

## A PERSPECTIVE OF METACOGNITION IN SOLVING MATH PROBLEMS IN VIETNAM SECONDARY SCHOOLS

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**ABSTRACT:** In an effort to innovate teaching and learning process to prepare for a new generation for the demands of the new era, many educators have discovered the value of metacognition. Students will need both mathematical skills and problem-solving skills, therefore, teachers should focus on both math content and thinking processes in students' math learning. This paper presents the importance of metacognition in solving mathematical problems. A project was conducted for students at the age of thirteen, and the findings suggest that students used four problem-solving steps highlighted by George Polya. However, students feel better when they adjust their thinking processes or use metacognitive skills in the process of solving math problems.

**KEYWORDS:** skill, metacognition, problem-solving, mathematical thinking.

### 1. INTRODUCTION

Mathematical education in the 21st century is how to confront with novel issues in the real world, to foster creative thinking skills and foster effective learning. In an effort to innovate teaching and learning to prepare for a new generation for the demands of the new era, many educators have discovered the value of metacognition.

The problem-solving experience that students encounter in schools is no longer relevant to today's world. Mathematical problem solving is how to find the direction from a particular situation in which the goals and solutions are clearly defined, especially the most challenging aspects of the problems encountered in many industries today relate to the development of useful ways of mathematical thinking on related relationships, models, and rules. From the point of view of [(LD03)] with the increasing importance in the global market change, mathematic now is the greater need for skilled mathematic and technological laborers. The processes such as: building, describing, explaining, predicting, representing as well as quantifying, coordinating and organizing data provide a basis for the development of capabilities. It is increasingly important because of the ability of collaborating on

multi-dimensional projects, planning, monitoring are essential to success.

Mathematics is always one of the most difficult subjects to students. Von Glasersfeld ([Gla95]) states that educators have found that many students, who are able to learn the necessary formulas and apply them to the limited range of textbooks and test cases, have not understood concepts and conceptual relationships when they face with novel issues.

Every person can become laggards in a constantly changing scientific world. To be as technologically savvy as they are, people have to keep up with changes. The education system of the 21st century is in the midst of great and promising reform efforts to restructure the education system in general and education in secondary schools in particular.

In mathematics, we are striving to pursue in many aspects and recognize that there are many ways to learn and understand mathematics such as using technology to support our learning, connecting math and experiences that students accumulate outside the school. Students have many opportunities to explore, investigate and explore their own models, ideas and even algorithms.

Regardless of the reform efforts in this area, it is still true that students' math performance is unlikely to reach the desired level to allow students to succeed in life after graduation. They have the need to use their understanding and ability to control and adjust their learning in accordance with the current situation. The present need of thinking skills is partly the result of cognitive development while the needs of society are changing. Poor learning result is due to learning habits that inhibit metacognitive abilities ([Bai98]). This expectation will be driven by the need to develop forms of mathematical thinking. The process of thinking, which is the key to learning, can be controlled and directed. Metacognition is defined as teaching a person's brain to control thinking processes for the purpose of directing it toward its learning management ([Mar06]).

After knowing that thinking can be controlled and directed, students will be aware of their own

thinking power, becoming purposeful learners. Schools often aim to educate students about perceptions, responsibilities, and ability to learn independently ([Bai98]). And if they are developing metacognition, they will believe that they can learn, will know how to evaluate their position in the classroom, and can see themselves as continuous learners and thinker.

Educators recognize the lack of professional metacognitive skills of students as one of the contributing factors to this problematic situation. A study was conducted to investigate the effect of metacognitive skills on problem solving ability of students. This paper sheds light on the importance of metacognition in solving math problems in Vietnam secondary schools.

## 2. OBJECTIVES OF THE STUDY

The main focus of this study is to clarify the importance of metacognition in the context of problem solving, which is one of the foundations that helps students learn mathematics well and solve real world problems.

## 3. RESEARCH QUESTIONS

This study aims to answer the questions

- What is the perception of metacognitive thinking and the basic function of metacognitive thinking?
- How important is metacognitive thinking in solving math problems in secondary school?

