On the Evaporation Mechanism in the Ant Colony Optimization Algorithms

Prasanna Kumar
Department of Mathematics, Birla Institute of Technology and Science Pilani - K.K.Birla Goa Campus, Zuarinagar, Goa, India, 403726

Raghavendra G. S.
Department of Computer Science, Birla Institute of Technology and Science Pilani - K.K.Birla Goa Campus, Zuarinagar, Goa, India, 403726

ABSTRACT: Ant colony optimization is an evolutionary algorithm in which ant colonies assist in locating shortest routes to food sources. One of the factors influencing the ant's behavior and performance is the pheromone evaporation. In this paper, a new approach towards pheromone evaporation mechanism in ACO through the exponential generating function is discussed. The convergence of the formula we derived with respect to the time, shows the advantages of this approach over more traditional ones given in the literature.
KEYWORDS: Algorithm, Pheromone evaporation, Generating function.

Tibiscus

Introduction

Ant Colony Optimization (ACO) is a metaheuristic proposed by Dorigo, that has been inspired by the foraging behavior of ant colonies [DS04]. In fact, this technique imitates the behavior of a colony of ants and their ability to collectively solve some combinatorial hard problems. In the last few years, ACO has proved its importance in the resolution of several different NP-hard combinatorial optimization problems. Birattari et al. developed Ant programming [BDD00], while Gutjahr presented a convergence proof for a particular ACO algorithm called Graph Based Ant System (GBAS) [Gut00].

Later, Gutjahr demonstrated for a time-dependent modification of the GBAS, that its current solutions converge to an optimal solution with probability exactly equal to 1 [Gut02]. Stutzle and Dorigo presented a short convergence proof for a class of ACO algorithms [SD02], where the global best pheromone update is used, and a lower limit on the range of the feasible pheromone trail is forced. They proved that the probability of finding the optimal solution could be made arbitrarily close to 1 if the algorithm is run for a sufficiently large number of iterations.

1. Pheromone Evaporation

It has been observed that a colony of ants is able to find the shortest path to a food source. As an ant moves and searches for food, it lays down a chemical substance called pheromone along its path. As the ant travels, it looks for pheromone trails on its path and prefers to follow trails with higher levels of pheromone deposits. If there are multiple paths to reach a food source, an ant will lay the same amount of pheromone at each step regardless of the path chosen. However, it will return to its starting point quicker when it takes the shorter path, which contains more pheromone. It is then able to return to the food source to collect more food. Thus, in an equal amount of time, the ant would lay a higher concentration of pheromone over its path if it takes the shorter path, since it would complete more trips in the given time. The pheromone is then used by other ants to determine the shortest path to find food as described in [DS04].

During the process, another factor affects on the amount of pheromone deposition namely, evaporation of pheromone, which can be seen as an exploration mechanism that delays faster convergence of all ants towards a suboptimal path. The decrease in pheromone intensity favors the exploration of different paths during the whole search process. In real ant colonies, pheromone trail also evaporate, but as we have seen, evaporation does not play an important role in real ant's shortest path finding. But on the contrary, the importance of pheromone evaporation in artificial ants is probably due to the fact that the optimization problems tackled by artificial ants are much more complex than those real ants can solve. A mechanism like evaporation that by favoring the forgetting of errors or of poor choices done in the past plays the important function of bounding the maximum value achievable by pheromone trails. In S-ACO, the pheromone evaporation is interleaved with pheromone deposit of ants. After each ant

Tibiscus

has moved to a next node according to ant's search behavior, pheromone trails are evaporated by applying the following equation to all arcs;

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij}, \qquad \forall i, j \in A$$

where $\rho \in [0, 1]$ is a parameter, A is the set of all nodes in the problem and τ_{ij} is the artificial pheromone trail associated with each arc (i, j). The value of pheromone evaporation lies between 0 and 1. For further details one can refer [DS04].

As pheromone evaporation plays some role in the efficiency of the algorithm, an effective formula for finding the rate at which the evaporation occurs is needed. We have come up with such a new formula, which improves all such attempts done by the researchers in the past. In fact, here we discuss in detail, the mathematical aspects and also more importantly, the convergence of the formula to authenticate its validity. Now we prove our results, which are motivated by a paper due to E. Foundas and A. Vlachos [FV06].

Theorem 1. *Let the pheromone evaporation at time t be ρ_t, where the value of ρ_t lies in the closed interval [0, 1]. Now the recurrence relation for the evaporation of pheromone at time $t+1$ is given by*

$$\rho_{t+1} = \alpha\rho_t + \beta(1-\rho_t) = k\rho_t + \beta \qquad (1)$$

where α, β are two constants such that $0 \le \alpha, \beta \le 1$ and $k = \alpha - \beta$.

Proof. It is obvious that the formula given above is well defined at $t+1$. By hypothesis, ρ_t at time t must lie in the interval [0, 1]. Thus if we show the value of ρ_{t+1} is always in [0, 1], the proof is done.

