IMPROVED VARIANCE ESTIMATOR USING LINEAR COMBINATION OF TRI-MEAN AND QUARTILE AVERAGE

J.O. Muili [1], A. Audu [1], R.V.K. Singh [2], A.B. Odeyale [2], A. Adebiyi[2]

[1, 2] Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria
[2]Department of Mathematics, Kebbi State University of Sci. and Tech., Aliero, Nigeria

Corresponding Author: J.O. Muili, jamiubidemi@gmail.com

ABSTRACT: In this paper, we proposed a ratio cum product estimator for estimating finite population variance. The proposed estimator was obtained by transforming [MS17] estimator. The bias and mean square error (MSE) of the proposed estimator has been obtained and the conditions for it efficiency over some existing variance estimators have been established. The efficiency of proposed estimator based on optimal value of the constant, exhibit significant improvement over the estimators considered in the study. The numerical illustration was also conducted to corroborate the theoretical results. The results of the empirical study show that the proposed estimator is more efficient over existing estimators.
KEYWORDS: Tri-mean, Quartile, Mean Square Error, Efficiency, Variance

1. INTRODUCTION

Various fields of life have been facing the problem in estimating the finite population variance. Population variance is one of the parameters which require efficient estimators because of its great significance in various fields of lifelike genetics, agriculture, biology and medical studies such as in matters of health: variations in body temperature, pulse beat and blood pressure which are the basic guides of diagnosis when prescribed treatment is designed to control their variation. An agriculturist requires sufficient knowledge of climatic variation to devise appropriate plan for cultivating his crop. A manufacturer needs constant knowledge of the level of variations in people's reaction to his product to be able to know whether to reduce or increase his price or improve the quality of his product. A fair understanding of variability is vitally important for better results in different walks of life.

In this paper, an improved linear combination of a ratio cum product estimator for estimating finite population variance has been proposed with objective to produce efficient estimator and properties have been established.

1.1 Nomenclature

Let $\Omega = (1, 2, 3, \ldots, N)$ be a population of size $N$and Y, X be two real valued functions having values$(Y_i, X_i) \in \mathbb{R}^+ > 0$ on the i^{th} unit of $U(1 \leq i \leq N)$. Let S_y^2 and S_x^2 be the finite population variance of Y and X respectively and s_y^2 and s_x^2 be respective sample variances based on the random sample of size n drawn without replacement. The following are the other notations used throughout this paper.

N : Population size
n : Sample size
Y: Study variable
X : Auxiliary variable
$\bar{y}, \bar{x}$: Sample means of study and auxiliary variables
$\bar{Y}, \bar{X}$: Population means of study and auxiliary variables
ρ: Coefficient of correlation
C_y, C_x : Coefficient of variations of study and auxiliary variables
Q_1 : The lower quartile
Q_3 : The upper quartile
Q_r : Inter-quartile range
Q_d : Semi-quartile range
Q_a : Semi-quartile average
Q_c : Coefficient of quartile deviation
$\beta_{2(y)}$: Coefficient of kurtosis of study variable
$\beta_{2(x)}$: Coefficient of kurtosis of auxiliary variable
TM : Tri-Mean
M_d : Median of the auxiliary

A

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i, \quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad \gamma = \frac{1-f}{n}, \quad Q_a = \frac{(Q_3 - Q_1)}{2},$$

$$TM = \frac{(Q_1 + 2Q_2 + Q_3)}{4}, s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2,$$

$$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2, S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2,$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

1.2 Existing Estimators

The sample variance estimator of the finite population variance is defined as

$$t = s_y^2 \qquad (1)$$

which is an unbiased estimator of finite population variance (S_y^2) and its variance is

$$Var(t) = \gamma S_y^4(\beta_{2(y)} - 1) \qquad (2)$$

[Isa83] proposed a ratio type variance estimator for the finite population variance (S_y^2) when the finite population variance S_x^2 of auxiliary variable X is known. The bias together with its mean squared error is given below:

$$S_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \qquad (3)$$

$$Bias\ (\hat{S}_R^2) = \gamma S_y^2[(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \qquad (4)$$

$$MSE\ (\hat{S}_R^2) = \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + (\beta_{2(x)} - 1) \\ -2(\lambda_{22} - 1) \end{bmatrix} \qquad (5)$$