## 4. RESEARCH METHODS

This research uses the following methods: theoretical research (analysis, synthesis, systematization, generalization, ...); Investigation, observation (designing survey forms, questionnaires, observations); Interviews with teachers and students in secondary school.

A study was conducted with one of its objectives to investigate the impact of metacognitive problem solving. An integrated approach using in this study involves the collection and analysis of qualitative and quantitative data. The researchers found that all methods had their strengths and weaknesses and the strengths of a single approach could complement other methodological weaknesses. In this study, understanding gained from quantitative analyzes has been extended and developed through qualitative analyzes.

The study was conducted with 350 students, randomly selected from 10 regional secondary schools.

The tools of this study included a set of difficult math problems solved by students in the survey to determine their ability to solve problems and a set of questions to be completed by one by one student to collect personal student data and information related to the problem solving process while answering math problems. Then, I analyze the data to determine student problem solving behavior.

## 5. DISCUSSION

### 5.1. The perception of metacognition

The concept of metacognition was first identified in the seventies. It seems that metacognition is the result of research on cognitive development, memory and reading. Many mathematicians have shown great interest in mathematical research and metacognition. Following are some metacognitive concepts:

Flavell ([Fla76]) defined metacognitive as the knowledge of a person involved in the process of his or her own perception and results or anything related to them. Metacognition refers to the monitoring of activities and the regulation of results and coordination of processes related to cognitive objects, usually in the services of a specific object; According to Thorpe and Satterly ([TS90]) metacognition is often described multidimensionally and is used as a general term for a range of different levels of cognitive skills; According to ([BS15]) argued that metacognition is one of the possibilities for knowing what you know and what you do not know. It is also the ability to use your knowledge to plan a strategy for giving information, taking the necessary steps in problem solving and to reflect a person's level of thought about a special concern; Martinez ([Mar06]) stated that metacognition is defined as teaching a person's brain to control thought processes in order to control their learning. Knowing that thinking can be controlled and directed, students will be aware of their own thinking power, then they become purposeful learners.

According to Baird ([Bai98]), schools encourage students to learn about cognitive, accountable and independent learning. If they are developing metacognition, they will believe that they can learn, will know how to evaluate their position in the classroom and can see themselves as constant learners, thinkers.

It seems that metacognition is the result of research on the development of cognition, memory and reading. Many mathematical educators have shown great interest in this field as they realize that purely mathematical analysis of mathematical performance is insufficient to study the problem.

Brown ([Bro87]) stated that the main concern is understanding one's perception rather than perception. He defined metacognitive as operating skills, contributes to forecasting, testing, monitoring, testing, coordinating and controlling intentional attempts to understand or solve problems on the right time and at the right place.

Schoenfeld probably gave the most comprehensive analysis of metacognition. According to Schoenfeld ([Sch87, Sch92]), metacognition mentions about our thinking and it includes three important aspects: knowledge of our thinking processes, self-control or self-regulation, trust and intuition. The point in Schoenfeld ([Sch87]) is that students should divide their time wisely between (a) understanding the problem, (b) planning, (c) making decisions, and (d) implementing decisions within the timeframe. In the process of solving the problem, they must monitor the progress of the solution. When decisions seem unsuccessful, they should try alternatives or make some adjustments.

Despite the apparent importance of metacognition in mathematical operations, it has not been systematically studied by mathematical educators. A number of problem solving models have been created, based on Polya's four-phased model, which only assumes hidden metacognitive processes. Metacognitive-related phenomena are considered to be too difficult to identify by many psychologists.

Metacognitive skills are "management activities related to problem solving" ([Bro78]). They involve planning, monitoring and evaluating the components of metacognition. It is also called 'cognitive rules' which refer to activities and actions taken by individuals to control their own perception ([CS09]). In keeping with Schoenfeld, I think metacognition is thinking about thought and operating skills, contributes to forecasting, testing, monitoring, testing, coordinating and controlling intentional attempts to understand or solve problems on the right time and at the right place. Thus, in order to learn effectively, students should divide the time wisely between (a) understanding the problem, (b) planning, (c) making decisions, and (d) implementing decisions within the timeframe. In the process of solving a problem, students should be monitored for progress. When the decision seem unsuccessful, the student should try to make a substitute or make some adjustments toward a successful problem solving.

