

## **GeoGebra: Another way of looking at Mathematics**

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**ABSTRACT:** In our modern society, where computer science implication in each domain is growing very fast, it became imperative to find ways of introducing it into the teaching process. World-wide trend is to integrate technology in order to develop teaching materials. As free dynamic mathematical software, GeoGebra represents a great opportunity for teachers, allowing them to create interactive learning environments and to visually explain the theoretical aspects included in the curricula.

**KEYWORDS:** GeoGebra, educational software, CAS.

### **Introduction**

The fast changes which affect modern society have a great influence in the educational process in what concerns both the pupil/student, who is more and more oriented to practical aspects and to the usefulness of the information he/she receives, and the teacher/professor who is involved in a continuous process of improvement and modernization of the teaching-learning-assessment activities. The very elaborated curricula and their global non-correlation, the auditorium's lack of interest on one hand, altogether with the requests related to the increasing of the quality and efficiency of the educational act, on the other hand, were correlated with the pressures made by the informational society which led to the building of different tools which are supposed to meet as many requirements as possible.

One of the main global objectives within the technical revolution of the educational process is the creation of technologists, mathematicians, IT engineers and trainers communities who are supposed to find the best way to integrate ITC in the educational process. GeoGebra is the first ITC worldwide instrument under evaluation in order to unitarily introduce ITC within the educational act.

### **1. GeoGebra - dynamic mathematics for everyone**

GeoGebra is a dynamic, free, open-source software which allows the exposure, the view and the practice of mathematics knowledge in order to rapidly share and understand the information. This software is characterized by versatility, dynamics, possibility to use it in an increasing number of languages, different versions of installation for using it online or offline and also by the possibility of spreading the files on the web – for everyone's benefit.

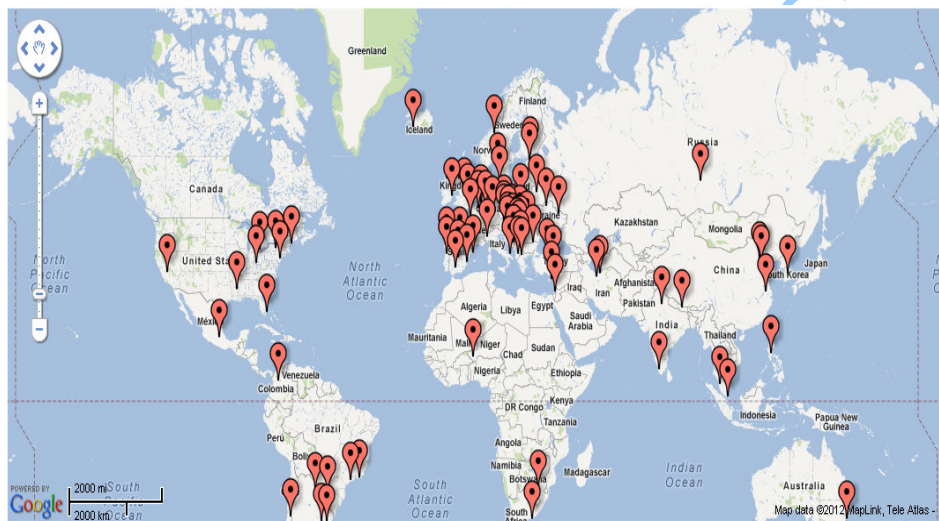
By looking at the open-source initiatives which have been developed in recent years, GeoGebra has developed accordingly since 2002; it stands for a dynamic educational and mathematical software which gathers geometry, algebra and mathematical analysis knowledge [HL10]. Designed like a combination between dynamic geometrical software and CAS; GeoGebra also includes facilities for several representations of mathematical objects, algebra, geometry and spreadsheet which are integrated in an easily to install and to use application. Among other advantages one can notice that it is a free, independent of platform and easily integrated in presentations software (it is based on Java).

Being in a continuous process of development, improvement and adaptation to the user's needs, GeoGebra contributed to the formation of an international community which is very active and which consists of teachers, researchers and IT engineers- all coordinated by the GeoGebra International Institute, with the purpose of promoting learning and teaching mathematics.