[KC06] proposed a class of ratio type estimators for finite population variance by imposing Coefficient of variation and Coefficient of kurtosis on the work of Isaki (1983) as:

$$S_{kc_1}^2 = s_y^2\left(\frac{S_x^2+C_x}{s_x^2+C_x}\right) \qquad (6)$$

$$S_{kc_2}^2 = s_y^2\left(\frac{S_x^2+\beta_{x(2)}}{s_x^2+\beta_{x(2)}}\right) \qquad (7)$$

$$S_{kc_3}^2 = s_y^2\left(\frac{S_x^2\beta_{x(2)}+C_x}{s_x^2\beta_{x(2)}+C_x}\right) \qquad (8)$$

$$S_{kc_4}^2 = s_y^2\left(\frac{S_x^2C_x+\beta_{x(2)}}{s_x^2C_x+\beta_{x(2)}}\right) \qquad (9)$$

$$Bias\ (\hat{S}_{kc_i}^2) = \\ = \gamma A_i S_y^2[A_i(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \qquad (10)$$

Where i=1, 2, 3, 4

$$MSE\ (\hat{S}_{kc_i}^2) = \\ = \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + A_i^2(\beta_{2(x)} - 1) \\ -2A_i(\lambda_{22} - 1) \end{bmatrix} \qquad (11)$$

Where i=1, 2, 3, 4

$$A_1 = \frac{S_x^s}{s_x^2+C_x}, A_2 = \frac{S_x^2}{S_x^2+\beta_{2(x)}},$$

$$A_3 = \frac{S_x^2\beta_{2(x)}}{S_x^2\beta_{2(x)}+C_x}, A_4 = \frac{S_x^2C_x}{S_x^2C_x+\beta_{2(x)}}$$

[SK12] proposed ratio type estimators for finite population variance using quartile and functions of quartiles of auxiliary variable as:

$$S_{sk_1}^2 = s_y^2\left(\frac{S_x^2+Q_1}{s_x^2+Q_1}\right) \qquad (12)$$

$$S_{sk_2}^2 = s_y^2\left(\frac{S_x^2+Q_3}{s_x^2+Q_3}\right) \qquad (13)$$

$$S_{sk_3}^2 = s_y^2\left(\frac{S_x^2+Q_r}{s_x^2+Q_r}\right) \qquad (14)$$

$$S_{sk_4}^2 = s_y^2\left(\frac{S_x^2+Q_d}{s_x^2+Q_d}\right) \qquad (15)$$

$$S_{sk_5}^2 = s_y^2\left(\frac{S_x^2+Q_a}{s_x^2+Q_a}\right) \qquad (16)$$

$$Bias\ (\hat{S}_{sk_i}^2) = \\ = \gamma R_i S_y^2[R_i(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \qquad (17)$$

$(i = 1, 2, 3, 4, 5)$

$$MSE\ (\hat{S}_{sk_i}^2) = \\ = \gamma S_y^2 \begin{bmatrix} (\beta_{2(y)} - 1) + R_i^2(\beta_{2(x)} - 1) \\ -2R_i(\lambda_{22} - 1) \end{bmatrix} \qquad (18)$$

$(i = 1, 2, 3, 4, 5)$

where

$$R_1 = \frac{S_x^2}{S_x^2+Q_1}, \quad R_2 = \frac{S_x^2}{S_x^2+Q_3}, R_3 = \frac{S_x^2}{S_x^2+Q_r},$$

$$R_4 = \frac{S_x^2}{S_x^2 + Q_d}, R_5 = \frac{S_x^2}{S_x^2 + Q_a}$$

[KS13] proposed a ratio-type estimator using correlation coefficient between study and auxiliary variable and the third quartile of the auxiliary variable as:

$$S_{ks}^2 = s_y^2\left(\frac{S_x^2\rho+Q_3}{s_x^2\rho+Q_3}\right) \qquad (19)$$

$$Bias\ (\hat{S}_{ks}^2) = \\ = \gamma W S_y^2[W(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \qquad (20)$$

$$MSE\ (\hat{S}_{ks}^2) = \\ = \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + W^2(\beta_{2(x)} - 1) \\ -2W(\lambda_{22} - 1) \end{bmatrix} \qquad (21)$$

Where $W = \frac{S_x^2\rho}{S_x^2\rho+Q_3}$

[SK15] proposed a generalized modified ratio type estimator for finite population variance using the known parameters of the auxiliary variable as:

$$S_{jG}^2 = s_y^2\left(\frac{S_x^2+\alpha w_i}{s_x^2+\alpha w_i}\right) \qquad (22)$$

$$Bias\ (\hat{S}^2_{jG}) = \\ = \gamma A_{jG} S_y^2 [A_{jG}(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \quad (23)$$

$$MSE\ (\hat{S}^2_{jG}) = \\ = \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + A^2_{jG}(\beta_{2(x)} - 1) \\ -2A_{jG}(\lambda_{22} - 1) \end{bmatrix} \quad (24)$$

