GRAPHICAL SOLUTION OF NON LINEAR PROGRAMMING PROBLEM

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ABSTRACT: *In this paper, a graphical solution of the nonlinear programming problem was presented the result satisfies the graphical solution of the nonlinear programming problem and we also solved some NLPP question in this paper by graphical method.*

KEYWORDS: *Non linear programming problem, Objective function, Constraint, Graphical solution, Feasible region*

1. INTRODUCTION

As we are familiar with the graphical of solving the non linear programming problem thus by the some way the graphical method can be employed to solve the two variable NLP problem also.

In linear programming problem the solution point is generally a corner point of the convex solution space. While in NLPP the solution point is not necessarily a corner point or an edge. We will illustrate the solution of NLPP by graphical method by some examples.

2. GRAPHICAL METHOD

One of the solving methods of non-linear programming problem is graphical method which is also called geometrical method.

As we know there some more method for solving of non-linear programming problem like Wolf, Beal and separable method but the graphical method is the easiest method for solving NLPP is defend on well-defined sets of logical steps. NLPP which contain two variables can be easily solved by graphical method. As we can abserve that from the characteristic of the curve we can achieve more information and from these information we can obtain the optimal solution of the given problem.

But in the case of NLP problem the optimum solution or may not be occur at one of the extreme points of the solution space, generated by the constraints and the objective Function of the given problem.

3. SOME STEPS IN SOLVING NLP PROBLEM GRAPHICALLY

The following steps are used in the graphical solution of the nonlinear problem. We will apply these steps in some examples.

a. Formulate. First of all we must write the NLP problem in standard form. Which have the objective function and constraint set.
b. In this step we construct the graphs and plot the constraint curve. Where the constraint curve represent the limitation of the available resource.
c. Identified the convex region of the (solution space) generated by the objective function and constraints of the given problem.
d. Determine the point in the convex region in which the optimum solution always accrues and the most attractive corner is the last point in feasible region.
e. Determine the optimal solution by algebraically calculating coordinates of the most attractive point.
f. For the optimal solution of the problem we must determine the value of the objective function.

Example1. Solve the following NLP problem graphically.

$$Max\ z = 2x_1 + 3x_2$$

Subject to the constraints

$$x_1x_2 \le 8$$
$$x_1{}^2 + x_2{}^2 \le 20$$
$$x_1, x_2 = 0$$

Solution. Let Ox_1 and Ox_2 be the set of rectangular Cartesian coordinate axes in the plane.
Clearly, the feasible region will lie in the first quadrant only, because $x_1 \ge 0\ , x_2 \ge 0$. Now we plot the curve of the given constraint $x_1{}^2 + x_2{}^2 = 20$ and$xy = 8$.

Since ${x_1}^2 + {x_2}^2 = 20$ represent a circle of radius $\sqrt{20}$ with center at the origin and $x_1 x_2 = 8$ represent a rectangular hyperbola whose asymptotes are the coordinate's axes.

Solving the equations ${x_1}^2 + {x_2}^2 = 20$ and $xy = 8$

Since we know

$$(x_1 + x_2)^2 = {x_1}^2 + {x_2}^2 + 2x_1x_2(x_1 + x_2)^2 = 20 + 2(8) = 36 \quad (1)$$

Also $(x_1 - x_2)^2 = {x_1}^2 + {x_2}^2 - 2x_1x_2$

$$(x_1 - x_2)^2 = 20 - 2(8) = 4 \quad (2)$$

Adding (1) and (2) we get

$x_1 = 4 \; and \; x_2 = 2$

Thus the intersection coordinate of these two curves are $B(4,2)$ and $D(2,4)$ in the figure

As shown in the figure, the point (x_1, x_2) lying in the first quadrant shaded by the horizontal lines satisfies the constraints

$${x_1}^2 + {x_2}^2 \leq 20\,, x_1 \geq 0\,, x_2 \geq 0$$

Where the points (x_1, x_2)lying in the area shaded by vertical lines satisfy the constraint $x_1x_2 = 8\,, x_1 \geq 0\,, x_2 \geq 0$

Thus the desired solution point (x_1, x_2)may be somewhere in the non-convex feasible region $OABCDE$ shaded by both the horizontal and vertical lines.

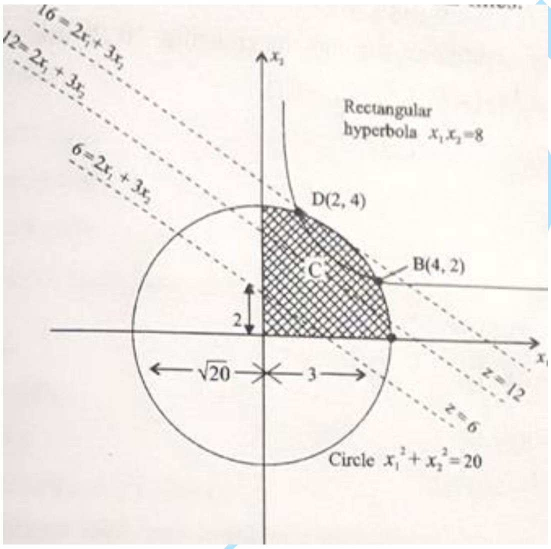

Figure (1) - show the graph and feasible region of the given problem.