The 85 GeoGebra National Institutes, along with that from Timișoara, have set a few objectives for its activity, among which we mention activities of:

- *Training and support*: it coordinates, supports and offers opportunities of professional development for the teachers' training (pre-service) and improvement (in-service)

- *Improvement and dissemination*: of the information presented in the workshops, of the educational materials used in the classroom along with the continuous improvement and extent of the educational software GeoGebra
- *Research and cooperation*: coordination and research support in GeoGebra area which is focused on teaching and learning Maths with the purpose of informing and improving the development activities, altogether with promoting the cooperation between the International Institute and the local institutes GeoGebra.



**Figure 1. The worldwide GeoGebra Institute net**

At international level, there are many studies and research articles which evaluate the benefits of using GeoGebra within the process of teaching and learning mathematics. The curricula for secondary level has suffered changes in a few European countries, so that GeoGebra has been included as an instrument for both teaching and learning activities. Respecting the idea of sharing the knowledge for the benefit of education, there are support materials for the teacher's professional development, users' guides and a vast collection of examples for all categories of users. Research is conducted related to a good inclusion of this software in the educational process and also studies related to particular cases; there are available applications in domains like: physics, chemistry, geography, economy or music. At university level, besides the use of GeoGebra within the disciplines of superior mathematics, one works for the evaluation and improvement of teachers' professional development.

## 2. Didactic project with GeoGebra - the trigonometric circle

Solomon Marcus, the academician, said: "to the extent that the students will learn to embrace the beauty of mathematics, sciences, arts and literature and when they will feel the need to study them, they won't suffer anymore from boredom and temptation of ridiculous activities, sometimes antisocial will decrease".

Having in mind these words and starting from some basic principles of teaching and evaluation such as "the teacher must create diverse methods of learning that must reach the objectives goals", "learning means continuous investigations" and "the evaluation must apply a great variety of methods" the authors understood that the students need in order to learn in an interactive mode the notions of mathematics such as the trigonometric circle. In this context, GeoGebra offers the chance to put in application this plan.

As starting point for this lesson, a coordinate system is considered in two dimensions and a unit circle is built. Some intuitive conventions are made: a point moving on the circle in a direct sense if its movement is clockwise retrograde. The same point is moving inverse if its motion is clockwise. This unit circle, altogether with the coordinate reference fixed and the direct and inverse motion established is called the trigonometric circle.

One of the most difficult aspects for students is to understand the reduction formula to the first quadrant. Using trigonometric functions periodicity, the problem of determination the values of this functions is reduced to the determination of the values for the restrictions of the functions in the interval  $[0, 2\pi]$ .

