AN EFFICIENT EXPONENTIAL TYPE ESTIMATOR FOR ESTIMATING FINITE POPULATION MEAN UNDER SIMPLE RANDOM SAMPLING

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ABSTRACT: In this paper, an improved exponential type estimator for estimating the population mean is proposed under simple random sampling scheme. The proposed estimator was obtained by combination of conventional product and exponential-type ratio estimators with aim of obtaining estimator with higher efficiency. The bias and mean squared error (MSE) of the proposed estimator were obtained up to the first order of approximation using binomial and exponential expansion techniques and the optimum value of the unknown constant of the estimator was derived by means of partially differentiating the mean squared error and equating to zero. Also, the conditions under which the proposed estimator is more efficient than the conventional estimators in the literature are established. An empirical study was carried out to support the fact that the proposed estimator is better than the existing ones, as the proposed estimator has a minimum mean squared error at the optimum value of the unknown constant and has higher percentage relative efficiency (PRE). This implies that the proposed estimator is more efficient than the conventional product and exponential-type ratio estimators considered in the study.
KEYWORDS: Simple random sampling, Auxiliary information, Mean squared error, Efficiency

1. INTRODUCTION

The auxiliary information in sampling theory is used for improved estimation of parameters enhancing the efficiencies of the estimators. The problem of estimating the population mean in the presence of auxiliary variable has been widely discussed in finite population sampling literature. The use of auxiliary information is well known to improve the precision of the estimate of the population mean and other parameters of the study variable in survey sampling. Ratio, product and difference methods of estimation are good examples in this context. Ratio method of estimation is quite effective when there is high positive correlation between study and auxiliary variables. However, if correlation is negative and very high, the product method of estimation can be employed effectively. In recent years, a number of research papers on ratio type, exponential ratio type and regression type estimators have appeared, based on different types of transformations (see Bahl and Tuteja [6], Murthy [3], Sisodia and Dwivedi [1], Singh et al [9], Singh and Tailor [5], Singh et al. [8], Yadav and Kadilar [4], Kadilar and Cingi [10], Singh HP et al. [11], Sahai A et al. [12], Srivastava SK et al. [13], Ahmed A et al. [14], Audu A et al. [15], Audu A et al. [16], Muili JO et al. [17], Singh HP et al. [18], Sisodia BVS et al. [19], Khoshnevisan M et al. [20], Singh and Audu [21], Ahmed A et al.[22] and Audu A et al.[23], Das AK et al. [24], Das AK et al. [25], Patel PA et al. [26], Rajyaguru A et al. [27], Rajyaguru A et al. [28], Archana V et al. [29], Singh R et al. [30], Audu A. et al. ([31]-[36], [38]), Singh R. [37], Muili J. O. [39], Ishaq O. O. [40]).

In the present study, we proposed an improved exponential type estimator for estimating the population mean of the study variable Y that is more efficient than the existing conventional product and exponential-type ratio estimators.

2. EXISTING ESTIMATORS IN LITERATURE

Consider a finite population U= $U_1, U_2, \ldots, U_N$ of N units. Let Y and X denote the variable under study and auxiliary variable respectively. Let (y_i, x_i), i=1, 2, 3, …., n denote the n pair of sample observations for the study and auxiliary variables, respectively, drawn from the population size N using simple random sampling without replacement (SRSWOR). Let $\bar{X}$ and $\bar{Y}$ be the population means of auxiliary and study variables, respectively, and let $\bar{x}$ and $\bar{y}$ be the respective sample means.

The usual sample mean estimator is defined as:

$$\bar{y} = n^{-1}\sum_{i=1}^{n} y_i \qquad (2.1)$$

The associated bias and variance of the sample mean estimator are given by:

$$Bias(\bar{y}) = 0 \quad (2.2)$$

$$Var(\bar{y}) = \gamma \bar{Y}^2 C_y^2 \quad (2.3)$$

Where $\gamma = n^{-1}(1-f), f = n^{-1}N, C_y^2 = S_y^2\left(\bar{Y}^2\right)^{-1}$, and S_y^2 is the variance of the study variable.

