0.5$, $\rho_{xy} = \pm 0.8$ and $\rho_{xy} = \pm 0.99$ using multivariate normal with the following parameters:

$N = 200,\ \bar{Y} = 1.51,\ \bar{X} = 2.02,\ \bar{Z} = 3.05,\ S_y = 0.241, S_x = 0.387,\ S_z = 0.239$

Table 1: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy} = 0.2$ and $\rho_{xy} = 0.5$

Estimators	$\rho_{xy} = 0.2$			$\rho_{xy} = 0.5$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	6.92X10^{-4}	10.75X10^{-4}	107.1	23.2X10^{-4}	5.41X10^{-4}	157.8
$\bar{y}_{FTAA}^{\beta_2(d)}$	4.81X10^{-4}	10.02X10^{-4}	113.6	31.7X10^{-4}	4.87X10^{-4}	175.2
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	21.2X10^{-4}	8.52X10^{-4}	133.7	13.9X10^{-4}	5.08X10^{-4}	168.1
$\bar{y}_{FTAA}^{\beta_2(d)}$	16.1X10^{-4}	8.41X10^{-4}	135.3	17.2X10^{-4}	4.52X10^{-4}	188.9

Table 2: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy}=0.8$ and $\rho_{xy}=0.99$

Estimators	$\rho_{xy}=0.8$			$\rho_{xy}=0.99$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	$19.8X10^{-4}$	$2.07X10^{-4}$	320.7	$28.9X10^{-4}$	$1.31X10^{-4}$	507.4
$\bar{y}_{FTAA}^{\beta_2(d)}$	$24.7X10^{-4}$	$1.99X10^{-4}$	333.2	$18.4X10^{-4}$	$1.23X10^{-4}$	524.1
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	$3.82X10^{-4}$	$2.84X10^{-4}$	233.6	$11.9X10^{-4}$	$1.58X10^{-4}$	421.1
$\bar{y}_{FTAA}^{\beta_2(d)}$	$8.42X10^{-4}$	$2.75X10^{-4}$	241.1	$1.77X10^{-4}$	$1.02X10^{-4}$	653.4

Table 3: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy}=-0.2$ and $\rho_{xy}=-0.5$

Estimators	$\rho_{xy}=-0.2$			$\rho_{xy}=-0.5$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	$2.13X10^{-4}$	$9.03X10^{-4}$	125.9	$23.2X10^{-4}$	$4.41X10^{-4}$	193.9
$\bar{y}_{FTAA}^{\beta_2(d)}$	$2.34X10^{-4}$	$7.79\ X10^{-4}$	146.2	$31.7X10^{-4}$	$4.23X10^{-4}$	202.0
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	$2.24X10^{-4}$	$7.93\ X10^{-4}$	143.6	$3.16X10^{-4}$	$3.84X10^{-4}$	224.9
$\bar{y}_{FTAA}^{\beta_2(d)}$	$2.28X10^{-4}$	$4.62\ X10^{-4}$	246.2	$5.31X10^{-4}$	$2.34X10^{-4}$	249.3

Table 4: Bias, MSE and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$, $\bar{y}_{FTAA}^{\beta_2(d)}$ when $\rho_{xy}=-0.8$ and $\rho_{xy}=-0.99$

Estimators	$\rho_{xy}=-0.8$			$\rho_{xy}=-0.99$		
	Bias	MSE	PRE	Bias	MSE	PRE
CASE I						
$\bar{y}_{FTAA}^{\beta_1(d)}$	$37.4X10^{-4}$	$3.51X10^{-4}$	279.5	$42.9X10^{-4}$	$1.58X10^{-4}$	421.3
$\bar{y}_{FTAA}^{\beta_2(d)}$	$45.1X10^{-4}$	$3.33X10^{-4}$	294.8	$45.4X10^{-4}$	$1.52X10^{-4}$	438.2
CASE II						
$\bar{y}_{FTAA}^{\beta_1(d)}$	$3.82X10^{-4}$	$4.47X10^{-4}$	210.2	$6.30X10^{-4}$	$2.85X10^{-4}$	232.8
$\bar{y}_{FTAA}^{\beta_2(d)}$	$8.42X10^{-4}$	$4.35X10^{-4}$	225.6	$4.76X10^{-4}$	$2.53X10^{-4}$	262.1

6. RESULTS AND DISCUSSION

Tables 1 - 4 show the biases, MSEs and PRE of $\bar{y}_{FTAA}^{\beta_1(d)}$ and $\bar{y}_{FTAA}^{\beta_2(d)}$ under simple random sampling scheme when the study and auxiliary variables are negatively and positively correlated with $\rho_{xy}=\pm0.2$, $\rho_{xy}=\pm0.5$, $\rho_{xy}=\pm0.8$ and $\rho_{xy}=\pm0.99$ coefficients respectively. These properties (Bias, MSE and PRE) were computed under cases I and II. The simulation results of the analysis revealed that $\bar{y}_{FTAA}^{\beta_2(d)}$ have minimum MSEs and high PRE and hence, more efficient and most preferred to $\bar{y}_{FTAA}^{\beta_1(d)}$.

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