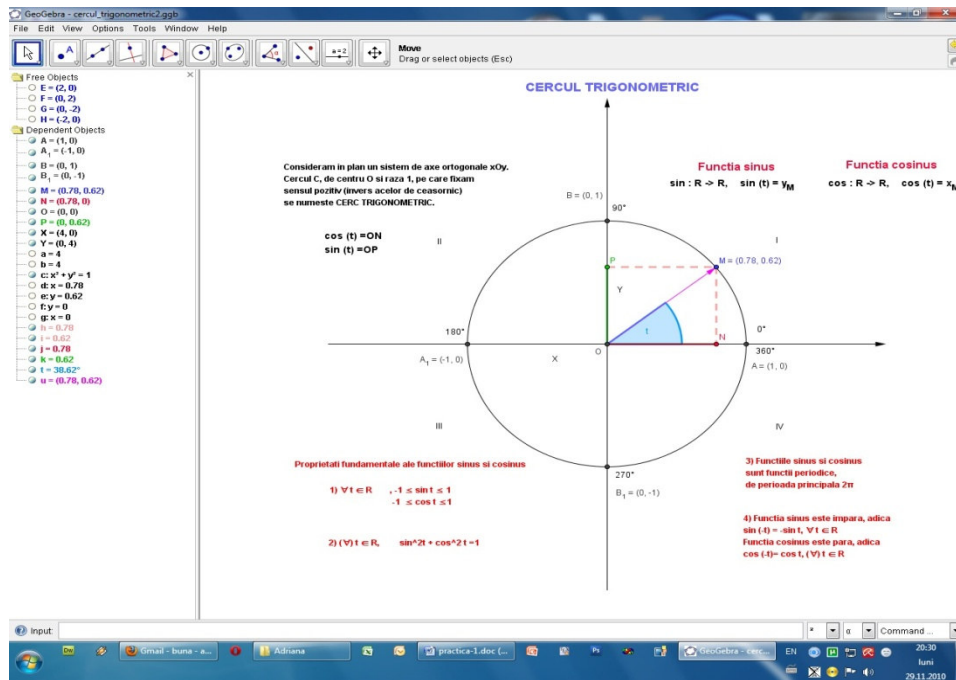


Figure 2. The construction of the trigonometry circle in GeoGebra

Using the symmetry of the trigonometric circle with respect to the coordinate axis and the origin of the geometric reference  $xOy$ , the problem of determining the trigonometric functions values can be reduced at the first quadrant (see Figure 3). By using the same properties, Figure 4 and Figure 5 reveal the transformations crossing from quadrant III, respectively quadrant IV up to first quadrant. Due to the dynamic character of GeoGebra, the simple movement of the M point around the circle will produce a visual result both in the geometric pane and on the algebraic pane.

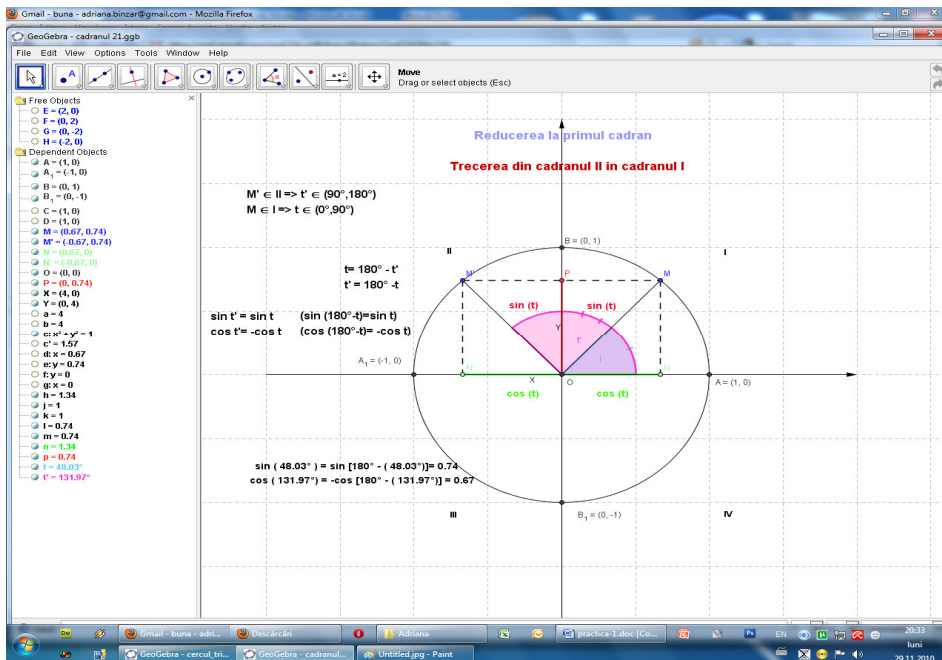


Figure 3. Transformation from the 2<sup>nd</sup> to the 1<sup>st</sup> quadrant in GeoGebra

