COMPARISON OF DIFFERENT TESTS FOR DETECTING HETEROSCEDASTICITY IN DATASETS

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ABSTRACT: *Heteroscedasticity occurs mostly because of beneath mistakes in variable, incorrect data transformation, incorrect functional form, omission of important variables, non-detailed model, outliers and skewness in the distribution of one or more independent variables in the model. All analysis were carried out in R statistical package using Imtest, zoo and package.base. Five heteroscedasticity tests were selected, which are Park test, Glejser test, Breusch-Pagan test, White test and Goldfeld test, and used on simulated datasets ranging from 20,30,40,50,60,70,80,90 and 100 sample sizes respectively at different level of heteroscedasticity (that is at low level when sigma = 0.5, mild level when sigma = 1.0 and high level when sigma = 2.0). Also, the significance criterion alpha = 0.05. However, each test was repeated 1000 times and the percentage of rejection was computed over 1000 trials. For Glejser test, the average empirical type I error rate for the test reject more than expected while Goldfeld has the least power value. Therefore, Glejser test has the highest capacity to detect heteroscedasticity in most especially on simulated datasets.*

KEYWORDS: *Park test, Glejser test, Breusch-Pagan test, White test, Goldfeld test*

1. INTRODUCTION

One of the assumptions of classical linear regression model states that the disturbances u_i featuring in the population regression function are of the same variance; meaning that they are homoscedatic. In other words, the variance of residuals should not increase with fitted values of response variable (SelvaPrabhakara, 2016). When this assumption fails, its consequence is what is termed heteroscedasticity which can be expressed as:

$E(u_i^2) = \sigma^2$ i= 1, 2 …n

There are several reasons why we can have variability in the variance of u_i. The study on error learning models shows that as people learn over time, their errors rates reduce drastically such that σ_i^2 is expected to decrease. Practically, as one increases his or her number of typing hours, the rate of errors committed decreases while the variance naturally decreases. Also, as income grows, one has choices on how to dispose the income, consequently, σ^2 is more likely to increase with the income. Also, a company with a large profit will give more dividends to their shareholders than a company that was newly established. In a situation where data collecting technique improves, σ_i^2 is more likely to decrease. In the same sense, banks with good equipment for processing data are less prone to make error when processing the monthly statement of account of their customers than banks that does not have good facilities. Heteroscedasticity can also come to play due to outliers. Outliers are observations that are much different in a population. It can either be too large or small in relation to the observation in such sample. It can also be as a result of violation of the assumption that regression model is correctly specified. Not only that, it can be as a result of skewness in the distribution of one or more regressors included in the model. Skewness is when a distribution is right skewed. Heteroscedasticity may be as a result of incorrect data transformation and incorrect functional form. Heteroscedasticity is not a property that is necessarily restricted to cross sectional data or time series data where there occurs an external shock or change in circumstances that created uncertainty about y [7]. Cross sectional data involves data that deal with members of a population at a particular time. Here, members may be of different types, size, and completion while time series data are similar in order of magnitude [6]. More often than not, heteroscedasticity arises when some variables that are important are omitted from the model or superfluous of variables in a model (model specification error).

1.1 Model specification

Applied econometrics is based on understanding intuition and skill. Users of economic data must be able to give model that other people can rely on [10]. Assumption of Classical Linear Regression Model states that the regression model used in the analysis is correctly specified. Other problem called **model specification error or model specification bias** is encountered.

Users of economic data must have the following in their mindset:

a. able to have a standard to choose a model for empirical analysis,
b. recognize the model specification error types that can be encountered in practice,
c. have in mind the result of specification error,
d. how to discover specification error or recognize tools that can be used,
e. discover the cure and good effect that can be used in detecting specification error,
f. know how to judge the strength of competing models.

1.2 How to identify heteroscedasticity

The residual from linear regression are

$$e = Y - \hat{Y} = Y - X\hat{\beta}$$

which is used in place of unobservable errors ε [3]. Residual are used to detect the behaviour of variance with datasets. Residual plots, where residuals ε_i are plotted on the y-axis against the dependent variable $\hat{y}_i$ on the x axis are the most commonly used tool [2]. But when heteroscedasticity is present, most especially when the variance is proportional to a power of the mean, the residuals will appear in form of a fan shape.