Cochran [2] defined ratio estimators as

$$\bar{y}_R = \frac{\bar{y}\bar{X}}{\bar{x}} \quad (2.4)$$

The associated bias and mean squared error of the ratio estimator are given by:

$$Bias(\bar{y}_R) = \gamma \bar{Y}\left(C_x^2 - \rho C_y C_x\right) \quad (2.5)$$

$$MSE(\bar{y}_R) = \gamma \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_y C_x) \quad (2.6)$$

Where
$\gamma = n^{-1}(1-f), f = n^{-1}N, C_y^2 = S_y^2(\bar{Y}^2)^{-1}, C_x^2 = S_x^2(\bar{X}^2)^{-1}$, and
S_y^2, S_x^2, are the variance of the study and auxiliary variables. ρ is the correlation coefficient.

Murthy [3] defined conventional product estimator as:

$$\bar{y}_P = \frac{\bar{y}\bar{x}}{\bar{X}} \quad (2.7)$$

The associated bias and mean squared error of the product estimator are given gy:

$$Bias(\bar{y}_P) = \gamma \bar{Y} \rho C_y C_x \quad (2.8)$$

$$MSE(\bar{y}_P) = \gamma \bar{Y}^2 (C_y^2 + C_x^2 + 2\rho C_y C_x) \quad (2.9)$$

Where
$\gamma = n^{-1}(1-f), f = n^{-1}N, C_y^2 = S_y^2(\bar{Y}^2)^{-1}, C_x^2 = S_x^2(\bar{X}^2)^{-1}$, and
S_y^2, S_x^2, are the variance of the study and auxiliary variables. ρ is the correlation coefficient.

Bahl and Tuteja [6] proposed an exponential ratio and product type estimator for the population mean as:

$$t_1 = \bar{y}_{BTratio} = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \quad (2.10)$$

$$t_2 = \bar{y}_{BTproduct} = \bar{y}\exp\left[\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right] \quad (2.11)$$

The associated biases and mean squared errors of the estimators are given by:

$$Bias(t_1) = \bar{Y}\gamma\left[\frac{3C_x^2}{8} - \frac{\rho C_y C_x}{2}\right] \quad (2.12)$$

$$Bias(t_2) = \bar{Y}\gamma\left[\frac{\rho C_y C_x}{2} - \frac{C_x^2}{8}\right] \quad (2.13)$$

$$MSE(t_1) = \bar{Y}^2\gamma\left[C_y^2 + \frac{C_x^2}{4} - \rho C_y C_x\right] \quad (2.14)$$

$$MSE(t_2) = \bar{Y}^2\gamma\left[C_y^2 + \frac{C_x^2}{4} + \rho C_y C_x\right] \quad (2.15)$$

Where
$\gamma = n^{-1}(1-f), f = n^{-1}N, C_y^2 = S_y^2(\bar{Y}^2)^{-1}, C_x^2 = S_x^2(\bar{X}^2)^{-1}$, and
S_y^2, S_x^2, are the variance of the study and auxiliary variables. ρ is the correlation coefficient.

3. MATERIALS AND METHODS

3.1 Proposed estimator

Following Murthy [3] and Bahl and Tuteja [6], an improved exponential estimator for estimating the population mean $\bar{Y}$ is proposed and defined as:

$$T_M = 2^{-1}\bar{y}\left[\left(\frac{\bar{x}}{\bar{X}}\right)^{\alpha} + \exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]\right] \quad (3.1)$$

To derive the bias and mean squared error (MSE) of the proposed estimator T_M, the following properties are defined:

$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, e_1 \frac{\bar{x}-\bar{X}}{\bar{X}}$, Then the two relations can also be written as: $\bar{y} = \bar{Y}(1+e_0), \bar{x} = \bar{X}(1+e_1)$ such that $E(e_0) = E(e_1) = 0$ and $E(e_0^2) = \gamma C_y^2, E(e_1^2) = \gamma C_x^2, E(e_0 e_1) = \gamma\rho C_y C_x$.

In calculus, the expansion of e^x or exp(x) is defined as:

$$exp(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots \quad (3.2)$$

Using (3.2) in (3.1), requires approximation to power two, then equation (3.1.2) reduces to:

$$\exp(x) = 1 + x + \frac{x^2}{2} \quad (3.3)$$

Also, from calculus, the binomial expansion for negative and fractional power is defined as:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \cdots \quad (3.4)$$

Using (3.4) in (3.1), requires approximation to power two, then equation (3.4) reduces to:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 \quad (3.5)$$

Substituting for $\bar{y}$ and $\bar{x}$ using the properties defined above, (3.1) becomes:

$$T_M = 2^{-1}\bar{Y}(1+e_0)\left[\left(\frac{\bar{X}(1+e_1)}{\bar{X}}\right)^{\alpha} + exp\left[\frac{\bar{X}-\bar{X}(1+e_1)}{\bar{X}+\bar{X}(1+e_1)}\right]\right] \quad (3.6)$$

