ON THE BURR X-TOPP LEONE DISTRIBUTION

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ABSTRACT: In this study, we construct a continuous probability distribution as an improvement of Topp-Leone distribution. However, density function and the cumulative function of the constructed distribution (Burr X-Topp Leone distribution) were obtained. Also, we show the validity of the new distribution.
KEYWORDS: Burr X G family, Topp-Leone distribution, Probability density function, cumulative density function.

1. INTRODUCTION

Probability distributions are used to explain many of the differences observed in real life phenomenon and to show how they can be described in simple numerical terms. The distribution shows the probability of an event occurring in one trial. It may be a test of human memory ([Sic85]), the patient's survival from protest cancer ([RSD74]) before the time of the next phone call.

Thus, many new classes of distributions have been developed in the literature by improving some common families of distributions by addition of one or more shape parameters to the baseline distribution. Also, introducing additional parameters to the existing probability distributions have been shown to improve the efficiency and goodness of fits of the distribution against the perception of model parsimony ([Y+16]). So, several researchers have proposed some methods of adding a parameter to distributions in the literature. As such, an adequate distribution will provide useful information for accurate decision sound conclusions. As such, when there is more flexible distributions, researchers tend to be sway to the new distribution ([SB16]).

Cordeiro *et al.*, ([C+11]) proposed a new distribution named beta generalized Rayleigh distribution. The authors derived some properties of their proposed distribution. However, the flexibility of the new distribution was shown through two real data sets which show the flexibility in analyzing positive data when compared to the generalized Rayleigh and Rayleigh distributions.

Eugene *et al.*, ([ELF02]) introduced a general class of distribution. The authors discussed shape properties of both beta-normal and the estimation of parameters of the beta-normal distribution with the aid of method of maximum likelihood. Thus, their methods show more flexibility in handling symmetric heavy-tailed distributions and also skewed and bimodal distributions. They concluded by applying the proposed distribution to two real data sets and compared the results to existing used methods.

Oguntunde *et al.*, ([O+17]) proposed Burr X-Exponential distribution using the Burr X family of distributions. The authors identified the basic statistical properties and the method of maximum likelihood in estimating the model parameters. However, they applied the proposed distribution to three different data sets to obtain its flexibility over its baseline distribution.

The main aim of this study is to construct a continuous probability distribution Burr X-Topp Leone Distribution.

2. METHODOLOGY

In this study, we considered Topp Leone as a baseline distribution. However, if a random variable T is distributed as the Topp-Leone and bouded on [0,1] ([SB16]). Let X be a continuous random variable with pdf $f(x)$. Topp Leone distribution has pdf givten by;

$$f(x) = \beta(2-2x)(2x-x^2)^{\beta-1} \quad (1)$$

Then the corresponding cdf is

$$F(x) = (2x-x^2)^{\beta} \quad (2)$$

Where $\beta > 0$ is a shape parameter.

The following is the cdf and pdf of the Burr X family of distribution is given by

$$G(x) = \left[1-\exp\left\{-\left(\frac{F(x)}{1-F(x)}\right)^2\right\}\right]^{\theta} \quad (3)$$

$$g(x) = \frac{2\theta f(x)F(x)}{[1-F(x)]^3}\exp\left[-\left(\frac{F(x)}{1-F(x)}\right)^2\right]\left[1-\exp\left\{-\left(\frac{F(x)}{1-F(x)}\right)^2\right\}\right]^{\theta-1} \quad (4)$$

Respectively ([O+17]), for $x > 0, \theta > 0$.
Where; θ is a shaped parameter whose role is to vary the tail weight.
While $F(x)$ and $f(x)$ are the cdf and pdf of the corresponding baseline distribution respectively.

3. PROPOSED BURR X –TOPP LEONE DISTRIBUTION

In this research, the Burr X distribution is used as a generator of baseline distribution as suggested by Yousof *et al.*, ([Y+16]).
The pdf of the proposed BXTLD is obtained by substituting the baseline distribution. That is, equations 1 and 2 into equation 4 respectively. As such, the proposed pdf is given as follows:

$$g(x) = \frac{2\theta\beta(2-2x)(2x-x^2)^{\beta-1}(2x-x^2)^{\beta}}{[1-(2x-x^2)^{\beta}]^3} \exp\left[-\left(\frac{(2x-x^2)^{\beta}}{1-(2x-x^2)^{\beta}}\right)^2\right]\left[1-\exp\left\{-\left(\frac{(2x-x^2)^{\beta}}{1-(2x-x^2)^{\beta}}\right)^2\right\}\right]^{\theta-1} \quad (5)$$