The required point is obtained by moving parallel to the objective line $2x_1 + 3x_2 = c$ for some constant $z = c$, i.e. we go on moving parallel to the objective line $2x_1 + 3x_2 = 6$ (for $c = 6$ say)away from the origin so long as the line $c = 2x_1 + 3x_2$ touches the extreme boundary of the feasible region.

In this problem the boundary point $D(2,4)$ gives the maximum value of the z.

Hence the graphical solution of the problem finally obtained as $x_1 = 2\,, x_2 = 4$ and $\max z = 16$.

Example 2. Solve the following NLP problem graphically.

$$Max\; z = 8x_1 - x_1^2 + 8x_2 - x_2^2$$

Subject to the constraint

$$x_1 + x_2 \leq 12$$
$$x_1 - x_2 \geq 4$$
$$and\; x_1, x_2 \geq 0$$

Solution. Let Ox_1 and Ox_2 be the set of rectangular Cartesian coordinate axes in the plane of the paper. Since $x_1 \geq 0$ and $x_2 \geq 0$, the feasible region will lie in the first quadrant only, the feasible region is shown by the shaded region in the figure.

Thus the optimal point $((x_1, x_2)$ must somewhere in the convex region ABC. However the desired point will be that at which a side of the convex region is tangent to the circle

$$z = 8x_1 - x_1^2 + 8x_2 - x_2^2.$$

The gradient of the tangent to this circle can be obtained by differentiating the equation

$$z = 8x_1 - x_1^2 + 8x_2 - x_2^2$$

With respect to x_1 i.e

$$8 - 2x_1 + 8\frac{dx_2}{dx_1} - 2x_2\frac{dx_2}{dx_1} = 0$$

$$\text{Or} \quad \frac{dx_2}{dx_1} = \frac{2x_1 - 8}{8 - 2x_2} \quad (3)$$

The gradient of the line $x_1 + x_2 = 12$ and $x_1 - x_2 = 4$ is

$$dx_1 + dx_2 = 0 \text{ Or}$$
$$\frac{dx_2}{dx_1} = -1$$

And

$$dx_1 + dx_2 = 0 \text{ Or}$$
$$\frac{dx_2}{dx_1} = 1$$

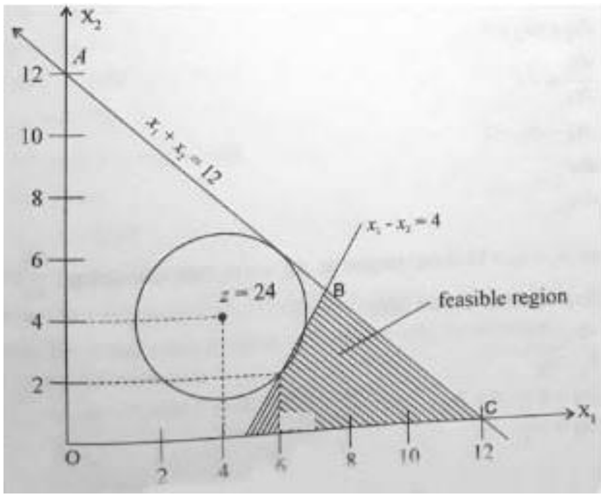

Figure (2), show the graph and feasible region of the given problem.

If the line $x_1 + x_2 = 12$ is the tangent to the circle, then substituting $\frac{dx_2}{dx_1} = -1$ from equation (2) in equation (1), we have

$$\frac{2x_1 - 8}{8 - 2x_2} = -1$$

Or $2x_1 - 8 = -8 + 2x_2$
Or $2x_1 = 2x_2$
Or $x_1 = x_2$

And here for $x_1 = x_2$, the equation $x_1 + x_2 = 12$ gives

$$(x_1, x_2) = (6,6)$$

This means that the tangent of the line $x_1 + x_2 = 12$ is at (6,6)

Similarly, if the line $x_1 - x_2 = 4$ is the tangent to the circle, then substituting $\frac{dx_2}{dx_1} = 1$ from equation (2) in equation (1), we have

$$\frac{2x_1 - 8}{8 - 2x_2} = 1$$

Or $2x_1 - 8 = 8 - 2x_2$
Or $2x_1 + 2x_2 = 16$
Or $x_1 + x_2 = 8$

Adding the equations $x_1 + x_2 = 8$ and $x_1 - x_2 = 4$, we have

$$(x_1, x_2) = (6,2)$$

This means that the tangent of the circle to the line $x_1 - x_2 = 4$ is at (6,2).

This point lies in the feasible region and satisfies with the constraints. Thus the optimal solution is

$$x_1 = 6\,, x_2 = 2\,, \max z = 24.$$

CONCLUSION

The graphical method is proposed to solve the NLP problem. This based on the plot of the curve of the constraints. The graphical solution can help us to make any decision and determining a particular plan of action from amongst several alternatives in a short time. The graphical method is the best method to make any decision for modern game theory, dynamic programming problem, economics and management.

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