Simplifying equation (3.6) gives:

$$T_M = 2^{-1}\bar{Y}(1+e_0)\left[(1+e_1)^{\alpha} + exp\left[\frac{-\bar{X}e_1}{2\bar{X}+\bar{X}e}\right]\right] \quad (3.7)$$

$$T_M = 2^{-1}\bar{Y}(1+e_0)\left[(1+e_1)^{\alpha} + exp\left[\frac{-e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right]\right] \quad (3.8)$$

Using equation (3.1.3) in equation (3.1.8) gives:

$$T_M = 2^{-1}\bar{Y}(1+e_0)\left[\left(1+\alpha e_1+\frac{\alpha(\alpha-1)}{2}\right) + exp\left[\frac{-e_1}{2}\left(1-\frac{e_1}{2}+\frac{e_1^2}{4}\right)\right]\right] \quad (3.9)$$

Expanding the second term in the parenthesis to second degree, this gives:

$$T_M = 2^{-1}\bar{Y}(1+e_0)\left[\left(1+\alpha e_1+\frac{\alpha(\alpha-1)}{2}\right) + exp\left[\frac{-e_1}{2}+\frac{e_1^2}{4}\right]\right] \quad (3.10)$$

Using equation (3.5) in equation (3.10), leaving the expansion to second degree, gives:

$$T_M = 2^{-1}\bar{Y}(1+e_0)\left[\left(1+\alpha e_1+\frac{\alpha(\alpha-1)}{2}e_1^2\right) + \left(1-\frac{e_1}{2}+\frac{3e_1^2}{8}\right)\right] \quad (3.11)$$

$$T_M = \bar{Y}(1+e_0)\left[1+\left(\frac{\alpha}{2}-\frac{1}{4}\right)e_1+\left(\frac{\alpha(\alpha-1)}{4}+\frac{3}{16}\right)e_1^2\right] \quad (3.12)$$

Expanding equation (3.12) up to second degree of approximation, gives:

$$T_M = \bar{Y}+\bar{Y}\left[\left(\frac{\alpha}{2}-\frac{1}{4}\right)e_1+\left(\frac{\alpha(\alpha-1)}{4}+\frac{3}{16}\right)e_1^2+e_0+\left(\frac{\alpha}{2}-\frac{1}{4}\right)e_0e_1\right] \quad (3.13)$$

Subtract $\bar{Y}$ and take expectation on both sides of equation (3.13) to obtain the bias of the proposed estimator as:

$$E(T_M-\bar{Y}) = \bar{Y}\left[\left(\frac{\alpha}{2}-\frac{1}{4}\right)E(e_1)+\left(\frac{\alpha(\alpha-1)}{4}+\frac{3}{16}\right)E(e_1^2)+E(e_0)+\left(\frac{\alpha}{2}-\frac{1}{4}\right)E(e_0e_1)\right] \quad (3.14)$$

$$Bias(T_M) = \bar{Y}\gamma\left[\left(\frac{\alpha(\alpha-1)}{4}+\frac{3}{16}\right)C_x^2+\left(\frac{\alpha}{2}-\frac{1}{4}\right)\rho C_yC_x\right] \quad (3.15)$$

To obtain the mean squared error (MSE) of the proposed estimator, it is defined as:

$$MSE(T_M) = E(T_M-\bar{Y})^2 \quad (3.16)$$

Substituting equation (3.13) in equation (3.16) to first order of approximation, gives:

$$MSE(T_M) = \bar{Y}^2E\left(\left(\frac{\alpha}{2}-\frac{1}{4}\right)e_1+e_0\right)^2 \quad (3.17)$$

Expanding and taking expectation gives the mean squared error (MSE) of the proposed estimator to first order of approximation as:

$$MSE(T_M) = \bar{Y}^2\gamma\left(C_y^2+\left(\frac{\alpha}{2}-\frac{1}{4}\right)^2C_x^2+2\left(\frac{\alpha}{2}-\frac{1}{4}\right)\rho C_yC_x\right) \quad (3.18)$$

Differentiating equation (3.18) partially with respect to α and equate to zero to obtain the optimum value of α as:

$$\alpha^{opt} = \frac{1}{2}-\frac{2\rho C_y}{C_x}$$

Substituting the optimum value of α into equation (3.18), to obtain the minimum MSE of the proposed estimator T_M as:

$$MSE(T_M)_{\min} = \gamma\bar{Y}^2C_y^2(1-\rho^2) \quad (3.19)$$

It follows from equation (3.19) that the proposed estimator T_M at its optimum condition is as efficient as that of the usual linear regression estimator.

