About a property of correctness in the fuzzy process space

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REZUMAT. Lucrarea prezintă câteva aspecte legate de proprietățile de corectitudine relativă şi absolută în spațiul proceselor fuzzy. Sunt introduse două definiții alternative ale relației de rafinare a proceselor fuzzy (din punctul de vedere al testării, respectiv al implementării) şi sunt demonstrate propozițiile care demonstrează echivalența acestor definiții.

1 Introduction

A process can describe a device through an agreement between the device and its environment as regarding the executions. All the executions may appear, although some can be interdicted by the agreement. Some of the executions are not allowed to appear, a "blame" either on the device or on the environment being assigned.

Considering E the set of all the executions and let $\Delta : E \to [0,1]$ and $\Gamma : E \to [0,1]$ two fuzzy subsets of E. As follows, we note with

$X = \{x \in E \mid \Delta(x) > 0\}$, $Y = \{x \in E \mid \Gamma(x) > 0\}$ and $B = \{x \in E \mid \Delta(x) = \Gamma(x) = 0\}$

and we, respectively, use the following terminology:

X - the ***accessible*** set of executions;

Y - the ***acceptable*** set of executions;

B - the ***blocking*** set.

Moreover, we use the notation $\Delta_X = \Delta_{/X}$, $\Gamma_X = \Gamma_{/X}$

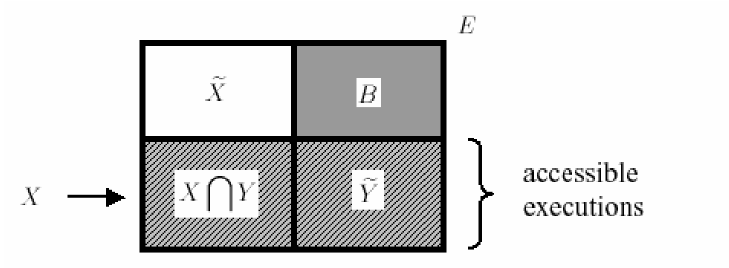


Figure 1: Accessible executions

Definition 1.1 *The pair* $\pi = (\Delta_X, \Gamma_X)$, *where* Δ_X *and* Γ_X *are defined as above, is called a* ***fuzzy process*** *over E.*

The set of all the fuzzy processes over a pair of subsets crisp X and Y of E, as above, is called ***the fuzzy space processes of (X, Y)***, *and the set of all the fuzzy processes over E is called* ***the fuzzy space processes of E.*** ■

A fuzzy process $\pi = (\Delta_X, \Gamma_X)$ represents an agreement between the device and its environment: the device guarantees that only the executions of *X* can appear, while the environment guarantees that only the executions of *Y* can appear.

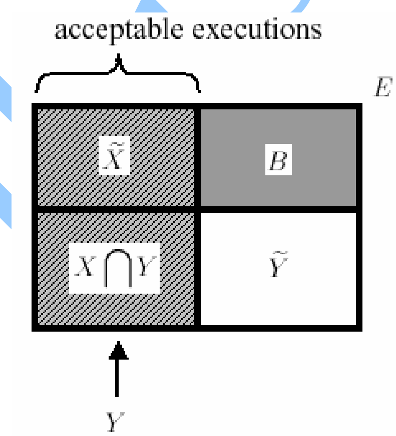


Figure 2: Acceptable executions

The executions of *X* are called *accessible,* the executions of *Y, acceptable,* and the executions of *B*, as we mentioned above, are called blockings and they do not answer our purpose. Δ_X and Γ_X give, respectively, the grade of accessibility and acceptability. The device can access, respectively, accept an execution. An execution $x \in E$ is ***(X,Y)-***

completely accessible if $\Delta(x) = 1$ and it is (X,Y)- ***completely acceptable*** if $\Gamma(x) = 1$.