### 5.2. Basic functions of Metacognition

According to J. Wilson (Wil98), metacognition has three basic functions: the awareness function, the evaluation function, and the regulation function:

- Awareness function refers to the ability of each person to understand cognitive processes, learning strategies and inherent knowledge; your self-awareness of your own cognitive ability. According to ([Far13]), the awareness function of metacognition includes: know what to know; set learning goals; review resources, learning conditions; think about what the task poses; find out how to evaluate performance; recognize the advantages and disadvantages of learning.

- Evaluation function addresses the tracking of thought processes and assessment of the strengths and weaknesses of a person's thinking in specific situations. Each person can make his or her comment about the effectiveness of thinking and the choice of strategies. Through the evaluation criteria, the learner looks at his or her learning process and knows the level of completion of the cognitive task that has been set. This function monitors the effectiveness of the plan and the used strategies. ([BL13]) has raised a number of questions that have helped the assessment process: Have I ever done the same task before? How did I accomplish that task? Why do I find it easy or difficult? What did I learn? What do I do to complete the task? How should I do it? Should I follow the same way as I did before? ...

- Regulation function occurs when individuals adjust their thinking processes. They use the skills of metacognition to control knowledge and thinking. At the same time, they reflect on their thinking and knowledge processes and make the necessary changes. ([Sch98]) introduced a question system to promote the adjustment process: What is the nature of the task? What is my job? What kind of information and strategies do I need to use? How much time will I need? Do I understand that task clearly? Do I need to change something? Have I met my goal? What have I done and have not done yet? What will I do next?...

Thus, the functions of metacognition help each individual to be aware of their perceptions, tasks, task performance, evaluation and adjustment to perform tasks more effectively.

### 5.3. Strategy for solving math problems

Strategies in solving math problems are essential in mathematical education. It has always been a challenge for educators to teach students how to solve problems.

“Problem solving is the process by which a student arrives at a solution to a problem. Integral to this are students’ thinking, planning, reasoning, and executing of the plan as they progress from the initial problem state to the fulfillment of their goal” ([Wil00, Del11]).

In fact, every student has the opportunity to apply knowledge to the realities of life. Lesh ([LZ07]) states that problem solving is a task, an activity that leads to a specific goal and becomes an exercise when the instructor applies an effective way of thinking into a particular situation. Therefore, thinking in an effective (methodical) way requires the instructor to have the skills to express the problem in mathematical language. Usually in sequence: express, try again, review results from different directions.

According to Erbas & Okur ([EO12]), "Problem solving is not just a mathematical approach, but also an important part of math learning where students gain deeper understanding of mathematic concepts by analyzing and synthesizing their knowledge" ([\*\*00, \*\*03, Pol73]). This implies that students should learn to adjust their knowledge to become successful learners. And junior and high school standards call on students to apply mathematical thinking to real problems and challenges. For students mathematical understanding and procedural skills are equally important, and both can be assessed using mathematical formulas.

Understanding and procedural skills are what includes metacognition. Polya emphasizes that effective problem-solving consists of four major phases: problem understanding, planning, implementing, and reviewing ([Pol73]). According to Erbas and Okur ([EO12]), "successful students use meta-verification to make sure they have found the problem". Passing this step seems to be a typical behavior of many students although each problem-solving or modeling framework emphasizes the importance of problem understanding. Although students have difficulty in each stage of the problem solving process, they were able to use metacognitive skills to detect errors or missing parts of the process and adapt independently to make the necessary changes" ([EO12]).

From the point of view of problem solving in mathematics, I think that solving problems in mathematics is the process of finding a solution to a problem when the method is not known to a problem solver. Then, the problem solver must use strategic skills to select the appropriate techniques for a solution.