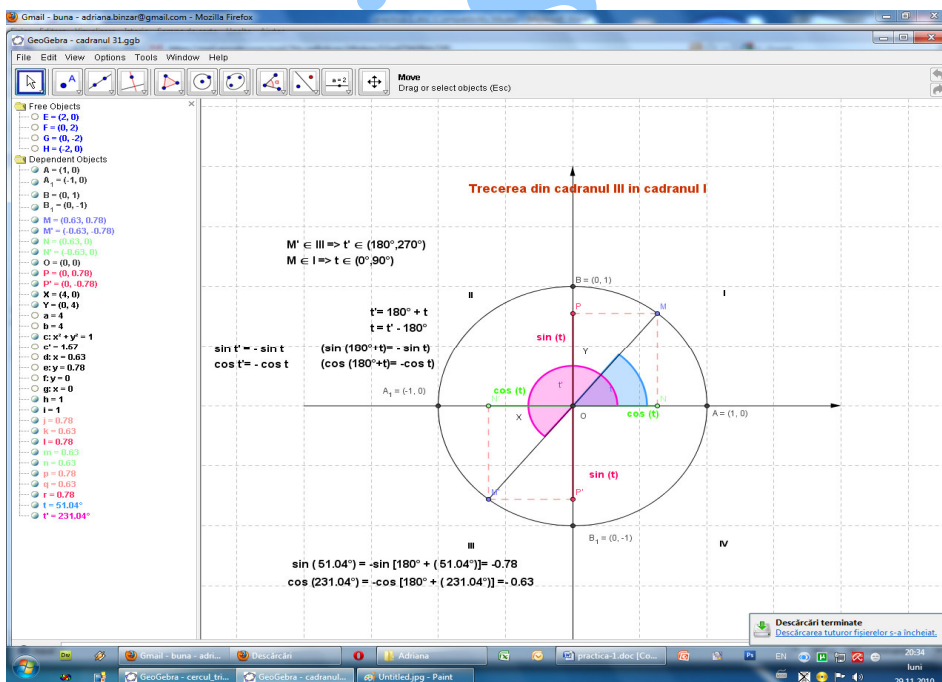


Figure 4. Transformation from the 3<sup>rd</sup> to the 1<sup>st</sup> quadrant in GeoGebra

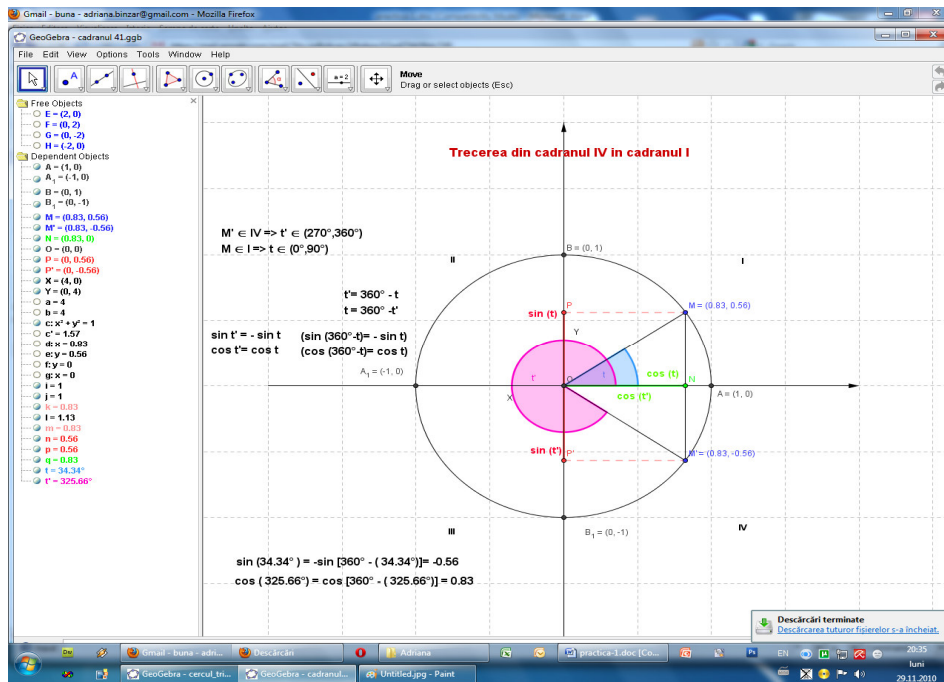


Figure 5. Transformation from the 4<sup>th</sup> to the 1<sup>st</sup> quadrant in GeoGebra

The advantages of a computer assisted lesson were obvious. The student can see the definition of the sinus and cosine function on the screen and also their major properties. They can observe the updated values of these functions by the movement of the M point on circle also in graphics and algebraic panes. It is very useful in learning new notions on the study of the trigonometric circle in the 9<sup>th</sup> grade and also in the 12<sup>th</sup> grade for the final exams.