From the viewpoint of the classic (crisp) theory of sets, a fuzzy process parts the E set of all the executions in four disjunctive subsets: $\widetilde{X}$, $\widetilde{Y}$, $X \cap Y$ and B, where:

$$\widetilde{X} = \{x \in E | \Delta(x) = 0 \quad şi \quad \Gamma(x) > 0\},$$

$$\widetilde{Y} = \{x \in E | \Gamma(x) = 0 \quad şi \quad \Delta(x) > 0\}$$

Obviously, $\widetilde{X} \cap \widetilde{Y} = \varnothing$

We call the elements of $\widetilde{X}$ ***escapes*** and the device must avoid them. We call the elements of $\widetilde{Y}$ ***rejections*** and the environment must avoid them. The elements of $\widetilde{X} \cup \widetilde{Y}$ together are called ***violations.***.

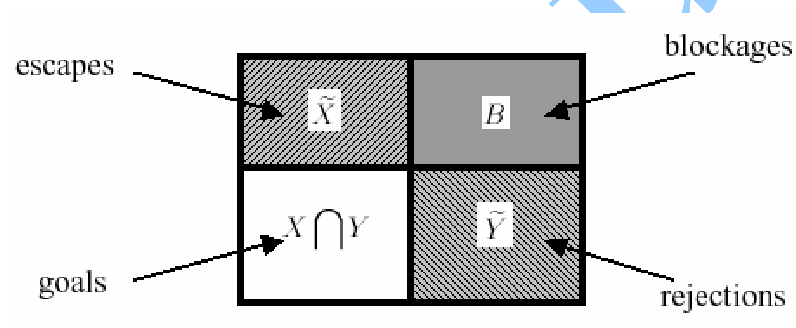


Figure 3: Types of executions

The agreement stipulates that only the executions in $X \cap Y$ are permitted to appear in the presence of the device. For this reason, $X \cap Y$ is also called the ***contract set***, and the executions $x \in X \cap Y$ are also called ***goals***, because they are linked both for the device and for the medium. The set X contains the executions for which the device respects the agreement (and the environment can respect it or not); the Y set contains the executions for which the environment respects the agreement (and the device can respect it or not). In the set $X \cup Y$ we still can have executions with the grade of acceptability or the grade of accessibility equal to zero, but not both to zero for a given execution.

2 The pattern correctness conditions

The property of absolute correctness in the fuzzy process space formalizes the fact that a device operates by itself, namely the device does not inflict any demand to the environment. In that sense, the device is called robust. In the terms of the avoided executions, this property leads to an empty set of refusals for the corresponding process.

The symmetric property is that the device does not offer any guarantee to the environment. In the terms of the avoided execution, this leads to an empty set for the correspondent process. In this sense, the device is called ***chaotic.***

Definition 2.1 *The fuzzy process* $\pi = (\Delta_X, 1_E)$ *is called* ***robust****, while the fuzzy process* $\pi = (1_E, \Gamma_Y)$*, is called* ***chaotic.*** ■

As follows, we denote by $\mathcal{R}_E$ and $\mathcal{H}_E$ the set of robust processes, respectively, chaotic over *E*. $(1_E(x) = 1, \forall x \in E, \quad 0(x) = 0, \forall x \in E)$.

The process $\pi = (1_E, 1_E)$ is the only fuzzy process that is robust and chaotic at the same time.

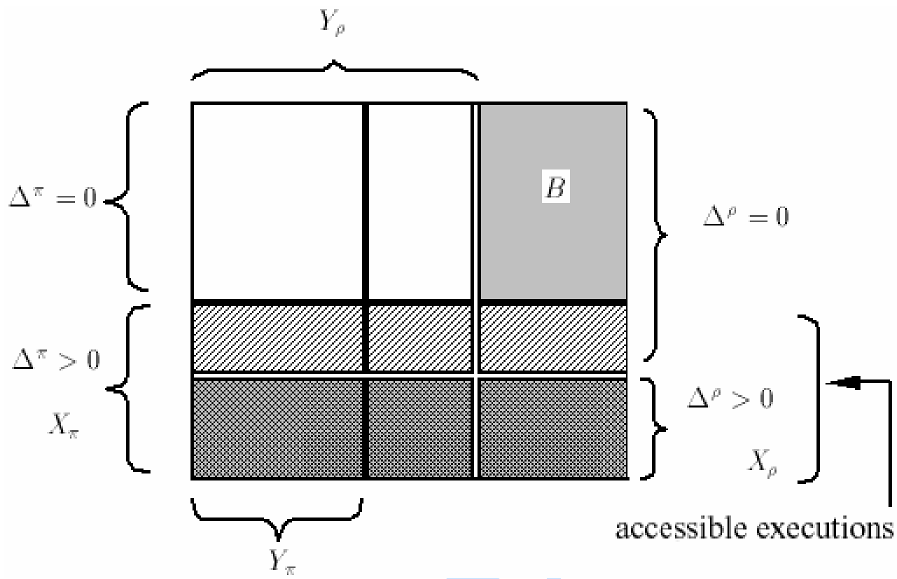
Definition 2.2 *The void fuzzy process,* Ω*, is given by* $\Omega = (1_E, 1_E)$ ■