This may not be the best method for detecting heteroscedasticity as it difficult to interpret, particularly when the positive and negative residuals do not exhibit the same general pattern [3]. Cook and Weisberg suggested that plotting the square residuals, e_i^2 to account for this. Then, a wedge shape bounded below by 0 would indicate heteroscedasticity.

However, as [2] pointed out, squaring residuals that are large in magnitude creates scaling problems; resulting to a plot where patterns in the rest of the residuals are difficult to see. They instead advocate for plotting the absolute residuals. This way, we do not need to identify positive and negative patterns, and do not need to worry about scaling issues. A wedge shape of the absolute residual also indicates heteroscedasticity where the variance increase with the mean. This is the plotting method that is used for identifying heteroscedasticity in the study.

2. COMMON HETEROSCEDASTICITY PATTERNS

There exists some likely assumptions about heteroscedasticity, they include:

Pattern 1:

The error variance is proportional to X_i^2, i.e

$$E(u_i^2) = \sigma^2 X_i^2$$

Graphical methods or Park and Glesjer approaches reveal that, if it is true that the variance of u_i is proportional to the square of the explanatory variable X_j the original model can be transformed as follow:

$$E(Y_i) = \beta_1 + \beta_2 X_i \qquad 2.0$$

Divide the original model by X_i

$$\frac{Y_i}{X_i} = \frac{\beta_1}{X_i} + \beta_2 + \frac{u_i}{X_i} \qquad 2.1$$

$$= \beta_1 \frac{1}{X_i} + \beta_2 + v_i \qquad 2.2$$

Where v_i is the transformed disturbance term, equal to u_i/X_i. Now it is easy to verify that:

$$E(v_i^2) = E\left(\frac{u_i}{X_i}\right)^2 = \frac{1}{X_i^2} E(u_i^2) \qquad 2.3$$

$$= \sigma^2 \text{ (by using } E(u_i^2) = \sigma^2 X_i^2) \qquad 2.4$$

It should be noted that in the regression transformation, the intercept term β_2 is the slope coefficient in the original equation and the slope coefficient β_1 is the intercept term in the original model. Therefore, to get to the original model we shall have to multiply the estimated equation 2.0 by X_i.

Pattern 2:

The error variance is proportional to X_i. The square root transformation

$$E(u_i^2) = \sigma^2 X_i \qquad 2.5$$

It is believed that the variance of u_i, instead of being proportional to the squared X_i, is proportional to X_i itself, then the original model can be transformed as:

$$\frac{Y_i}{\sqrt{X_i}} = \frac{\beta_1}{\sqrt{X_i}} + \beta_2\sqrt{X_i} + \frac{u_i}{\sqrt{X_i}} \qquad 2.6$$

$$= \beta_1 \frac{1}{\sqrt{X_i}} + \beta_2\sqrt{X_i} + v_i \qquad 2.7$$

Where $v_i = u_i/\sqrt{X_i}$ and where $X_i>0$

Having given pattern 2, one can readily verify that $E(v_i^2) = \sigma^2$, a homogeneous situation. However, one may proceed to apply Ordinary Least Square to equation 2.3 to regressing $Y_i/\sqrt{X_i}$ on $1/\sqrt{X_i}$ on $1/\sqrt{X_i}$and $\sqrt{X_i}$.

It is worthy of note that an important feature of the transformed model that states there no intercept term. However, regression-through-the-origin model to estimate β_1 and β_2 have to be used. Having run through equation 2.3, one can get back to the original model simply by multiplying equation 2.3 by $\sqrt{X_i}$.

Pattern 3:

The error variance is proportional to the square of the mean value of Y.

$$E(u_i^2) = \sigma^2[E(Y_i)]^2 \quad 2.8$$

Equation 2.4 asserts that the variance of u_i is proportional to the square of the expected value of Y.