$$g(x)_{BXTLD} = \frac{2\theta\beta(2-2x)(2x-x^2)^{2\beta-1}}{[1-(2x-x^2)^{\beta}]^3} \exp\left[-\left(\frac{(2x-x^2)^{\beta}}{1-(2x-x^2)^{\beta}}\right)^2\right]\left[1-\exp\left\{-\left(\frac{(2x-x^2)^{\beta}}{1-(2x-x^2)^{\beta}}\right)^2\right\}\right]^{\theta-1} \quad (6)$$

Also, the cdf of BXTLD is obtained by substituting equation 2 in to equation 3. Then it is given as:

$$G(x)_{BXTLD} = [1-\exp\left\{-\left(\frac{(2x-x^2)^{\beta}}{1-(2x-x^2)^{\beta}}\right)^2\right\}]^{\theta} \quad (7)$$

4. MODEL VALIDITY CHECK OF THE PROPOSED BXTLD

In this section, we shows the validation of the proposed BXTLD. Thus, each distribution should be valid $\int_{-\infty}^{\infty} g(x)dx = 1$. As such;

$$\int_0^1 g(x)_{BXTLD}dx = 1 \quad (8)$$

Using the pdf of proposed BXTLD, we shows that $\int_0^1 g(x)_{BXTLD}dx = 1$

$$g(x) = \int_0^1 \frac{2\theta\beta(2-2x)(2x-x^2)^{\beta-1}(2x-x^2)^{\beta}}{[1-(2x-x^2)^{\beta}]^3} exp\left\{-\left(\frac{(2x-x^2)^{\beta}}{1-(2x-x^2)^{\beta}}\right)^2\right\}\left[1-\exp\left\{-\left(\frac{F(x)}{1-F(x)}\right)^2\right\}\right]^{\theta-1} dx \quad (9)$$

Let

$$y = (2x-x^2)^{\beta}, \, {dy}/{dx} = \beta(2x-x^2)^{\beta-1}(2-2x)$$
$$=> dx = \frac{dy}{\beta(2x-x^2)^{\beta-1}(2-2x)}$$

Substituting the above terms into equation 9, we obtained equation 10.

$$= \int_0^1 \frac{2\theta\beta(2-2x)(2x-x^2)^{\beta-1}y}{(1-y)^3} \exp\left\{-\left(\frac{y}{1-y}\right)^2\right\}\left[1-\exp\left\{-\left(\frac{y}{1-y}\right)^2\right\}\right]^{\theta-1} \frac{dy}{\beta(2x-x^2)^{\beta-1}(2-2x)} \quad (10)$$

Also, let

$$m = \left(\frac{y}{1-y}\right)^2, \, {dm}/{dy} = 2\left(\frac{y}{1-y}\right)\frac{1}{(1-y)^2}, dy$$
$$= \frac{(1-y)(1-y)^2}{2y} = \frac{(1-y)^3 dm}{2y}$$

From the above terms, it resulted to equation 11:

$$= \int_0^{\infty} \frac{2\theta y}{(1-y)^3} e^{-m}(1-e^{-m})^{\theta-1}\frac{(1-y)^3 dm}{2y} \quad (11)$$
$$= \int_0^{\infty} \theta e^{-m}(1-e^{-m})^{\theta-1}dm \quad (12)$$

Let

$$w = 1-e^{-m}, \, {dw}/{dm} = e^{-m}, \, dm = \frac{dw}{e^{-m}}$$

substitute these into equation 12, we obtained equation 13.

$$= \int_0^1 \theta e^{-m} w^{\theta-1}\frac{dw}{e^{-m}} \quad (13)$$
$$= \int_0^1 \theta\, w^{\theta-1}dw \quad (14)$$

Integrating equation 14, we have:

$$\theta\left[\frac{w^{\theta-1+1}}{\theta-1+1}\right]_0^1 => [w^{\theta}]_0^1 => 1-0 = 1.$$

Hence, the proof.

5. CONCLUSION

In this paper, a continuous probability distribution as an improvement of Topp-Leone distribution is constructed. As such, we obtained the density function and the cumulative function of the constructed distribution. Also, the validity of the new distribution was illustrated.

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