This process has no escapes and no refusals. Therefore, it does neither offer any guarantees, nor inflict any restrictions to the environment. Its behavior is similar to the vacuum.

The property of ***relative correctness*** in the fuzzy process space formalizes the fact that a ρ fuzzy process is a suitable substitute for a fuzzy process π. ρ should inflict less demands to the environment and offer more guarantees than π. In terms of the avoided execution, this means that ρ has a bigger set of acceptable executions and a smaller set of accessible executions, comparing to those corresponding to π.

Definition 2.3 *Considering two given fuzzy processes for the same set E of executions,* $\pi = (\Delta^{\pi}_{X_\pi}, \Gamma^{\pi}_{X_\pi})$ *and* $\rho = (\Delta^{\rho}_{X_\rho}, \Gamma^{\rho}_{X_\rho})$*, we say that the fuzzy process* π *is refined by the fuzzy process* ρ *and we write* $\pi \sqsubseteq \rho$ *if and only if*

$$(\Delta^{\pi}(x) \geq \Delta^{\rho}(x)) \wedge (\Gamma^{\pi}(x) \leq \Gamma^{\rho}(x)), \forall x \in E. \quad \blacksquare$$



$$X_\pi = \{x \in E \mid \Delta^{\pi}(x) > 0\} \qquad X_\rho = \{x \in E \mid \Delta^{\rho}(x) > 0\}$$

$$\Delta^{\pi}(x) \geq \Delta^{\rho}(x), \forall x \in E, \quad X_\pi \supseteq X_\rho$$

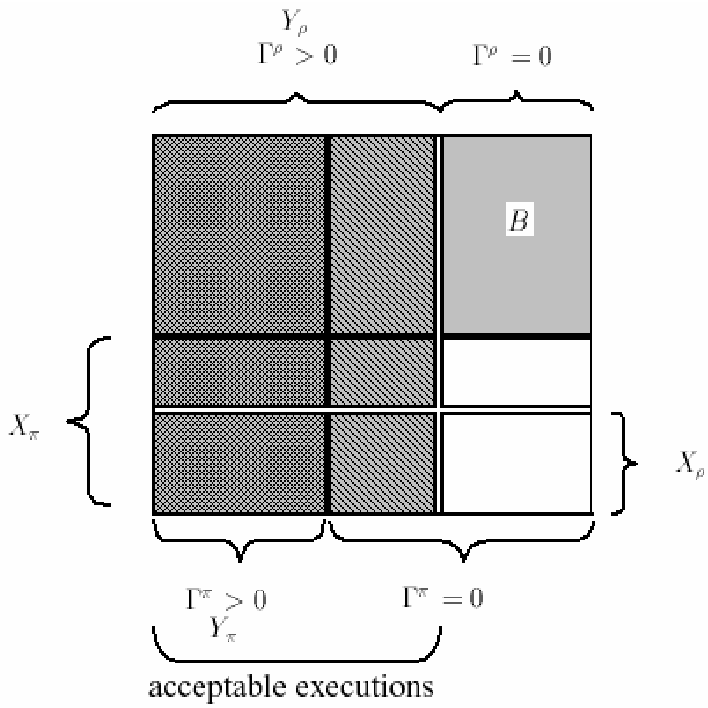
Figure 4: Refinement – accessible executions

The above condition involves that:

$$(X_\pi \supseteq X_\rho) \wedge (Y_\pi \subseteq Y_\rho).$$

If the accessibility grade for an execution is smaller, the behavior of the device is more particular, meaning the device has less freedom and reacts to tighter restrictions.