Therefore,

$$E(Y_i) = \beta_1 + \beta_2 X_i \quad 2.9$$

Now, if the original equation is transformed as follows:

$$\frac{Y_i}{E(Y_i)} = \frac{\beta_1}{E(Y_i)} + \beta_2 \frac{X_i}{E(Y_i)} + \frac{u_i}{E(Y_i)} \quad 2.10$$

$$= \beta_1 \left(\frac{1}{E(Y_i)}\right) + \beta_2 \frac{X_i}{E(Y_i)} + v_i$$

Where $v_i = (u_i/E(\hat{Y}_i))$. It can be seen that $E(v_i^2) = \sigma^2$, that is, the disturbances v_i are homogenous. Hence, it is regression of equation 2.5 that will satisfy the homoscedastic assumption of the classical regression model.

The transformation in equation 2.5 is not operational because E(Y_i) depends in β_1 *and* β_2, which are unknown. Of course, we have $\hat{Y}_i$. Then, using the estimated $\hat{Y}_i$, we transform our model to:

$$\frac{Y_i}{\hat{Y}_i} = \beta_1 \left(\frac{1}{\hat{Y}_i}\right) + \beta_2 \left(\frac{X_i}{\hat{Y}_i}\right) + v_i \quad 2.11$$

Where $v_i = (u_i/\hat{Y}_i)$. Though $\hat{Y}_i$ are not exactly $E(Y_i)$, they are consistent estimators, that is, as the sample size increase indefinitely, they converge to true E(Y_i). Hence, the transformation in equation 2.4 will perform satisfactory in practice if the sample size is reasonably large.

Pattern 4:

The log transformation such as

$$In\, Y_i = \beta_1 + \beta_2\, In\, X_i + u_i \quad 2.12$$

More often than not reduces heteroscedasticity compared with the regression

$$Y_i = \beta_1 + \beta_2 X_i + u_i \quad 2.13$$

The result arises because log transformation is good at compressing the scales in which the variables are measured, thus, reduces a tenfold differences between two values to a twofold difference. For instance, figure 90 is ten time of figure 9 but In 90 (=4.4998) is about twice as large as In 9 (=2.1972).

Another benefit of the log transformation is that the slope coefficient β_2 measures the elasticity of Y with respect to X, that is, the percentage change in Y for a percentage change in X.

For example, if Y is consumption and X is income, β_2 in equation 2.5 will measure income elasticity, whereas in the original model β_2 measures only the rate of change of mean consumption for a unit change in income. It is one reason why the log models are quite popular in empirical econometrics [6].

3. HETEROSCEDASTICTY TESTS

Park Test:

Park test made an assumption in his work that variance of the error term is proportional to the square of the independent variable [8].

It also validates the graphical method by suggesting that σ_i^2 serves the purpose of the explanatory variable X_i. He hereby suggested $\sigma_i^2 = \sigma^2 X_i^{\beta} e^{vi}$ as the functional form.

or $In\sigma_i^2 = In\sigma^2 + \beta In X_i + v_i$

wherev_i is the non-probabilistic disturbance term.

Since σ_i^2 is generally not known, Park suggested using $\hat{u}_i^2$ as a proxy and running the following regression:

$$In\ \hat{u}_i^2 = In\sigma^2 + \beta In X_i + v_i \quad 3.0$$

$$= \alpha + \beta In X_i + v_i \quad 3.1$$

If β reveals that it is statistically significant, it is an evidence that heteroscedasticity is present in the data,otherwise, we may accept the assumption of homogeneity. However, the Park test is a two-stage procedure. For the first stage, we run OLS regression not minding the heteroscedasticity issue. We obtain $\hat{u}_i$ from the regression and then run the regression in the second stage.

It should be noted that empirically pleading, the Park test has some difficulties. Goldfield and Quandt have argued that the error term v_i entering into $In\hat{u}_i^2 = In\sigma^2 + \beta In X_i + v_i$ may not meet the Ordinary least square assumptions and may itself be heterogeneous. Nevertheless, as a powerful explanatory method, we may use the Park Test.

Park test is used for heteroscedasticity when one has some variable Z that one thinks might explain the different variances of the residuals.

There are different forms of Park test, the log form is the commonest of all and is the one described here, where: log (Residual^2) = b(intercept)+slope log(X), yet, another form can still be used theoretically, just like the linear form Residual^2= b+m(X). The linear form is closely similar to the Breuch Pagan test.

Steps for running Park Test

Step 1: On the data, run ordinary least squares and be sure that the regression produces a table of residuals.