Where $A_{jG} = \frac{S_x^2}{S_x^2 + \alpha w_i}$

[MS17] proposed a ratio type variance estimator by using linear combination of Tri-mean and population semi inter-quartile average of the auxiliary variable as:

$$S^2_{sj} = s_y^2 \left(\frac{S_x^2 + (TM + Q_a)}{s_x^2 + (TM + Q_a)} \right) \quad (25)$$

$$Bias\ (\hat{S}^2_{sj}) = \\ = \gamma A_{sj} S_y^2 [A_{sj}(\beta_{2(x)} - 1) - (\lambda_{22} - 1)] \quad (26)$$

$$MSE\ (\hat{S}^2_{sj}) = \\ = \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + A^2_{sj}(\beta_{2(x)} - 1) \\ -2A_{sj}(\lambda_{22} - 1) \end{bmatrix} \quad (27)$$

Where $A_{sj} = \frac{S_x^2}{S_x^2 + (TM + Q_a)}$

Many other researchers like [ASK12, AAS16, Sri66, GS08, TS12, SSB81, SS13, OK14, KO13, US06, TSS96, YK13, YMK17] have suggested both ratio and product estimators for population variance.

2. PROPOSED ESTIMATOR

Motivated by the work of [MS17], we proposed an improved ratio cum product estimator of finite population variance using tri-mean and inter-quartile range of auxiliary variable as:

$$S^2_{sj} = s_y^2 \left(\alpha \left(\frac{S_x^2 + (TM + Q_a)}{s_x^2 + (TM + Q_a)} \right) + (1 - \alpha) \left(\frac{s_x^2 + (TM + Q_a)}{S_x^2 + (TM + Q_a)} \right) \right) \quad (28)$$

2.1 Properties of Proposed Estimator

In order to obtain the Bias and MSE, we define $e_0 \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 \frac{s_x^2 - S_x^2}{S_x^2}$ such that $s_y^2 = S_y^2(1 + e_0)$ and $s_x^2 = S_x^2(1 + e_1)$, from the definition of e_0 and e_1, we obtain

$$\left. \begin{array}{l} E(e_0) = E(e_1) = 0,\ E(e_0^2) = \gamma(\beta_{2(y)} - 1) \\ E(e_1^2) = \gamma(\beta_{2(x)} - 1), E(e_0 e_1) = \gamma(\lambda_{22} - 1) \end{array} \right\} \quad (29)$$

Expressing $\widehat{S}^2_{MJ}$ in terms of $e_0$and e_1, we have

$$\hat{S}^2_{MJ} = S_y^2(1 + e_0) \begin{bmatrix} \alpha \left(\frac{S_x^2 + (TM + Q_a)}{S_x^2(1 + e_1) + (TM + Q_a)} \right) \\ +(1 - \alpha) \left(\frac{S_x^2(1 + e_1) + (TM + Q_a)}{S_x^2 + (TM + Q_a)} \right) \end{bmatrix} \quad (30)$$

Simplifying (30) up to first order approximation, it reduces to (31) as:

$$\hat{S}^2_{MJ} = S_y^2 \begin{bmatrix} 1 + e_0 + A_{MJ}(1 - 2\alpha)e_1 + \alpha A^2_{MJ} e_1^2 \\ +A_{MJ}(1 - 2\alpha)e_0 e_1 \end{bmatrix} \quad (31)$$

Where $A_{MJ} = \frac{S_x^2}{S_x^2 + (TM + Q_a)}$

Subtracting S_y^2 from both sides of (31)

$$\hat{S}^2_{MJ} - S_y^2 = S_y^2 \begin{bmatrix} e_0 + A_{MJ}(1 - 2\alpha)e_1 + \alpha A^2_{MJ} e_1^2 \\ +A_{MJ}(1 - 2\alpha)e_0 e_1 \end{bmatrix} \quad (32)$$

Taking Expectation of both sides of (32) and applying the results of (29) obtaining the $Bias(\hat{S}^2_{MJ})$ as:

$$Bias(\hat{S}^2_{MJ}) = \gamma S_y^2 \begin{bmatrix} \alpha A^2_{MJ}(\beta_{2(x)} - 1) \\ +A_{MJ}(1 - 2\alpha)(\lambda_{22} - 1) \end{bmatrix} \quad (33)$$