Intuitively, the refining represents a relative notion of correctness: $\pi \sqsubseteq \rho$ means that π may be replaced by ρ without producing any undesirable results.



$$Y_\pi = \{x \in E \mid \Gamma^\pi(x) > 0\} \qquad Y_\rho = \{x \in E \mid \Gamma^\rho(x) > 0\}$$

$$\Gamma^\pi(x) \geq \Gamma^\rho(x), \forall x \in E, \quad Y_\pi \subseteq Y_\rho$$

Figure 5: Refinement –acceptable executions

A better process accepts more executions, consequently it is less exposed to an inadequate utilization; and a better process accesses fewer executions, consequently it is less exposed to an inadequate utilization by the other processes. The following property shows that the refinement is a partial order.

Proposition 2.4 *The $\sqsubseteq$ relation is a partial order relation in the fuzzy process set, namely for any fuzzy processes π, ρ and σ over the set of the execution E:*

[R] $\pi \sqsubseteq \pi$

[AS] $\pi \sqsubseteq \rho \;\wedge\; \rho \sqsubseteq \pi \;\Rightarrow\; \pi = \rho$

[T] $\pi \sqsubseteq \rho \;\wedge\; \rho \sqsubseteq \sigma \;\Rightarrow\; \pi \sqsubseteq \sigma$

Proof. *If* $\pi = (\Delta^{\pi}_{X_\pi}, \Gamma^{\pi}_{X_\pi})$, $\rho = (\Delta^{\rho}_{X_\rho}, \Gamma^{\rho}_{X_\rho})$ *and* $\rho = (\Delta^{\rho}_{X_\rho}, \Gamma^{\rho}_{X_\rho})$, *then the above properties immediately result from the definition 3.3.*

For instance, to illustrate the anti-symmetry property

$$\pi \sqsubseteq \rho \quad \wedge \quad \rho \sqsubseteq \pi \quad \Rightarrow \quad \pi = \rho$$

is equivalent with

$$((\Delta^{\pi} \geq \Delta^{\rho}) \wedge (\Gamma^{\pi} \leq \Gamma^{\rho})) \quad \wedge \quad ((\Delta^{\rho} \geq \Delta^{\pi}) \wedge (\Gamma^{\rho} \leq \Gamma^{\pi}))$$

namely with

$$((\Delta^{\pi} = \Delta^{\rho}) \wedge (\Gamma^{\pi} = \Gamma^{\rho}))$$

that is

$$\pi = \rho \quad \blacksquare$$

Proposition 2.5 *Considering two fuzzy processes* π *and* ρ *over the set of the executions E, then if*

$$\pi \in \mathcal{R}_E \text{ şi } \pi \sqsubseteq \rho \;\Rightarrow\; \rho \in \mathcal{R}_E$$

Proof. $\pi \in \mathcal{R}_E \;\Rightarrow\; \pi = (\Delta^{\pi}_{X_\pi}, 1_E)$ *and* $\pi \sqsubseteq \rho$, $\rho = (\Delta^{\rho}_{X_\rho}, \Gamma^{\rho}_{X_\rho})$

$\Rightarrow (\Delta^{\pi} \geq \Delta^{\rho}) \wedge (1_E \leq \Gamma^{\rho})$

that is

$$\Gamma^{\rho} = 1_E \text{, therefore } \rho = (\Delta^{\rho}_{X_\rho}, 1_E) \;\Leftrightarrow\; \rho \in \mathcal{R}_E \quad \blacksquare$$

The above proposition determines that if the robustness is considered a predicate over the processes, than it is monotonous as to the refinement. Its interpretation is that, if π is "correct by itself" and ρ "at least as good" as π, then ρ is, also, "correct by itself".