Step 2: The residuals from step 1 should be squared

Step 3: The natural log of the squared residuals from step 2 should taken

Step 4: The natural log of Z should be taken and the variable which you suspect is causing the heteroscedasticity behaviour

Step 5: Run Ordinary least square once again for the natural log of Z (step 4) against the natural log of the squared residuals (step 3). In other words, log of

Z is your independent variable and log (residuals') is the dependent variable for the regression.
Step 6: Calculate the t-Statistic for the Z variable and note that large t-statistic indicates the presence of heteroscedasticity.

Glejser Test:
The test is just like Park test. After one must have obtained the residuals $\hat{u}_i$ from the ordinary least square regression, Glejser suggested regressing the absolute values of $\hat{u}_i$ on the X variables that are thought to be closely related to σ_i^2. The Glejser test places weaker restrictions on the shape of error distribution under the null hypothesis than Breusch and Pagan (1979) test based on normal likelihood function (Machado, 2000). It uses the following functional forms:

$$|\hat{u}_i| = \beta_1 + \beta_2 X_i + v_i \quad 3.2$$

$$|\hat{u}_i| = \beta_1 + \beta_2 \sqrt{X_i} + v_i \quad 3.3$$

$$|\hat{u}_i| = \beta_1 + \beta_2 \frac{1}{X_i} + v_i \quad 3.4$$

$$|\hat{u}_i| = \beta_1 + \beta_2 \frac{1}{\sqrt{X_i}} + v_i \quad 3.5$$

$$|\hat{u}_i| = \sqrt{\beta_1 + \beta_2 X_i} + v_i \quad 3.6$$

$$|\hat{u}_i| = \sqrt{\beta_1 + \beta_2 {X_i}^2 + v_i} \quad 3.7$$

Thus, v_i is the error term.
Also, as an empirical matter is concerned, Glejser test approach can be used. But Goldfeld and Quandt pointed out that error term v_i has some problems in that its expected value is nonzero, it is serially correlated and at the same time ironically heterogeneous.
Another problem of Glejser method is that the models such as:

$$|\hat{u}_i| = \beta_1 + \beta_2 X_i + v_i \quad 3.8$$

and

$|\hat{u}_i| = \sqrt{\beta_1 + \beta_2 {X_i}^2} + v_i$ are nonlinear in the parameters, therefore, it cannot be estimated with the usual Ordinary least square procedure. For large samples, Glejser has found that the first four of the proceeding models give generally pleasant result in detecting heteroscedasticity. Practically, Glejser technique may be used for both large and small sample strictly as a qualitative device to be aware of heteroscedasticity.

Breusch-Pagan Test
The accomplishment of the Goldfeld-Quandt test depends both on the value of c (the number of central observations to be omitted) and identification of the correct X variable with which to order the observations. The Breusch-Pagan tests the null hypothesis that the residuals' variances are unrelated to a set of explanatory variables versus the alternative hypothesis that the residuals' variances are parametric function of the predictor variable (Andreas .G, 2016). We can do away with the limitations of this test provided we consider the Breusch-Pagan Godfrey (BPG) test.
This test can be illustrated by considering the k-variable linear regression model.

$$Y_i = \beta_i + \beta_2 X_{2i} + \ldots + \beta_k X_{ki} + v_i \quad 3.9$$

Assume that the error variance σ_i^2 is described as: $\sigma_i^2 = f(\alpha_1 + \alpha_2 Z_{2i} + . \; . \; . + \sigma_m Z_{mi})$
where σ_i^2 is a linear function of the Z's.
If $\alpha_2 = \alpha_3 = \ldots \; = \sigma_m = 0,\; \sigma_i^2 = \alpha_1$
which is a constant. Therefore, to test whether σ_i^2 is homogeneous, one can test the hypothesis that $\alpha_2 = \alpha_3 = \ldots = \alpha_m = 0$. This is the brain behind the Breusch-Pagan Test. The actual test procedure is as follows.
Step 1: Calculate $Y_i = \beta_1 + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + u_i$ by ordinary least square and obtain the residuals $\hat{u}_1, \hat{u}_2, \; . \; . \; ., \hat{u}_n$

Step 2: Find $\bar{\sigma}^2 = \sqrt{\frac{\hat{u}_i^2}{n}}$.