Squaring and expanding (32), obtaining (34) as:

$$MSE(\hat{S}^2_{MJ}) = S_y^4 \begin{bmatrix} e_0^2 + A^2_{MJ}(1 - 2\alpha)^2 e_1^2 \\ ++2A_{MJ}(1 - 2\alpha)e_0 e_1 \end{bmatrix} \quad (34)$$

Taking expectation and apply the results of (29), obtaining $MSE(\hat{S}^2_{MJ})$ as:

$$MSE(\hat{S}^2_{MJ}) = \\ = \gamma S_y^4 \begin{bmatrix} (\beta_{2(y)} - 1) + A^2_{MJ}(1 - 2\alpha)^2(\beta_{2(y)} - 1) \\ +2A_{MJ}(1 - 2\alpha)(\lambda_{22} - 1) \end{bmatrix} \quad (35)$$

Obtaining the expression for the value of α, differentiate $MSE(\hat{S}^2_{MJ})$ partially with respect to α and equate to zero as:

$$\frac{\partial(MSE(\hat{S}^2_{MJ}))}{\partial \alpha} = \gamma S_y^4 \begin{bmatrix} -4A^2_{MJ}(1 - 2\alpha)(\beta_{2(x)} - 1) \\ -4A_{MJ}(\lambda_{22} - 1) \end{bmatrix} \quad (36)$$

Simplifying (36) for α, obtaining optimum value of α as:

$$\alpha = \frac{1}{2} \left[1 + \frac{(\lambda_{22} - 1)}{A_{MJ}(\beta_{2(x)} - 1)} \right] \quad (37)$$

Substituting (36) in (34)

$$MSE(\hat{S}^2_{MJ})_{min} = \gamma S_y^4 \left[(\beta_{2(y)} - 1) - \frac{(\lambda_{22} - 1)^2}{(\beta_{2(x)} - 1)} \right] \quad (38)$$

3. EFFICIENCY COMPARISONS

In this section efficiencies of the proposed estimators are compared with efficiencies of some estimators in the literature.

The $\hat{S}^2_{MJ}$ of estimator of the finite population variance is more efficient than $Var(t)$ if,

$$\left[\left(\beta_{2(y)}-1\right)-\frac{(\lambda_{22}-1)^2}{(\beta_{2(x)}-1)}\right]<\left(\beta_{2(y)}-1\right) \quad (39)$$

The $\hat{S}^2_{MJ}$ of estimator of the finite population variance is more efficient than $\hat{S}^2_R$ if,

$$MSE\left(\hat{S}^2_{MJ}\right)_{min} < MSE\left(\hat{S}^2_R\right)$$
$$\left[\left(\beta_{2(y)}-1\right)-\frac{(\lambda_{22}-1)^2}{\left(\beta_{2(x)}-1\right)}\right] < \left[\begin{matrix}\left(\beta_{2(y)}-1\right)+\left(\beta_{2(x)}-1\right)\\ -2(\lambda_{22}-1)\end{matrix}\right] \quad (40)$$

The $\hat{S}^2_{MJ}$ of estimator of the finite population variance is more efficient than $\hat{S}^2_{kc_i}$ if,

$$MSE\left(\hat{S}^2_{MJ}\right)_{min} < MSE\left(\hat{S}^2_{kc_i}\right)$$
$$\left[\left(\beta_{2(y)}-1\right)-\frac{(\lambda_{22}-1)^2}{\left(\beta_{2(x)}-1\right)}\right] < \left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_i^2\left(\beta_{2(x)}-1\right)\\ -2A_i(\lambda_{22}-1)\end{matrix}\right] \quad (41)$$

The $\hat{S}^2_{MJ}$ of estimator of the finite population variance is more efficient than $\hat{S}^2_{sk_i}$ if,

$$MSE\left(\hat{S}^2_{MJ}\right)_{min} < MSE\left(\hat{S}^2_{sk_i}\right)$$
$$\left[\left(\beta_{2(y)}-1\right)-\frac{(\lambda_{22}-1)^2}{\left(\beta_{2(x)}-1\right)}\right] < \left[\begin{matrix}\left(\beta_{2(y)}-1\right)+R_i^2\left(\beta_{2(x)}-1\right)\\ -2R_i(\lambda_{22}-1)\end{matrix}\right] \quad (42)$$