Corollary 2.6 *For a fuzzy process* π *over the set of executions E:*

$$\pi \in \mathcal{R}_E \;\Leftrightarrow\; \Omega \sqsubseteq \pi$$

Proof. *From the definition of the refinement relation and of the void process:*

$$\Omega \sqsubseteq \pi \;\Leftrightarrow\; (1_E \geq \Delta^{\pi}) \wedge (1_E \leq \Gamma^{\pi})$$

Since $1_E \geq \Delta^{\pi}$ *is trivial, we have that*

$$\Omega \sqsubseteq \pi \;\Leftrightarrow\; 1_E \leq \Gamma^{\pi} \;\Leftrightarrow\; 1_E = \Gamma^{\pi} \;\Leftrightarrow\; \pi = (\Delta^{\pi}_{X_\pi}, 1_E) \;\Leftrightarrow\; \pi \in \mathcal{R}_E \quad \blacksquare$$

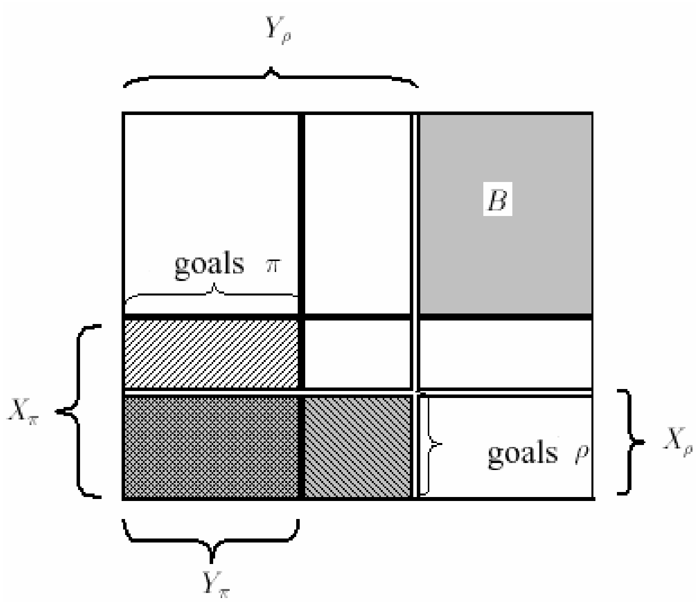


Figure 6: Refinement – goal executions

The interpretation is that ρ is "correct by itself", if and only if π is "at least as good" as a process which is so indifferent to its medium that it can be seen as a "vacuum".

Definition 2.7 *Considering a set of executions E, the fuzzy process* $(0_E, 1_E)$ *is called* ***top*** *and it is denoted* $\top$*, and the fuzzy process* $(1_E, 0_E)$ *is called* ***bottom*** *and it is denoted* $\perp$. ■

Let us note that the *top* fuzzy process is robust, and the *bottom* fuzzy process is chaotic, for a given set of executions.

Proposition 2.8 *For any fuzzy process* π *over the set of the executions E, we have that:*

$$\perp \sqsubseteq \pi \sqsubseteq \top$$

Proof. $\perp = (1_E, 0_E)$, $\top = (0_E, 1_E)$, $\pi = (\Delta^\pi_{X_\pi}, \Gamma^\pi_{X_\pi})$

$$(1_E \geq \Delta^\pi \geq 0_E) \wedge (0_E \leq \Gamma^\pi \leq 1_E) \quad ■$$

3 Results and conclusions

An alternative definition of refinement is to say that an "implementation" ρ is correct with respect to a "specification" π if ρ operates corectly in the environment of π. The question arises whether this alternative definition is equivalent to that in Definition 2.3. The following Proposition answers this question, and, by that, connects our relative and absolute notions of correctness.

Proposition 3.1 *Considering two fuzzy processes π and ρ over the set of the executions E, then*

$$\pi \sqsubseteq \rho \quad \Leftrightarrow \quad -\pi \otimes \rho \in R_E$$

Proof. *Let* $\pi = (\Delta^{\pi}_{X_\pi}, \Gamma^{\pi}_{Y_\pi})$ *and* $\rho = (\Delta^{\rho}_{X\rho}, \Gamma^{\rho}_{Y\rho})$ *(see figure 7)*

$$-\ \pi \otimes \rho \in \mathrm{R}_E \Leftrightarrow \Gamma^{\pi\otimes\rho} = 1_E \quad \Leftrightarrow$$

$$(Y_{-\pi} \cap Y_\rho) \cup (\widetilde{X}_{-\pi} \cap \widetilde{Y}_\rho) \cup (\widetilde{Y}_{-\pi} \cap \widetilde{X}_\rho) = E$$

$$(X_\pi \cap Y_\rho) \cup (\widetilde{Y}_\pi \cap \widetilde{Y}_\rho) \cup (\widetilde{X}_\pi \cap \widetilde{X}_\rho) = E$$