It should noted that OLS estimator is $\sum \frac{\hat{u}_i^2}{n-k}$
Step 3: Construct variables p_i defined as:

$$p_i = \hat{u}_i^2 / \bar{\sigma}^2$$

which is simply each residual square divided by $\bar{\sigma}^2$
Step 4: Regress p_i thus constructed on the Z's as $p_i = \alpha_1 + \alpha_2 Z_{2i} + \; \ldots + \; \alpha_m Z_{mi} + v_i$
wherev_i is the residual term of this regression.
Step 5: Obtain the explained sum of squares from $p_i = \alpha_1 + \alpha_2 Z_{2i} + \cdots + \; \alpha_m Z_{mi} + v_i$
and define as:

$$\theta = \frac{1}{2(ESS)}$$

Assuming u_i are normally distributed, one can show that if there is homoscedasticand if the sample size n increases indefinitely, then:
$\theta \sim \chi_{m-1}^2$
That is θ follows the chi-square distribution with (m-1) degree of freedom.
Therefore, if in an application the compound $\theta (= \chi^T 2)$ exceeds the critical χ^2 value at the chosen level of significance, one can reject the hypothesis of homogeneity, or else, do not reject it.

White Test
Do not forget that Goldfeld-Quandt test requires reordering of observations with respect to the X variable that supposedly bring about heteroscedasticity or the BPG test, which is eventually sensitive to the normality assumption while the general test of homoscedastic proposed by

White does not rely on the normality assumption and is easy to carryout.

Consider three-variable regression model,

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad 3.10$$

The White test proceeds is as follows:

Step 1: Having given the data, we calculate $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ and get the residuals, $\hat{u}_i$

Step 2: We can then run the following auxiliary regression.

$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \alpha_4 X_{2i}^2 + \alpha_5 X_{3i}^2 + \alpha_6 X_{2i} X_{3i} + vi$$

It means that the squared residuals from the original regression are regressed on the original X variables or regressors, the squared values and the cross product(s) of the regressors. Higher powers of regressors can also be introduced.

It is observed that there is a constant term in this equation even through the original regression may or may not contain it. Obtain the R^2 from this (auxiliary) regression.

Step 3: Under the null hypothesis that there is no heteroscedasticity, it can be shown that sample size(n) times that R^2 obtained from the auxiliary regression asymptotically follows the chi-square distribution with degree of freedom equal to the number of regressors (excluding the constant term) in the auxiliary regression. That is,

$$n.R^2 \sim \chi_{df}^2$$

Where df is defined previously.

Step 4: it should be noted that in as much as the chi-square value obtained in $n.R^2 \sim \chi_{df}^2$ is greater than the critical chi-square value at the chosen level of significance, it can hereby be concluded that there is heteroscedasticity. If it is not greater than the critical chi- square value, there is heteroscedasticity, which is to say that in the auxiliary regression

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i,$$
$$\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0$$

If a model has several regressors, then introducing all the regressors, their squared (or higher-powered) terms and their cross products can quickly consume degree of freedom. Therefore, one must use caution in using the test.

This test can either be a test of heteroscedasticity or specification error and sometimes both. It has been proved that if no cross-product terms are present in the White test procedure, then it is a test of pure heteroscedasticity. If cross-product terms are present, then it is a test of both heteroscedasticity and specification bias.

White test has the:

1. It will not force you to specify a model of the structure of the heteroscedasticity, if it is present.
2. It is based on the assumption that the errors are normally distributed.
3. It will tests if truly the presence of heteroscedasticity will cause the ordinary least square formula for the variances and the covariances of the estimates to be inconsistence.

Goldfeld-Quandt Test:

This test is used specifically it is assumes that the heteroscedasticity variance,σ_i^2 is positively related to one of the explanatory variance in the regression model. For clarity, let us consider the two-variable model:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad 3.11$$

Suppose σ_i^2 is positively related as Xi, $\sigma_i^2 = \sigma^2 X_i^2$

Where σ^2 is a constant.

Assume $\sigma_i^2 = \sigma^2 X_i^2$ reveals that σ_i^2 is proportional to the square of the X variable.