Polya ([Pol73]) proposed the following four problem-solving stages:

1. Understanding the Problem: Includes reading and clarifying a problem to identify the unknown, and the purpose of the problem.
2. Draft a plan: This phase is the selection of a strategy and a plan for a solution to the problem.
3. Implementing: Once the problem solver has a plan, the problem solver will implement this plan in a solution.

4. Revision: When the solution is ready, the problem solver needs to check the suitability of the solution to this problem.

However, when solving problems people will find that solving a problem is not just a simple top-down process of the above four stages.

In practice many stages can be performed at the same time, when done, each new discovery tends to add or change the overall plan.

The problem is often not fully understood until the problem solver has tried and failed to come up with a solution with different strategies. It is a series of strides and obstacles between four stages. Fernandez, Hadaway and Wilson ([HFW94]) provided a problem-solving paradigm, including managerial processes which are called metacognition by other educators such as Schoenfeld and Flavell. This number indicates that solving problem does not depend much on the problem solver's experience. Clockwise and counterclockwise of the process indicates that the problem solving process can be top down or bottom up with reference to Pólya's model. Management processes or metacognition will also activate the resolution.

#### 5.4. Some examples use metacognition in solving math problems in secondary schools

**Example 1.** Say you have this problem: A farm contains horses and chickens. **30** heads and **100** legs. How many chickens are there?

Here's how to solve any of this type quickly:

*Solution 1.*

Trial with a total of 30 heads:

+ 1 horse and 29 chickens, so the total number of legs is 62 feet (not satisfied).

+ 2 horses and 28 chickens, so the total number of legs is 64 feet (not satisfied).

...

+ 20 horses and 10 chickens, the total number of legs is 100 feet (satisfactory).

+ 21 horse (not satisfied).

Conclusion: 20 horses, 10 chickens.

*Solution 2.*

$30 \times 2 = 60$ . - Multiply number of heads by 2 (two legs per head for chickens).

$100 - 60 = 40$ . - Subtract legs gotten from part one from total legs.

$40/2 = 20$ . - Divide part two by two (difference of legs between chickens and horses)

$30 - 20 = 10$ . - Subtract part three from number of heads for answer.

Answer is 10.

*Solution 3.*

Even easier: Use algebraic formula:



(Animal Type 1:X)\*(Number of Legs on that type) +  
 (Animal Type 2:Y)\*(Number of Legs on that type) =  
 Total number of legs i.e.:  $X*4 + Y*2 = \# \text{ Legs}$

Also:  $X+Y = \text{Heads}$

Riddle Total legs 100, Total Heads 30, Horses (X) &  
 Chickens (Y)

Known true values:  $X*4 + Y*2 = 100, X+Y = 30$

Divide the equation  $4X + 2Y = 100$  by 2 on both  
 sides =  $2X+ Y = 50$

If  $X+Y = 30$ , and  $2X+Y = 50$ , then X must = 20  
 (subtract  $X+Y$  from one side and 30 from the other)

20 horses 10 chickens = 100 legs 30 heads

This formula works regardless of how many 'legs'  
 the creature type has. Just plug in the appropriate  
 value in the original equation to match the new  
 criteria.

**Example 2.** 120 litres of wine are stored in 141  
 bottles. Some of them have volume of 1 litre, the  
 other have volume of 0.7 litre. How many of each  
 bottles were used?

*Solution.*

Let x be the number of bottles of 1 liter; y is the  
 number of bottles of 0.7 liters.

Then  $x+y=141$  should infer  $y = 141-x$

$120 = x.1+y.(0.7)$

Or  $120 = x + (141 - x). (0.7)$ , we get  $x = 71$  and  $y =$   
 70.

## 6. FINDINGS

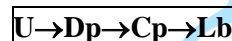
I conducted tests in 10 secondary schools and took  
 randomly 350 students representing these 10  
 schools. I proposed the above two issues and let  
 students participate in problem solving. After 45  
 minutes, I allowed the students to fill in their  
 thoughts in the process of solving the problem. The  
 composite showed that 84.85% understood the  
 problem, identified the unknown, and what was the  
 purpose of the problem. However, the percentage of  
 students who can make a problem-solving plan is  
 not high, accounting for 61.14% of the students who  
 have chosen the strategy and have a plan to address  
 the problem. 60% of students would solve the  
 problem following a plan, with the solution put  
 forward. Most notably, only 25.14% of the students  
 reviewed the activities in the problem solving  
 process and made adjustments in the resolution  
 process, and these students succeeded in addressing  
 the task.