We write $f(\rho_t) = \rho_{t+1}$. Then from (1) it follows that $f'(\rho_t) = \alpha - \beta$. But then $f'(\rho_t) = 0$ if $\alpha = \beta$, which leads us to conclude that the maximum value of ρ_{t+1} is just β, which is less than 1. It is quite obvious that the minimum value of ρ_{t+1} is achieved only at $\alpha = \beta = 0$, in which case $\rho_{t+1} = 0$. Thus we have $0 \le \rho_{t+1} \le 1$.

Now we establish a new expression for the pheromone evaporation through exponential generating function. In fact, we prove,

Hibiscus

Theorem 2. *Let the pheromone evaporation at time t be ρ_t, where the value of ρ_t lies in the closed interval [0, 1] and the rate at which the evaporation occurs be given by the formula (1) with the additional condition $\alpha \geq \beta$. Then*

$$\rho_t = \frac{\beta(1-k^t)}{1-k} \quad if \quad k \neq 1. \tag{2}$$

Proof. The exponential generating function for ρ_t is given by,

$$\begin{aligned} f(x) = \sum_{t=0}^{\infty} \rho_t \frac{x^t}{t!} &= \rho_0 + \sum_{t=1}^{\infty} (k\rho_{t-1} + \beta) \frac{x^t}{t!} \\ &= \rho_0 + k\sum_{t=1}^{\infty} \rho_{t-1} \frac{x^t}{t!} + \beta \sum_{t=0}^{\infty} \frac{x^t}{t!} \\ &= \rho_0 + k\sum_{t=0}^{\infty} \rho_t \frac{x(t+1)}{(t+1)!} + \beta e^x . \end{aligned}$$

But then,

$$f'(x) = k\sum_{t=0}^{\infty} \rho_t \frac{x(t)}{t!} + \beta e^x = kf(x) + \beta e^x .$$

Thus we have a first order, first degree, linear differential equation in x as follows;

$$\frac{dy}{dx} - ky = \beta e^x \tag{3}$$

Where $y = f(x)$. One can easily find its general solution which is given by

$$y = \frac{\beta e^x}{1-k} + ce^{kx},$$

where $k \neq 1$ and c being an arbitrary real constant. By taking $c = -\frac{\beta}{1-k}$, we get a particular solution of (3) as

$$y = \frac{\beta}{1-k}(e^x - e^{kx}).$$

Hence, we have established an identity

$$f(x) = \sum_{t=0}^{\infty} \rho_t \frac{x^t}{t!} = \frac{\beta}{1-k} \sum_{t=0}^{\infty} (1-k^t) \frac{x^t}{t!}.$$

The comparison of corresponding coefficients in the above two power series yields the desired formula (2).

Remark 1. The value of ρ_t must be taken as the initial pheromone evaporation value ρ_0, if $\alpha = 1$, $\beta = 0$ in other way if $k = 1$. Readers should observe that the above conclusion is not arbitrarily drawn. It can be verified, just by substituting the values $\alpha = 1$, $\beta = 0$ in identity (1).

Remark 2. One can easily see that the theorem 2 and the remark 1 can be combined to get the expression for all values of k such that $0 \le k \le 1$ as follows:

$$\begin{aligned} \rho_t &= \frac{\beta(1-k^t)}{1-k} \quad if \quad k \neq 1 \\ &= \rho_0 \quad if \quad k = 1. \end{aligned} \qquad (4)$$

Theorem 3. *The value of ρ_t given by (4) converges.*

Proof. From the theorem 2.2, we have, $\rho_t = \frac{\beta(1-k^t)}{1-k}$ if $k \neq 1$. We are aware of the fact that $t \to \infty \Rightarrow k^t \to 0$ if $k \neq 1$. Therefore,

$$\lim_{t\to\infty} \rho_t = \frac{\beta}{1-k} \quad if \quad k \neq 1.$$

Tibiscus

If k = 1, it is obvious that the sequence $\{\rho_t\}$ converges to ρ_0.

Remark 3. By giving appropriate values for the parameters α, β, we can get the limit of convergence of ρ_t equal to zero or close to zero. Then the pheromone evaporation becomes almost nil which is expected during the implementation of ACO. Therefore the formulae given above shall enhance the performance quality of the ACO algorithm.

References

[BDD00] **M. Birattari, G. Di Caro, M. Dorigo** - *For a formal foundation of the ant programming approach to combinatorial optimization: The problem, the representation, and the general solution strategy*, Technical Report, TR- H-301, ATR-Human Information Processing Labs, Kyoto, Japan, 2000.

[DS04] **M. Dorigo, T. Stuetzle** - *Ant colony optimization*, MIT Press/Bradford Books, Cambridge, MA, 2004.

[FV06] **E. Foundas, A. Vlachos** - *New approaches to evaporation in ant colony optimization algorithms*, Jl.Interdis.Math., Vol. 9, 179-184, 2006.

[Gut00] **J. Walter Gutjahr** - *A graph-based ant System and its convergence*, Fut.Gen.Comp.Syst., Vol. 16, 873-888, 2000.

[Gut02] **J. Walter Gutjahr** - *ACO algorithms with guaranteed convergence to the optimal solution*, Inform.Proc.Let., Vol. 82, 145-153,2002.

[SD02] **T. Stutzle, M. Dorigo** - *A short convergence proof for a class of ant colony optimization algorithms*, IEEE Transaction on Evolutionary Computation, 6, 358-365, 2002.

Tibiscus