If $\sigma_i^2 = \sigma^2 X_i^2$ is appropriate, it would mean σ_i^2 would be larger, the larger the values of X_i. If that is the result, it indicates that heteroscedasticity is most likely to be present in the model. To verify this assertion, Goldfeld and Quandt suggest the following steps:

Step 1: Start the ranking of the observations according to the values of X_i right from the beginning with the lowest X value.

Step 2: Ignore c central observations, where c is specific a priori and divide the remaining (n-c) observations into two groups each of $\frac{(n-c)}{2}$ observations.

Step 3: Fit separately ordinary least square regressions to the first $\frac{(n-c)}{2}$ observations and the last $\frac{(n-c)}{2}$ observations and find the respective residual sums of squares RSS_1 and RSS_2, RSS_1 representing the RSS from the regression corresponding to the smaller X_i value (the small variance group) and RSS_2 that from the larger X_i values (the large variance group). These RSS each have:

$$\frac{n-c}{2} - k \quad \text{or} \left(\frac{n-c-2k}{2}\right) df$$

Where k is the number of parameters to be estimated including the intercept.

Step 4: Compute the ratio

$$\lambda = \frac{RSS_2}{df} \Big/ \frac{RSS_1}{df}$$

If u_i are assumed to be normally distributed (which we usually do) and if the assumption of homoscedasticis valid, then it can shown that λ follows F distribution with numerator and denominator df each of $\left(\frac{n-c-2k}{2}\right)$.

If in an application, the computed λ exceeds that of the critical F at the chosen level of significance, we can reject the hypothesis of homoscedasticity, meaning that heteroscedasticity is more like to be present.

It should be noted that c central observations are omitted to sharpen the difference between the small variance group (i.e RSS_1) and the large variance group (i.e RSS_2). But the ability of the Goldfield-Quandt test to do this successfully depends on how c is chosen.

Practically, the power of the test depends on how c is chosen. In statistics, the power of a test is measured by the probability of rejecting the null hypothesis when it is false [i.e by 1-Prob(type II error)].

However, it may be noted that in case there is more than one X variable in the model, the ranking of observations, the first step in the test can be done according to any one of them. Thus in the model:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i$$

This test is applicable for large sample and it assumes that the observations must be at least twice as many as the parameters to be estimated. The test also assumes normality and serial independent error terms. It compares the variance of error terms across discrete subgroup [5].

4. ANALYSIS AND RESULTS

Homoscedasticdatasets of 20, 30, 40, 50, 60, 70, 80 90 and 100 observations were simulated respectively and the number of times in percentage that Breusch-Pagan test, White test, Goldfeld test, Park test and Glejser test reject null hypothesis were noted using power test when alpha = 0.05 and the test with the highest number of percentage indicate the best test to detect homogeneity.

In the same vein, Homoscedasticdatasets of 20, 30, 40, 50, 60, 70, 80 90 and 100 observations were simulated respectively and the number of time that Breusch- Pagan test, White test, Goldfeld test, Park test and Glejser test reject null hypothesis were noted, for low level of heteroscedasticity, when alpha = 0.05, for mild level of heteroscedasticity and high level of heteroscedasticity. The test with the highest percentage indicates the best test among the selected tests. The results obtained from the simulation study are in percentage as shown below:

Table 4.1: Empirical type I Error at alpha = 0.05

Sample size	Breusch –Pagan	White	Goldfield	Park	Glejser
20	4.2	3.1	4.3	5.3	7.1
30	5.1	5.7	5.7	4.6	8.8
40	4.6	5.5	5.2	5.7	8.5
50	6.1	4.7	4.7	4.8	10.0
60	4.4	4.6	4.8	4.9	8.6
70	4.0	5.2	3.8	4.9	9.6
80	4.4	4.2	5.3	3.5	8.7
90	5.5	5.4	5.4	4.5	9.8
100	5.4	4.3	4.8	5.9	10.5