I picked out two students and continued to test,  
 solving the second problem mentioned above.

Case I: Do Le Son is one of the students with weak  
 mathematical abilities. Figure 2 below shows the  
 timeline diagram of Do Le Son working on solving  
 the problem in the first example. Do Le Son went  
 down from the top in four stages of problem solving  
 proposed by Polya ([Pol73]). He spent less time

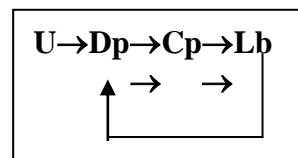
clarifying this issue before choosing the wrong  
 strategy and implementing it with a solution. He  
 never went back to clarify the problem and thought  
 of an alternative strategy to solve the problem. In  
 other words, there is no sign of metacognitive skills  
 or transformation of the problem solving phase of  
 Do Le Son in the whole problem-solving process.  
 With the lack of basic mathematical skills necessary  
 for successful problem solving, Do Son has never  
 had a adequate solution.

Note: Understand (U); Devise a plan (Dp); Carry  
 out the plan (Cp); Look back (Lb)



**Figure 1: Diagram of Do Le Son working on solving the problem**

Case II: Le Anh Tuan is considered one of the best  
 students in mathematics from one of the  
 participating schools. Figure 2 below shows Le Anh  
 Tuan's timeline diagram working on the same  
 problem in the first example as Do Le Son. Le Anh  
 Tuan has gone through all four stages of problem  
 solving. After spending a lot of time reading and  
 understanding, he started planning for a solution. Le  
 Anh Tuan stepped back and went back to the four  
 stages of problem solving whenever he felt he did  
 not fully understand the problem, the strategy was  
 chosen incorrectly or the sentence was not correct,  
 or he did not believe with his solution. In other  
 words, Le Anh Tuan used some metacognitive skills  
 by constantly monitoring his actions while planning  
 a solution, implementing his plan, and checking his  
 solution. That is the process of positive thinking that  
 leads to his right solution.



Or  $U \rightarrow Dp \rightarrow Cp \rightarrow Lb \rightarrow Dp \rightarrow Cp \rightarrow Lb$

**Figure 2: Diagram of Le Anh Tuan working on solving the problem**

### Discussion:

Recognizing that, no matter how students were not  
 excited, the results of the analysis of the factors  
 indicate that they used the four stages of problem  
 solving proposed by Polya ([Pol73]). However,  
 problem-solving skills are not enough for students to  
 succeed in solving math problems. There are three  
 main factors that cause failure in obtaining a  
 solution by Do Le Son, which coincides with  
 Schoenfeld's proposal ([Sch85]):

Do Le Son committed to a wrong approach. He  
 never questioned his accomplishments in solving the

problem. Alternative approaches have never been proposed.

Do Le Son embarked on his strategy with the wrong mathematical skills for a solution that he was not sure and did not bother about right or wrong.

The case of Le Anh Tuan illustrates the importance of metacognition in bringing success in solving mathematical problems. This finding is consistent with Schoenfeld's conclusion ([Sch85]) that a problem solver is constantly questioning his achievement. By taking careful steps such as chasing the lead and abandoning the idea, reviewing the unsuccessful solution, he or she will solve the problem successfully.

## 7. CONCLUSIONS

The student's metacognitive abilities can thrive in an environment where thought processes are really an important part of teaching and communicating. To create such an environment, teachers and students must develop a language of thought and use it continuously. In addition, teachers should use innovative strategies such as writing review reports, collaborative learning to enhance and develop students' own thinking skills as well as direct students' own thinking. This study firstly asserts that students with good metacognitive skills will solve the math problems better, promoting the progress of each student themselves.

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