4. RESULTS AND DISCUSSIONS

4.1 Efficiency comparisons

In this section, the MSE of the conventional estimators $\bar{y}, \bar{y}_R, \bar{y}_P, t_1, t_2$ are compared with the MSE of the proposed estimator T_M. From equations (2.3), (2.6), (2.8), (2.14), (2.15), and (3.19)

$$[Var(\bar{y})-MSE(T_M)_{\min}] = \lambda\bar{Y}^2C_y^2\rho^2>0 \quad (4.1)$$

$$[MSE(\bar{y}_R)-MSE(T_M)_{\min}] = \lambda\bar{Y}^2(C_x^2-\rho C_y)^2>0 \quad (4.2)$$

$$[MSE(\bar{y}_P)-MSE(T_M)_{\min}] = \lambda\bar{Y}^2(C_x^2+\rho C_y)^2>0 \quad (4.3)$$

$$[MSE(t_1)-MSE(T_M)_{\min}] = \lambda\bar{Y}^2\left(\frac{C_x^2}{2}-\rho C_y\right)^2>0 \quad (4.4)$$

$$[MSE(t_2)-MSE(T_M)_{\min}] = \lambda\bar{Y}^2\left(\frac{C_x^2}{2}+\rho C_y\right)^2>0 \quad (4.5)$$

It is observed that T_M is always efficient than the conventional estimators $\bar{y}, \bar{y}_R, \bar{y}_P, t_1, t_2$, because the condition from (4.1) to (4.5) are always satisfied.

4.2 Empirical study

The appropriateness of the proposed estimator has been verified with the help of the following data sets in table 1.

Table 1. Statistics of Population

Parameters	**Population 1** (Cochran [7])	**Population 2** (Murthy [41])
N	10	80
N	4	20
$\bar{Y}$	5.920	11.264
$\bar{X}$	3.590	51.826
ρ	0.680	0.941
C_y	0.144	0.750
C_x	0.128	0.354
$\beta_{2(x)}$	0.381	0.063

The explanation of the data sets in table 1 from sources is given as follows:

Population 1: Source, Cochran [7]: The auxiliary variable X is the number of rooms and the study variable Y is the number of persons.

Population 2: Source, Murthy [41]: The auxiliary variable X is the output of the 80 factories and the study variable is the fixed capital.

The percentage relative efficiency is computed as:

$$PRE(G_i) = \frac{Var(\bar{y})}{MSE(G_i)} \quad \text{for} \quad i = 1,2,3,4,5,6 \quad \text{and}$$

$$G_1 = \bar{y}, G_2 = \bar{y}_R, G_3 = \bar{y}_P, G_4 = t_1, G_5 = t_2, G_6 = T_M$$

The mean squared errors (MSEs) and percentage relative efficiencies (PREs) of the different estimators of the population mean with respect to the sample mean based on populations 1 and 2 are given in table 2.

Table 2. MSEs and PREs of Proposed and Conventional estimators for population 1 and 2

Estimators	**MSE popn.1**	**PRE popn.1**	**MSE popn.2**	**PRE popn.2**
$\bar{y}$	12.6366	100	26.7633	100
$\bar{y}_R$	1.7874	157.5875	8.9518	298.9715
$\bar{y}_P$	54.4632	33.9480	56.4996	47.3689
t_1	1.6172	161.3546	16.3669	163.5205
t_2	52.0376	56.3282	40.1408	66.6734
T_M (proposed)	1.4399	173.1528	3.0649	873.2175

Table 2 shows the numerical results of (MSE and PRE) of $\bar{y}, \bar{y}_R, \bar{y}_P, t_1, t_2, T_M$ estimators using population sets 1 and 2. Of all the estimators considered in the study, the proposed estimator has minimum MSE and maximum PRE for the population sets. This implies that yhe proposed estimator demonstrates high level of efficiency over others and can produce better estimate of the population mean.

5. CONCLUSIONS

From the results of the empirical study, it was obtained that the proposed estimator is more efficient than other estimators considered in the study and therefore, it is recommended for use for estimating the population mean in practice.

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