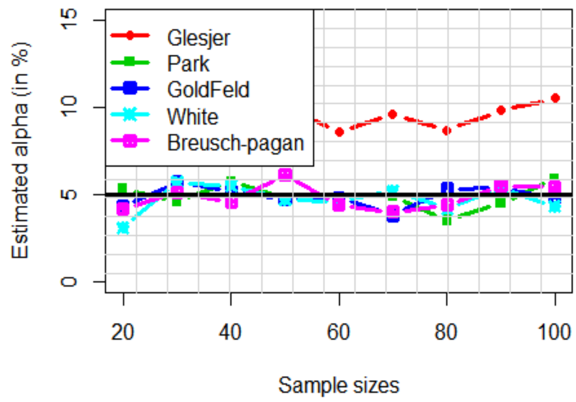

Figure 4.1: Empirical Type I Error at alpha = 0.05

Table 4.1 and figure 4.1 show the empirical type I error rate for the five tests. The scenario corresponds to the case where a homoscedastic model was simulated and each test was carried out on the model residual. At each sample size, each test was repeated 1000 times and the percentage of rejection was computed over the 1000 trials. On the average , Breusch-Pagan test, White test, Goldfeld test and Park test yield an empirical type I error rate of about 5% which shows that the tests are invalid since the values obtained are not close to the imposed 5% level. However, for Glejser test, the average empirical type 1 error rate for the test rejects more than expected. This suggests that care should be taken while using the Glejser test as a homoscedastic scenario may be considered heteroscedastic.

Table 4.2: Empirical Power at low heteroscedasticity level at alpha = 0.05

Sample size	Breusch –Pagan	White	Goldfield	Park	Glejser
20	22.4	14.2	5.7	19.5	28.9
30	37.8	26.5	5.9	30.1	49.6
40	50.1	37.2	5.8	40.7	62.2
50	61.9	51.2	6.1	47.6	75.5
60	71.9	63.1	7.2	58.2	85.4
70	78.3	73.2	5.6	65.9	91.7
80	85.0	82.0	6.7	71.7	95.7
90	89.9	88.2	6.6	80.0	96.9
100	93.6	93.1	4.5	83.4	98.6

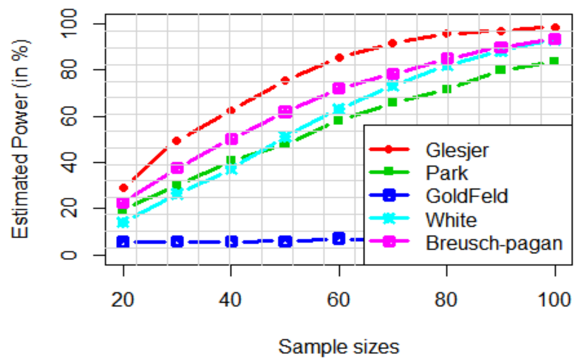

Figure 4.2: Empirical Power at low Heteroscedasticity level at alpha = 0.05

Table 4.2 and figure 4.2 show empirical power of the selected test at low heteroscedasticity level. Here, low heteroscedasticity model was simulated and each test was carried out on the model residual. Each test was repeated 1000 times at each sample size and the percentage of rejection was computed over the 1000 trials. On the long run, all the tests except Goldfeld increase as the powers increase, it shows that the test are valid. However, Glejser test is the best test among all because it has the highest power.

Table 4.3: Empirical Power at mild heteroscedasticity level at alpha = 0.05

Sample size	Breusch –Pagan	White	Goldfield	Park	Glejser
20	57.9	40.2	7.1	52.6	74.8
30	81.7	70.8	8.0	78.9	95.1
40	92.7	87.9	8.5	89.6	99.1
50	97.8	96.7	7.6	97.3	99.8
60	99.4	99.5	7.3	98.5	100.0
70	99.7	99.6	7.4	99.7	100.0
80	100.0	100.0	8.2	99.8	100.0
90	100.0	100.0	7.7	100.0	100.0
100	100.0	100.0	8.2	100.0	100.0