The figures with sinus and cosine calculations by reduction at the first quadrant are very instructive. In the past years such explanations were done on the blackboard with colored chalk but now, using the computers, they become more efficient and attractive and much simple to understand. A second advantage would be that the range of values we were using was reduced to the values of angles as 30°, 45° and 60°. In GeoGebra the students can visualize all the possible angles on the trigonometric circle. This application is very useful both for the teacher easing the teaching process and for students, offering them a real image of relation of the links between the values of sinus and cosine.

### 3. GeoGebra in other concrete problems

In the XVIII<sup>th</sup> century, the mathematician Leonhard Euler discovered that three of the centers of a triangle are always collinear. The line on which lied the points with this amazing property (the center of gravity of the triangle, the center of circumscribed circle of the triangle and the orthocenter) has been named the Euler's line. This is a complex and hard to solve problem, but which can be better understood if it is solved visually in GeoGebra (Figure 6). We mention that, because GeoGebra is a dynamic application, the auditorium can check the fact that this property is valid for any type of triangle.

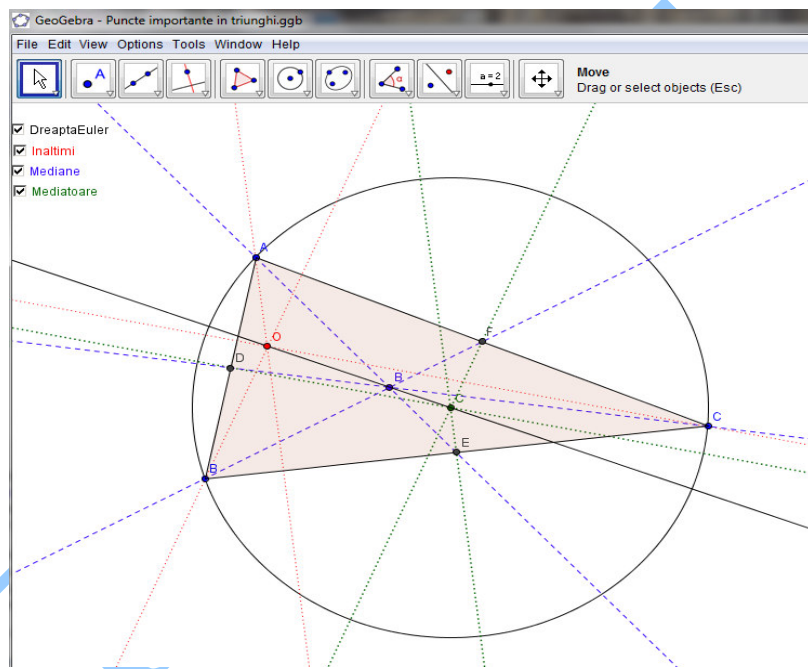


Figure 6. Euler's line in GeoGebra

This material was elaborated along with the professors who activate within the Socrates Excellence Center for teaching computer aided geometry, within one of the workshops initiated by the GeoGebra Institute of Timișoara which purpose was that of familiarization of the teachers from the pre-university level with the GeoGebra working environment. The problem can be solved by introducing the other important points within a triangle by defining the circle and one of the two points or by generalizing Euler's line.

## Conclusions

Taking into account the results obtained by the experienced international users and analyzing the findings obtained by the Romanian colleagues, after more two years of using GeoGebra in some schools, at a secondary level and one year in the for IT students, and from the individual studies of the researches, one can notice a better communication with the students, due to the interactive character of the lessons. The abstract notions that were taught can be more easily understood because they can be visualized through GeoGebra, lessons becoming more attractive for the audience. Also it should be mentioned that such lessons allow the presentation and explanation of a larger number of particular cases.

By this article the authors intend to point out the importance of implementing GeoGebra in the educational process in order to create modern lessons providing visual support for the theoretical concepts presented. Moreover, this software allows the user to include educational material through virtual lessons on the Internet, through various types of applications such as Java applets, mathlet sites, or sites mathcast using high level technology such as smart boards.

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