The $\hat{S}^2_{MJ}$ of estimator of the finite population variance is more efficient than $\hat{S}^2_{kS}$ if,

$$MSE\left(\hat{S}^2_{MJ}\right)_{min} < MSE\left(\hat{S}^2_{kS}\right)$$
$$\left[\left(\beta_{2(y)}-1\right)-\frac{(\lambda_{22}-1)^2}{\left(\beta_{2(x)}-1\right)}\right] < \left[\begin{matrix}\left(\beta_{2(y)}-1\right)+W^2\left(\beta_{2(x)}-1\right)\\ -2W(\lambda_{22}-1)\end{matrix}\right] \quad (43)$$

The $\hat{S}^2_{MJ}$ of estimator of the finite population variance is more efficient than $\hat{S}^2_{jG}$ if,

$$MSE\left(\hat{S}^2_{MJ}\right)_{min} < MSE\left(\hat{S}^2_{jG}\right)$$
$$\left[\left(\beta_{2(y)}-1\right)-\frac{(\lambda_{22}-1)^2}{\left(\beta_{2(x)}-1\right)}\right] < \left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_{jG}^2\left(\beta_{2(x)}-1\right)\\ -2A_{jG}(\lambda_{22}-1)\end{matrix}\right] \quad (44)$$

The $\hat{S}^2_{MJ}$ of estimator of the finite population variance is more efficient than $\hat{S}^2_{sj}$ if,

$$MSE\left(\hat{S}^2_{MJ}\right)_{min} < MSE\left(\hat{S}^2_{sj}\right)$$
$$\left[\left(\beta_{2(y)}-1\right)-\frac{(\lambda_{22}-1)^2}{\left(\beta_{2(x)}-1\right)}\right] < \left[\begin{matrix}\left(\beta_{2(y)}-1\right)+A_{sj}^2\left(\beta_{2(x)}-1\right)\\ -2A_{sj}(\lambda_{22}-1)\end{matrix}\right] \quad (45)$$

When conditions (39), (40), (41), (42), (43), (44), and (45) are satisfied, we can conclude that the proposed estimator is more efficient than some selected existing estimators.

4. NUMERICAL ILLUSTRATION

In order to investigate the merits of the proposed estimator, we have considered the real population as:

Data: [Mur67]
$N = 80$, $n = 20$, $S_x = 8.4542$, $S_y = 18.3569$, $A_2 = 0.9615$, $C_x = 0.7507$, $A_3 = 0.9964$, $\bar{X} = 11.2624$, $\bar{Y} = 51.8264$, $\beta_{2(x)} = 2.8664$, $\beta_{2(y)} = 2.2667$, $A_4 = 0.9493$, $A_{jG} = 0.8763$, $\rho = 0.9413$, $\lambda_{22} = 2.2209$, $Md = 7.5750$, $Q_1 = 9.318$, $C_y = 0.3542$, $A_1 = 0.9896$, $Q_2 = 7.575$, $Q_3 = 16.975$, $Q_d = 5.9125$, $Q_a = 11.0625$, $Q_r = 11.82$, $TM = 9.318$.

Table 1: Bias and MSE of Some Selected Existing and Proposed Estimators

Estimators	Bias	MSE	PRE
Sample variance	0	5393.89	100
Isaki ([Isa83])	10.88	3925.16	137.4183
Kadilar & Cingi ([KC06]) 1	10.44	3850.16	140.0952
Kadilar & Cingi ([KC06]) 2	9.29	3658.41	147.4381
Kadilar & Cingi ([KC06]) 3	10.72	3898.56	138.356
Kadilar & Cingi ([KC06]) 4	8.81	3580.83	150.6324
Subramani & Kumarapandiyan ([SK12]) 1	8.17	3480.55	154.9723
Subramani & Kumarapandiyan ([SK12]) 2	3.91	2908.65	185.4431
Subramani & Kumarapandiyan ([SK12]) 3	5.50	3098.41	174.0857
Subramani & Kumarapandiyan ([SK12]) 4	7.82	3427.19	157.3852
Subramani & Kumarapandiyan ([SK12]) 5	5.77	3133.33	172.1456
Khan and Shabbir ([KS13])	3.62	2878.56	187.3815
Subramani & Kumarapandiyan ([SK15])	6.12	3180.77	169.5781
Maqbool & Shakeel ([MS17])	3.03	2820.06	191.2686
Proposed Estimator	3.96	1993.07	270.6322

5. CONCLUSION

From results of Table 1, we infer that the proposed variance estimator is more efficient than the existing estimators in the sense of having lesser Mean Square Error (MSE) and highest PRE compare to other existing estimators considered in this research. We therefore recommend for use in estimating finite population variance.

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