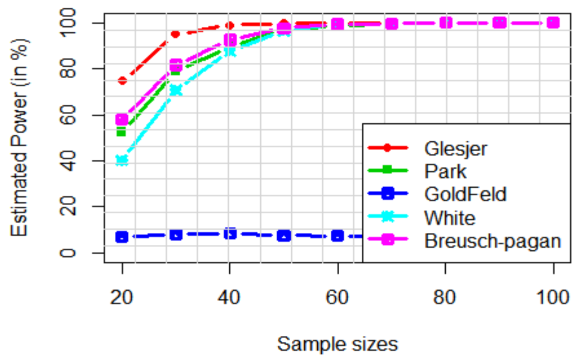

Figure 4.3: Empirical Power at Mild Heteroscedasticity level at alpha =0.05

Table 4.3 and figure 4.3 show empirical power of the selected tests at mild heteroscedasticity level. The scenario corresponds to the case where a mild heteroscedasticity model was simulated and each test was carried out on the model residual. On each sample size, each test was repeated 1000 times and the percentage of rejection was computed over the 1000 trials. All the tests except Goldfeld behave similarly with a minimum sample size 40 required to achieve a reasonable power of at least 80%. At the sample size of 40, the most powerful test of the four tests is Glejser and likewise on average the best test. However, care should be taken in considering the fact that Glejser test rejects more than expected which is the basis, the best test for mild heteroscedasticity is Breusch-Pagan for a reasonable sample size of 40. At sample size above 40, all the tests except Goldfeld behave similarly with their respective power.

Table 4.4: Empirical Power at high heteroscedasticity level at alpha = 0.05

Sample size	Breusch –Pagan	White	Goldfield	Park	Glejser
20	93.6	80.8	9.7	90.6	99.3
30	99.7	97.5	9.3	98.5	100.0
40	100.0	99.6	9.4	99.6	100.0
50	100.0	99.9	12.1	99.9	100.0
60	100.0	100.0	9.9	100.0	100.0
70	100.0	100.0	10.6	100.0	100.0
80	100.0	100.0	12.6	100.0	100.0
90	100.0	100.0	11.2	100.0	100.0
100	100.0	100.0	13.7	100.0	100.0

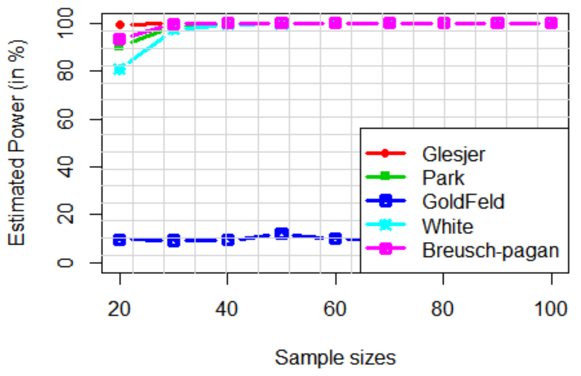

Figure 4.4: Empirical Power at High Heteroscedasticity level at alpha =0.05

Table 4.4 and figure 4.4 show empirical power at high heteroscedasticity level at alpha = 0.05, high heteroscedasticity model was simulated and each test was carried out on the model residual. At each sample size, each test was repeated 1000 times and the percentage of rejection was computed over 1000 trials. The power value of all the tests except Goldfeld test are considered to be adequate because they are more than 80%, it shows that we have 80% chance of detecting a different between population mean and the target when a difference actually exist. The highest sensitivity power of Glejser test makes it to be the best.

5. SUMMARY OF RESULTS

For table 4.1, the focus was to assess the validity of the test by comparing the empirical type I error with the imposed 0.05, the results show that the best test is the test with the highest power.
For table 4.2, the power at low heteroscedasticity level was compared and found out that at various sample sizes, the best test is the test with highest power at various level and sample sizes.
For table 4.3, the power at mild heteroscedasticity level was compared and found out that at various sample sizes, the best test is the test with highest power at various level and sample sizes.
For table 4.4, the power at high heteroscedasticity level was compared and found out that at various

sample sizes, the best test is the test with highest power at various level and sample sizes.

CONCLUSION

This research work focused on the uses of selected five tests, which are Breusch Pagan, White, Goldfeld, Park and Glejser tests for detecting heteroscedasticity in simulated datasets, ranging from 20, 30, 40, 50, 60, 70, 80, 90 and 100. The datasets was simulated at all levels of heteroscedasticity with low result when the sigma was set at 0.05, mild when sigma was at 0.10, and high when sigma was at 2.0. Also, all the tests were run on the simulated datasets to know the test at 5% threshold. The results show that Glejser test has the highest tendency to detect heteroscedasticity at all levels, while the Goldfeld test is the least of selected tests as far as the simulated datasets is considered.

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