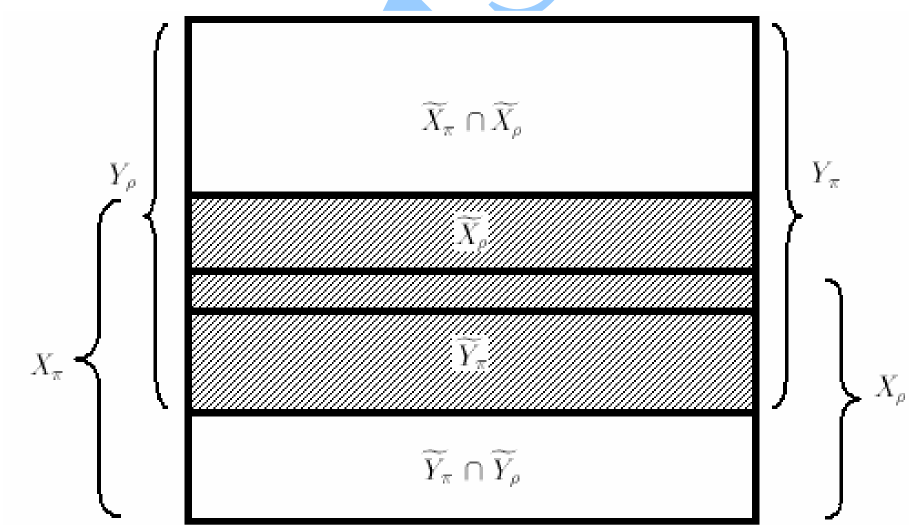


Figure 7. ■

Proposition 3.1 permits us to verify whether an implementation satisfies a specification by placing the implementation in the environment of the specification, and checking an absolute correctness condition on their product. Such approaches were taken in [Ebe91] and further developed in [Ver94].

Another alternative definition of refinement uses testing point of view: ρ is "better than as good as" π if ρ passes all tests that π passes. Passing a test σ can be viewed as the absence of rejects when the device is coupled with σ. Proposition 3.2 show that this testing definition of refinement is equivalent to Definition 2.3, and thereby provides another link between the absolute and relative notions of correctness in fuzzy process spaces.

Proposition 3.2 *Considering two fuzzy processes π and ρ over the set of the executions E,*

$$\pi \sqsubseteq \rho \quad \Leftrightarrow \quad \forall \sigma : (\sigma \otimes \pi \in R_E \ \Rightarrow \ \sigma \otimes \rho \in R_E)$$

Proof. *By proposition 3.1 we have*:

$$\pi \otimes \rho \ \in R_E \ \Rightarrow \ -\sigma \sqsubseteq \pi$$

and

$$\sigma \otimes \rho \ \in \mathrm{R}_E \ \Rightarrow \ -\sigma \sqsubseteq \rho$$

Thus, it is sufficient to prove

$$\pi \sqsubseteq \rho \ \Leftrightarrow \ \forall \sigma : (-\sigma \sqsubseteq \pi \ \Rightarrow \ -\sigma \sqsubseteq \rho)$$

By transitivity of refinement:

$$\pi \sqsubseteq \rho \ \wedge \ -\sigma \sqsubseteq \pi \ \Rightarrow \ -\sigma \sqsubseteq \rho$$

Reciprocal, let $\sigma = -\pi$. *That* $-\sigma = \pi$ *and by transitivity of refinement* $\Rightarrow -\sigma \sqsubseteq q$. *By hypotesis* $-\sigma \sqsubseteq \rho$. *Finally, since* $-\sigma = \pi$, *we have* $\pi \sqsubseteq \rho$ ■

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